A RESEARCH INTO YEAR FIVE PUPILS' PRE-ALGEBRAIC THINKING IN SOLVING PRE-ALGEBRAIC PROBLEMS

by

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Thesis submitted in fulfillment of the requirements for the degree of
Doctor of Philosophy

July 2008

ACKNOWLEDGEMENTS

First and foremost, I express my deepest appreciation to my supervisor, Assoc. Prof. Dr. Munirah Ghazali for her invaluable guidance and motivation. Her genuine interest in my study has inspired me to present and publish papers related to this study.

I am thankful to the Dean and lecturers in School of Educational Studies, USM, especially Assoc. Prof. Dr. Lim Chap Sam, Assoc. Prof. Dr. Fatimah Saleh, Dr. Hashimah Mohd. Yunus, Dr. Ong Saw Lan, Dr. Shuki Osman and Dr. Chew Cheng Meng. I am also grateful to Assoc. Prof. Dr. Vincent Pang of UMS, Dr. Paul Lau Ngee Kiong of UiTM (Sarawak), Dr. Souba Rethinasamy and Dr. Ting Su Hie of UNIMAS, Mr. Gan Teck Hock and Mr. Sia Pong Won of Institut Perguruan Sarawak, Mdm. Chuah Kim Hwa of Institut Perguruan Batu Lintang, and my sister We Nei, for their constructive comments and insightful suggestions.

I wish to thank EPRD and Sarawak State Education Department for the approval in conducting this study. I also wish to acknowledge the Headmasters, Senior Assistants and teachers of the participating schools. Very special thanks are accorded to the pupils who participated enthusiastically in this study.

I would like to express my greatest gratitude to my beloved parents, Gan Guan Eee and Tan Heng Tian, as well as my sisters, We Lee and Wel Na, for their unconditional understanding and continuous moral support along the course of my study. I also extend my sincere thanks to my colleagues and friends for their help, thoughtfulness and encouragement. Last but not least, I thank the staff from libraries of USM, SEAMEO-RECSAM, UM, UNIMAS, UKM and UUM, as well as Sarawak State Library and Sarawak State Education Department Resource Centre for their kind assistance.

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ABSTRAK

Kajian ini bertujuan untuk mengkaji bagaimana murid sekolah rendah menyelesaikan masalah praalgebra serta mengenalpasti pemikiran praalgebra dalam proses penyelesaian mereka. Dalam kajian ini, proses penyelesaian merangkumi strategi penyelesaian, mod perwakilan serta justifikasi matematik. Tiga kategori masalah praalgebra digunakan, iaitu masalah generalisasi yang melibatkan corak nombor, masalah generalisasi yang melibatkan corak geometri dan masalah berayat yang melibatkan kuantiti yang tidak diketahui.

Tiga belas orang pelajar Tahun Lima yang berumur 11 tahun dilibatkan dalam kajian ini. Mereka diberi sepuluh masalah praalgebra untuk diselesaikan secara individu dalam dua sesi yang berasingan. Data dikumpul melalui hasil penyelesaian bertulis, protokol menyuarakan fikiran, retrospeksi melalui temubual berasaskan tugas serta rakaman proses penyelesaian peserta kajian. Protokol menyuarakan fikiran dan semua sesi temubual dirakam secara audio untuk tujuan transkripsi verbatim.

Analisis rakaman video menghasilkan descripksi dan klasifikasi strategi penyelesaian serta justifikasi matematik di kalangan peserta kajian. Analisis hasil penyelesaian bertulis menghasilkan klasifikasi mod perwakilan. Analisis protokol lisan membolehkan pemikiran praalgebra di sebalik proses penyelesaian peserta kajian dikenalpasti.

Dapatan kajian menunjukkan bahawa strategi 'recursive' dan strategi 'berdasarkan bentuk rajah' paling kerap digunakan oleh peserta kajian untuk menyelesaikan masalah yang melibatkan corak nombor dan corak geometri masing-masing. Untuk masalah berayat, strategi 'unwinding' dan strategy aritmetik paling kerap digunakan. Di sebalik proses penyelesaian peserta kajian, pemikiran praalgebra yang berkaitan dengan mengecam dan

mengembangkan corak serta mengecam hubungan antara beberapa kuantiti dapat dikenalpasti.

Untuk mod perwakilan, perwakilan simbolik-aritmetik paling kerap digunakan dalam penyelesaian bagi masalah generalisasi yang melibatkan corak nombor serta masalah berayat. Perwakilan gambar paling kerap digunakan dalam penyelesaian masalah generalisasi yang melibatkan corak geometri. Pemikiran praalgebra dalam perwakilan peserta berkait rapat dengan keupayaan mereka untuk mewakilkan hubungan dengan gambarajah serta menggunakan pelbagai mod perwakilan, khususnya dalam masalah berayat yang mempunyai pelbagai penyelesaian (Soalan 10).

Semasa menjustifikasikan penyelesaian mereka untuk masalah generalisasi yang melibatkan corak nombor dan corak geometri, peserta kajian memaparkan penaakulan secara induktif semasa mengenalpasti ciri-ciri dalam sesuatu corak. Semasa menjustifikasikan penyelesaian mereka untuk masalah berayat, peserta lebih cenderung untuk menjelaskan sebab sesuatu kaedah boleh digunakan berdasarkan sebab penggunaan sesuatu operasi aritmetik serta verifikasi jawapan mereka. Pemikiran praalgebra yang dikenalpasti daripada sebilangan peserta kajian menunjukkan kebolehan mereka dalam menggambarkan serta mengeneralisasikan corak serta ‘bekerja’ dengan sifat-sifat operasi.

Dapatan kajian ini menunjukkan petanda awal pemikiran algebra di sesetengah peserta kajian. Bagi kebanyakan peserta kajian, pemikiran praalgebra di kalangan mereka boleh dipertingkatkan lagi melalui aktiviti pengajaran-pembelajaran Matematik yang relevan dalam bilik darjah.

A RESEARCH INTO YEAR FIVE PUPILS' PRE-ALGEBRAIC THINKING IN SOLVING PRE-ALGEBRAIC PROBLEMS

ABSTRACT

This study was undertaken to investigate how primary school pupils solve pre-algebraic problems and to infer their pre-algebraic thinking underlying their solution processes. In this study, the solution processes encompassed the solution strategy, mode of representation and mathematical justification. Three categories of pre-algebraic problems were used, namely generalization problems involving number patterns, generalization problems involving geometric patterns and word problems involving unknown quantities.

Thirteen 11-year-old Year Five pupils were involved as the participants of this study. They were given ten pre-algebraic problems to be solved individually in two separate sessions. Data were collected via the participants' written solutions, think-aloud verbal protocols, retrospection through task-based interview and videotaping of their solution processes. Their think-aloud protocols and all interview sessions were audio taped to be verbatim transcribed.

Analysis of the video transcripts led to description and classification of the participants' solution strategies and mathematical justifications. Analysis of their written solutions led to the classification of their modes of representation. Analysis of their verbal protocols led to the inference of their pre-algebraic thinking underlying their solution processes.

Findings of this study indicated that recursive strategy and 'based on shape of figure' strategy were most frequently used in solving problems involving number patterns and geometric patterns, respectively. For word problems, 'unwinding' and arithmetic strategies were most frequently used. Underlying the participants' strategies, pre-algebraic thinking

related to identification and extension of patterns and identification of relationships between quantities were inferred.

With respect to modes of representation, arithmetic-symbolic representation was dominantly used in the participants' solutions for generalization problems involving number patterns and the word problems. Pictorial representation was dominantly used for generalization problems involving geometric patterns. The pre-algebraic thinking embedded in the participants' representations was related to their ability to represent relationships with pictures and use multiple representations, particularly in word problems that allow multiple solutions (Problem 10).

In justifying their solutions for generalization problems involving number and geometric patterns, the participants demonstrated their inductive reasoning in identifying the characteristics of the patterns given. In justifying their solutions for word problems, they were inclined to explain why a particular method work based on reasons for operations used and verification of answers. The pre-algebraic thinking inferred from some participants' justifications reflected their ability to describe and generalize patterns and work with properties of operations.

Findings of this study indicated early signs of algebraic thinking among some participants. For most participants, their pre-algebraic thinking can further be enhanced through relevant teaching-learning activities in mathematics classrooms.

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Algebra is known to be important not only for academic purposes but also for the world of work. It is a prerequisite for further mathematics study and hence job opportunities in the global marketplace. There are certain professions that rely heavily on the understanding of algebraic concepts. For instance, to do computer graphics, a strong background in linear algebra is required in addition to knowledge of calculus (Urquhart, 2000). Perhaps that is why algebra has been described as a "gatekeeper" course (NCTM, 1999; Choike, 2000; Urquhart, 2000).

Thus, in the 80s, National Council of Teachers of Mathematics (NCTM) in the United States called for a focus on algebra across the grades, beginning as early as preschool, so that students could develop the algebraic skills and algebraic ways of thinking that are needed for success in high school and beyond. This recommendation which aimed at developing young children's capability for algebraic thinking had become an important strand of the recommendations in the Principles and Standards for School Mathematics (Blanton & Kaput, 2003). Consequently in 1989, NCTM's Curriculum and Evaluation Standards for School Mathematics promoted algebra as a K-12 enterprise, and algebraic thinking was to be included in elementary classrooms (Moses, 1997).

In 1994, the Algebra Working Group appointed by NCTM introduced the emerging view of algebra which acknowledged the dynamic nature of mathematics in general and of algebra in particular, treats mathematics as a human activity and puts students' thinking at the forefronts (Yackel, 1997). This group advocated that all children can learn algebra and children can develop algebraic thinking.

Developing algebraic thinking is a top priority in today's elementary mathematics curriculum since algebra is now second in importance after number and operations (Burns, 2002). Early exposure of algebra in the elementary grades enables students to have various opportunities to represent patterns, make generalizations and explore their conjectures (Chappell, 1997), thus form a solid base for algebra knowledge (Land & Becher, 1997). It is hoped that appropriate preparation in upper primary school may assist students to overcome obstacles experienced by beginning algebra students (Brockman & Hoffman, 2002). Urquhart (2000) even recommended algebra to be integrated into teaching as early as preschool. These views have emphasized an early start for the learning of algebra and development of algebraic thinking as early preparation for algebra learning in higher levels.

In Malaysia, algebra is only introduced formally in the first year of secondary education, after six years of primary education (Ministry of Education, 2003c). This scenario seems to suggest that algebraic thinking is distinct from the elementary curriculum (Ruapp, et. al., 1997). In fact, the Integrated Primary School Curriculum (*KBSR or Kurikulum Bersepadu Sekolah Rendah*) for Mathematics actually "contains" some elements of algebra. In Year 1, pupils are taught to solve simple word problems which involve finding missing addend, minuend or subtrahend within the range of 10 (Ministry of Education Malaysia, 2002). In Year 2, pupils are expected to find unknown numbers in number sentences involving addition and subtraction within the range of 1000 (Ministry of Education Malaysia, 2003a), such as "34 + ? = 60", "? + 27 = 136", "45 - ? = 20" and "? – 13 = 76". They are also expected to be able to find the unknown in number sentences involving multiplication and division within 2, 3, 4 and 5 times-tables (Ministry of Education Malaysia, 2003a), such as "9 × ? = 18", "? × 3 = 9", "16 ÷ ? = 8" and "? ÷ 3 = 5". In Year 3, pupils are expected to be able to find the unknown in number sentences involving multiplication and division within 6, 7, 8 and 9 times-tables (Ministry of Education Malaysia, 2003b). These activities of finding unknowns in open number sentences are actually algebraic in nature and challenge young children's ability to understand the properties of, and relationships between arithmetic operations.

Ability to work with arithmetic operations is called 'operation sense' by Slavit (1999) who claimed that this operation sense led to algebraic thinking. Therefore, it is not exaggerating that element of algebra is actually 'hidden' and embedded in the Malaysian primary school mathematics curriculum.

1.2 Problem Statement

1.2.1 Algebra and Algebraic Thinking: An Early Start?

Many young children may experience difficulties in learning algebra because symbolic mathematical notation is introduced to them prematurely (Edwards, 2000). Fearnley-Sanders (2000) seemed to support Edwards's view by saying that algebra is a form of generalized arithmetic, so an earlier start may not be appropriate.

However, early algebra is not about introducing the traditional and formal algebra in the primary school, but is about developing arithmetic thinking in conjunction with algebraic thinking (Warren, 2002). Boero (2001) called this as 'pre-algebra'. Pre-algebra involves a reconceptualization of arithmetic in the elementary school (Warren, 2002). Some of the core ideas of pre-algebra and the language of algebra, in concrete and meaningful forms can be introduced to primary school pupils. Pattern recognition, functional relationships, tables and graphs, properties of arithmetic, and inverse relationships of the operations are all related to the preparation for learning algebra (Kutz, 1991). Introduction of these pre-algebraic concepts at a level appropriate to the primary school pupils may lay the foundation for success in algebra which is the cornerstone of secondary school mathematics.

Early experiences with algebra typically are nothing more than generalized arithmetic. Generalization of an arithmetic relationship can be facilitated through the use of a variable as it shifts attention from algorithmic computation to generalization. Children may also acquire the idea of variable from numerical experiences through numerical problem solving (Osborne & Wilson, 1992). Thus, a key factor in getting ready for algebra is developing an understanding of variable. Osborne & Wilson stressed that teaching of arithmetic is incomplete and insufficient if it does not emphasize understanding of variable and does not have an orientation to generalization.

So, are young children able to generalize arithmetic relationship and acquire the idea and understanding of variable? To trace back a study conducted by Lam (1985) among Primary 1 children in Singapore, the study revealed young children's inability in solving reverse thinking open sentences like "? – 7 = 10", "5 + ? = 9", "10 - ? = 6", "? + 7 = 10" and "? + 5 = 8". Lam attributed the inability to lack of exposure and young children have not reached the maturity level to think in a reverse manner. In another study, Sufean (1986) pointed out that Malaysian Grade 3 and Grade 4 pupils found open sentences of the form "a × ? = c" and "? ÷ b = c" most difficult to be solved. Sufean then argued that these pupils' inability to solve multiplication and division open sentences was related to their ability to (a) read open sentences, (b) recall basic facts of multiplication and division, and (c) understand the concepts of multiplication and division.

As discussed at the end of section 1.1, these open number sentences are included in the Malaysian Mathematics curriculum since Year 1 in primary school. Even then, pre-algebraic activities like these may not guarantee pupils' ability to think algebraically if the way of solving these open number sentences are based on knowing by heart all possible combinations of two numbers that total up to 10 (Ministry of Education Malaysia, 2002) or emphasizing mental calculation (Ministry of Education Malaysia, 2003a, 2003b). Herscovics and Linchevski (1994) also argued that solving a missing addend problem such as '4 + ? = 9' may not be associated to doing algebra, as this problem can be solved using purely arithmetic means such as counting procedures or an inverse operation. So, how is it possible to include algebraic thinking in solving open number sentences in the primary school mathematics curriculum?

1.2.2 Arithmetic to Algebra: 'Conceptual Leap' or Transition?

Algebra can contribute to the understanding of abstractions in general, and this understanding is a basic and valuable cognitive tool in the learning of mathematics (Saul, 2001). However, for many students, algebra acts more like a wall than a gateway presenting an obstacle that they find too difficult to cross (Fearnley-Sanders, 2000). This may be supported by numerous studies about students' inability to move beyond arithmetic thinking to think algebraically (Lee & Wheeler, 1989; Kieran, 1992; Esty & Teppo, 1996; CSBE, 2000; MacGregor & Stacey, 1999; Palomares & Hernandez, 2002; Van Ameron, 2003).

In Malaysia, few local studies have been conducted to investigate Malaysian secondary school students' learning of algebra. A study by Ong (2000) on 139 Malaysian urban Form 4 students' understanding of algebraic notation revealed that most of them were unable to interpret letters as specific unknown, generalized number and variable. In another study involving 123 Form 4 students, Teng (2002) found most students' alternative conceptions in identifying linear equations, interpretation of notations and manipulation involving linear equations. Lim (2007) assessed ability in solving linear equations among nine Form 4 students' of varying levels of achievement. The findings revealed that the low achievers were unable to explain the linear relationship in a linear pattern while the moderate achievers, though able to explain the linear relationship verbally or arithmetically, were still unable to generalize the linear pattern in the form of algebraic expression or linear equation. Only the high achievers were able to describe and generalize linear patterns, apply linear concept and then analyze the elements (constant, coefficient and variable) in a linear equation.

Findings of these few local studies suggested incomplete and poor mastery of related concepts as well as inability to apply the relevant prior knowledge among the students. In fact, students' difficulty in learning and mastering algebraic concepts and skills is not a new issue (*Lembaga Peperiksaan*, 1995, 1996a, 1996b, 1997a, 1997b). Form 5 students' inability to expand algebraic expressions and solve equations continue to be reported (*Lembaga Peperiksaan*, 2002), indicating the seriousness of the existing problem in the teaching and learning of algebra.

Moreover, results from the Third International Mathematics and Science Study – Repeat (TIMSS-R) which involved 5,577 Malaysian Form 2 students indicated that Malaysia ranked 17 out of 38 participating countries in terms of average score according to the topic of algebra (*Bahagian Perancangan & Penyelidikan Dasar Pendidikan*, 2000). In TIMSS (Trend in Mathematics and Science Study) 2003, though Malaysia ranked 18 out of 49 participating countries in terms of average score according to the topic of algebra (Mullis, Martin, Gonzalez & Chrustowski, 2004), the average score for algebra items in 2003 is significantly lower than that in 1999. On top of that, Malaysian sample's score in the algebra items was also below the country average score for all the five content areas (Numbers, Algebra, Measurement, Geometry and Data) covered in the study. This situation seems to add the urgency to look into the teaching and learning of algebra in the Malaysian context.

In the interest of building a strong foundation in algebra, preparation towards algebra at the primary school years deemed important since algebra is a compulsory topic in secondary school mathematics. Since learning of algebra involves algebraic thinking which can be different from arithmetic thinking, it is therefore in the primary school years that the pupils have to lay down the basic of algebraic thinking, upon which their later achievement in algebra will crucially depend on. Algebraic experiences in the elementary schools are essential in building the thinking that is "an important precursor to the more formalized study of algebra in the middle and secondary schools" (NCTM, 2000, p. 159; Bay-Williams, 2001). This seemed to suggest that the basic of algebraic thinking may be developed from arithmetic thinking and transitioned into algebraic thinking. Warren (2000a) called this transition of thinking from arithmetic to algebraic thinking as pre-algebraic thinking. This transition from arithmetic thinking to algebraic thinking, or pre-algebraic thinking, has now become a question of importance in the teaching and learning of algebra (Warren, 2000a).

Despite the difficult transition from arithmetic to algebraic thinking, there exist various literatures that indicated 4^{th} to 6^{th} Grade pupils' ability to exhibit their algebraic thinking (Curcio & Schwartz, 1997; Cai, 1998; Slavit, 1999), algebraic reasoning (Lubinski & Otto, 1997) and use of algebraic symbolism (Land & Becher, 1997). With these literatures, it may be reasonable to expect the Malaysian primary school pupils to develop pre-algebraic thinking. The recent global awareness of developing algebraic thinking in the early grades has evoked an urging need to look into the pre-algebraic thinking among Malaysian primary school pupils who are not exposed to algebra formally and directly in the classroom.

In Malaysia, research into algebra learning seemed to focus on secondary school students (Ong, 2000; Teng, 2002; Lim, 2007) and students in institutes of higher learning, for instance Roselah (2001) who investigated six diploma students' problem-solving process for algebra problems. However, research investigating Malaysian primary school pupils' ability in solving problems related to pre-algebra has yet to be intensified. This could be due to the fact that algebra is not included explicitly in the primary school mathematics curriculum. Thus, the need arises to investigate how the primary school pupils solve problems related to pre-algebra based on their numeric and arithmetic understanding and their prior experience in solving arithmetic word problems.

1.3 Purpose of the Study

The purpose of this study is to investigate how primary school pupils solve pre-algebraic problems with respect to their solution strategies, modes of representation and mathematical justifications. From their problem-solving processes, their pre-algebraic thinking is then inferred.

There are two broad categories of pre-algebraic problems to be used in this study – generalization problems involving patterns and word problems involving unknown quantities. Generalization problems involving patterns are subdivided into generalization problems involving number patterns and generalization problems involving geometric patterns. This study therefore aims to investigate the solution strategies, modes of representation and mathematical justifications among primary school pupils in solving these three categories of pre-algebraic problems and to infer their pre-algebraic thinking from their problem-solving processes.

1.4 Research Questions

In line with the purpose of this study, the following research questions are formulated to guide the study:

Research Question 1

a) What solution strategies are used by primary school pupils when they solve

    i) generalization problems involving number patterns

    ii) generalization problems involving geometric patterns

    iii) word problems involving unknown quantities

b) What modes of representation are used by primary school pupils when they solve

    i) generalization problems involving number patterns

    ii) generalization problems involving geometric patterns

    iii) word problems involving unknown quantities

c) What are the mathematical justifications among primary school pupils when they solve

i) generalization problems involving number patterns

ii) generalization problems involving geometric patterns

iii) word problems involving unknown quantities

Research Question 2

What pre-algebraic thinking can be inferred from primary school pupils' solutions when they solve

i) generalization problems involving number patterns

ii) generalization problems involving geometric patterns

iii) word problems involving unknown quantities

1.5 Research Framework of the Study

With the intention to investigate the Malaysian primary school pupils' pre- algebraic thinking, this study focuses on the critical cognitive aspects of problem solving process which includes solution strategies, modes of representation and mathematical justifications (Silver, 1987; Charles & Silver, 1989). These three aspects are also related to the emphases in the teaching and learning of mathematics in Malaysia (Ministry of Education Malaysia, 2002, 2003a, 2003b, 2004). In the Malaysian primary mathematics curriculum, various problem solving strategies such as 'draw a diagram', 'identifying patterns', 'make a list', 'trial and error' and 'working backwards' are to be taught to the pupils as some of the common strategies of problem solving. Representations are also to be emphasized as a process of analyzing mathematical problems to enable pupils to find relationship between mathematical ideas. Justification in the context of this study can be related to the emphasis on communication in mathematics, as quoted from the Curriculum Specifications:

> "Communication is one way to share ideas and clarify the understanding of mathematics. Through talking and questioning, mathematical ideas can be reflected upon, discussed and modified. (…) Through effective communications, pupils will become efficient in problem solving and be able to explain concepts and mathematical skills to their peers and teachers." (Ministry of Education Malaysia, 2004, p. ix)

Through justifications, pupils explain their solutions and give reasons for choosing their solution strategies and modes of representation. This involves validation of their thinking and verification of their own line of reasoning. Thus, pupils' solution strategies, modes of representations and mathematical justifications may reflect their beginning ability to think pre-algebraically and communicate their pre-algebraic ideas in the pre-algebra language. This seems to suggest a possible association of the cognitive analysis of pupils' problem-solving processes to the inference of their pre-algebraic thinking.

Since algebra is not formally and directly taught to the primary school pupils, what sort of problems would deem appropriate to be used in this study? One of the ideas that led to algebra and hence algebraic thinking is recognizing and generalizing patterns as well as relationship between quantities (Hopkins, Gifford & Pepperell, 1999). With respect to generalizing patterns, generalizing number patterns can be viewed as a potential vehicle for transmitting students from numeric to algebraic thinking (Warren, 2000b; Lannin, Barker & Townsend, 2006) while generalizing geometric patterns can be related to early algebraic experiences (Warren, 2000a; 2000b). On the other hand, word problems have always been a part of mathematics that brings together arithmetic and algebraic thinking (Palomares & Hernandez, 2002; Van Amerom, 2003). Therefore, word problems that serve to form the link between arithmetic and algebra can also be 'pre-algebraic problems'.

Consequently, the pre-algebraic problems used in this study were confined to: (a) generalization problems concerning looking for, recognizing, describing, generalizing and extending both number and geometric patterns (Reys, Lindquist, Lambdin, Smith & Saydam, 2001), and (b) word problems concerning unknown quantities which require working with operations, hence using operation sense, as unknown and properties of operations are important aspects of pre-algebraic thinking (Lubinski & Otto, 2002).

However, requiring the primary school pupils to solve pre-algebraic problems raised the need to consider the ability of primary school pupils who are to be used as participants in this study. Their ability may be based on their previous mathematics learning experiences. How the primary school pupils cope with their attempts to solve these three categories of pre-algebraic problems, and think in pre-algebraic ways based on their prior mathematical knowledge and problem-solving experiences are of great interest in this study.

Figure 1.1 gives the research framework of this study. According to Lester (2005), a research framework is “a basic structure of the ideas that serves as the basis of phenomenon that is to be investigated” (p. 458). The research framework reflected the three categories of pre-algebraic problems to be used in this study, namely generalization problems involving number patterns, generalization problems involving geometric patterns and word problems involving unknown quantities. The problem-solving processes of these pre-algebraic problems are analyzed to identify the cognitive aspects of solution process, namely solution strategies, modes of representation and mathematical justifications. Their pre-algebraic thinking is then inferred via verbal protocol analysis (which will be discussed in Chapter 3).

Comparison is made upon the pupils’ solution strategies, modes of representation, mathematical justifications, as well as their pre-algebraic thinking in solving these three categories of pre-algebraic problems to obtain a general picture of how primary school pupils solve pre-algebraic problems, and hence the pre-algebraic thinking underlying the pupils’ solution processes.

From the findings of the study, the relationships between the three main components of solution process, namely solution strategy, mode of representation and mathematical justification are to be noted. On top of that, how the pupils’ pre-algebraic thinking is embedded in their solution strategies, modes of representation and mathematical justifications are of major interest to this study.

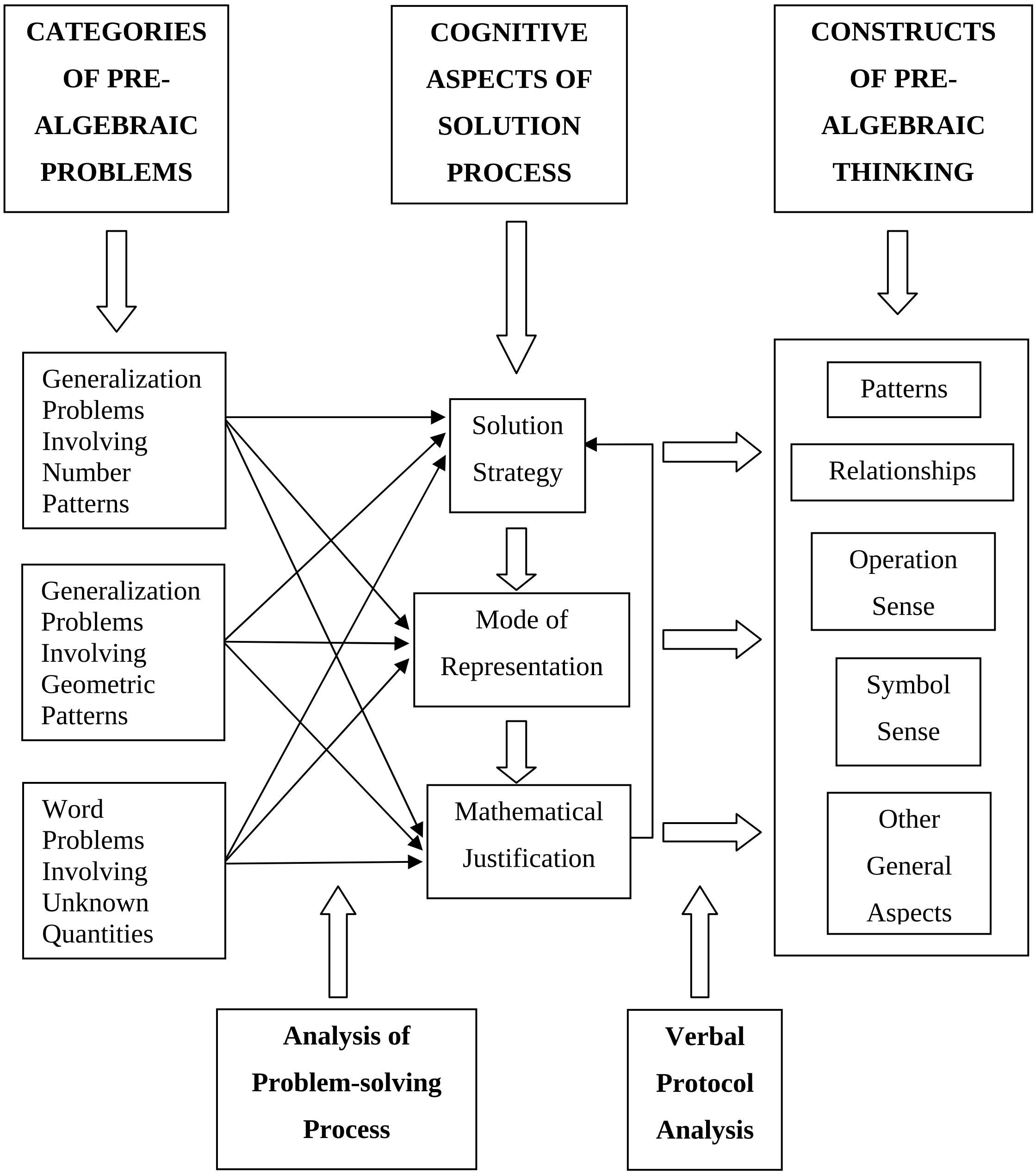

Figure 1.1. Research framework of the study.

1.6 Significance of the Study

Findings of this study hope to reveal various strategies and representations used by primary school pupils while solving pre-algebraic problems and the different ways they justify their solution processes. This may evoke awareness among mathematics educators that young children may be capable of using other strategies, including some informal or selfinvented strategies (De Corte & Verschaffel, 1989; Nathan & Koedinger, 2000) to solve pre-algebraic problems besides the standard school-taught methods, particularly the arithmetic method.

Inferences from the pupils' problem solving processes may lead the researcher to suggest specific ways that their pre-algebraic thinking may differ from the views of development of such thinking commonly held by teachers and views presented in frequently-used algebra textbooks (Nathan & Koedinger, 2000). The findings may then provide insights into children's highly-individualized pre-algebraic thinking, which often do not follow orthodox models of the classrooms and textbooks (Mulhern, 1989).

Findings related to the primary school pupils' pre-algebraic thinking may also served to inform the curriculum developers, educators and parents of the extent to which algebra is within the reach of primary school pupils. This in turn may lead to curriculum planning in the instructional design for the teaching and learning of pre-algebra before formal study of algebra in secondary school, particularly in deciding to what extent algebra can be included in primary school mathematics curriculum. This is because descriptions regarding how pupils solve pre-algebraic problems and their pre-algebraic thinking may shed some light on how pre-algebra can be incorporated into the teaching and learning of primary school mathematics in accordance to the ability of the pupils.

Evidence of primary school pupils' ability in solving pre-algebraic problems and their pre-algebraic thinking can inform our instructional decision making (Curcio & Schwartz, 1997). The originality, variety and depth of primary school pupils' pre-algebraic thinking may reveal their beginning ability in pre-algebraic ideas in mathematics. This may enable the curriculum planners to identify bridging methods that will help to facilitate a smooth transition from arithmetic to algebra among the primary school pupils. These bridging methods may in turn help both primary and secondary school mathematics teachers to guide their pupils to foster (particularly in the primary years) and develop algebraic thinking in the later stages of algebra learning.

Last but not least, the findings of this study may also have its implications on professional development among mathematics teachers and educators. They need to learn about algebraic thinking through their own experiences with it in order to make it a realistic part of their interactions with primary school pupils (Urquhart, 2000). Based on the findings regarding how primary school pupils solve pre-algebraic problems using their prior knowledge, and hence their pre-algebraic thinking, modules or intervention programs can be developed to be used in teachers' professional development activities on fostering pre-algebraic thinking in the primary mathematics classrooms.

1.7 Limitations of the Study

This study made use of techniques involving collecting and analyzing verbal protocols. One limitation of this method is that the process of collecting, coding and analyzing verbal protocol data is extremely labour intensive (Cai, 1995). Therefore, involvement of a large number of participants is not feasible for this study. Hence, the results of this study were merely indicative and could be used only to describe the pattern in the participants involved in this study.

The second limitation is the limited scope of the pre-algebraic problems since only generalization problems and word problems were used in this study to suit the ability of the primary school pupils, as discussed in the statement of problem in section 1.4. It is therefore important to recognize that the practical transferability (or generalizability) of the findings are constrained by the nature of the pre-algebraic problems chosen to be included in the instrument of this study.

The third limitation is the limited number of pre-algebraic problems used in this study. This is because the process of think-aloud can be very cognitively demanding for the participants. Too many problems may tire them, thus affecting their thought processes (Payne, 1994) and hence, the ‘value’ of data (Lee, 2001b).

The fourth limitation is related to the methodology of the study, particularly verbal protocol analysis which involves the use of participants’ own verbal reports as data. The validity of verbal reports as data may be doubted as these participants’ verbal reports necessarily would involve selectivity and interpretation by the researcher (Mulhern, 1989). Thus, the researcher must be very cautious and impartial in interpreting verbal reports. Mulhern suggested that protocol data must be integrated with more objective measures to achieve credibility (or validity) of the data. In this study, member checks and triangulation were used to achieve credibility of the data. In addition, experienced researchers were requested to verify part of the data analysis to ensure objectivity of the analysis.

1.8 Operational Definitions

This section presents the operational definition of terms used in the context of this study.

Pre-algebraic thinking refers to the transition between arithmetic and algebraic thinking (Warren, 2000a) which is associated with

a) looking for, recognizing, describing, generalizing, extending and creating patterns
b) looking for, recognizing and representing relationships
c) understanding number system, working with properties of operations and algorithm seeking (operation sense)

d) using variables and open structures to represent quantity and express relationships (symbol sense)

e) other general aspects such as justifying generalizations or conclusions, testing conjectures, using variety of representations, and operating on unknown quantities

Pre-algebraic problems refer to those which do not involve any algebraic formalization (Boero, 2001) but instead involve patterns and arithmetic manipulation of unknowns only. In this study, three categories of pre-algebra problems were used, namely

a) generalization problems involving number patterns

b) generalization problems involving geometric patterns

c) word problems involving unknown quantities

Solution strategy refers to plan used to solve problem, which may encompass

a) generalization strategies such as recursive or chunking strategies, and

b) strategies used to solve word problems which can be standard school-taught methods or informally adopted strategies

Mode of representation refers to representational tool used to organize, record and communicate mathematical ideas, which may involve the use of

a) oral explanations or written words (verbal representation)

b) diagrams or other visual illustrations such as number lines (pictorial representation)

c) standard algorithms or number sentences (arithmetic-symbolic representation)

d) symbols such as placeholders in open number sentence (algebraic-symbolic representation)

Mathematical justification is related to communication in mathematics which involves the following activities:

a) articulation of thinking and explain why a method worked

b) making and evaluating mathematical conjectures and arguments as a result of reasoning

c) justifying solutions

d) documenting or describing orally one's own thinking

e) validating one's own thinking or verifying one's own line of reasoning

f) providing multiple solution methods

Pattern in the context of this study focuses on number patterns and geometric patterns. There are two categories of patterns:

a) repeating or regular pattern, which is characterized by a common difference across all the numbers or shapes/figures

b) growing or irregular pattern, which 'grows' in an irregular but yet generalizable way

Generalization problem refers to "problems that are solvable by finding a pattern of quantitative relationship in a given problem situation" (Ishida & Sanji, 2002, p. 137). Two types of generalization problem are used in this study, namely near generalization problem and far generalization problem. According to them, near generalization requires the students to generate a number or figure immediately after the given numbers or figures, for example, write the fourth number in a numerical pattern based on the first three numbers given or draw the fourth figure based on the first three figures given in a geometrical pattern. If students need to write the fifth number based on the first three numbers given in a numerical pattern, or they are required to draw the fifth figure based on the first three figures given in a geometrical pattern, the process is called far generalization.

Word problem refers to mathematical problem phrased in words or pure prose. The word problems used in this study are based on rhetorical algebra (Lesser, 2000) and the process of organizing the arithmetic is needed to solve for the unknown (Choike, 2000).

1.9 Summary

Section 1.1 of this chapter gave the background of this study where importance of algebra for further studies and employment opportunities were briefly mentioned, leading to the need for early exposure of algebra and developing algebraic thinking in the primary school years. This section ended by arguing that the elements of early algebra are actually embedded in the Malaysian curriculum specifications for primary school mathematics.

Section 1.2 presented the problem statement of this study. Sub-section 1.2.1 briefly clarified the concepts of early algebra and pre-algebra. Two studies concerned young children's ability in solving 'open number sentence' problems were quoted. This inability evoked the question of how algebraic thinking is to be included in the primary mathematics teaching and hence, the relevance of an early start in learning algebra and developing algebraic thinking. Sub-section 1.2.2 began with an emphasis on the difficult transition from arithmetic thinking to algebraic thinking. This section also presented some local studies concerning the problems related to the learning of algebra and the generally poor mastery of algebraic concepts and skills among the secondary school students in the public examinations as well as in TIMSS-R and TIMSS 2003. This led to the awareness of a need to strengthen the foundation for algebra as early as primary school, which provided the baseline of the problem to be studied. In the problem statement, scarcity of studies concerning algebra learning particularly among the primary school pupils was highlighted and justified leading to the need of investigating how primary school pupils solve pre-algebraic problems based on their prior knowledge and experiences. The problem statement also justified the three categories of pre-algebraic problems to be used in this study based on primary school pupils' ability and experience in mathematics learning thus far.

Following from the statement of problem were sections 1.3 and 1.4 where the purpose and research questions for this study were stated and formulated respectively. This led to the construction of the research framework for the study as presented in section 1.5. The research framework showed that the process of investigation was based on the critical cognitive aspects of problem solving which seemed to be related to the emphases of the Malaysian primary mathematics curriculum.

Section 1.6 highlighted the significance of the study by discussing how the findings of this study may be used to write curriculum in order to support the transition of arithmetic to algebra among the primary school pupils. Contribution of the findings in enhancing teacher preparation programs with respect to fostering pre-algebraic thinking in the primary schools was also briefly mentioned. Section 1.7 touched on the limitations of the study. Operational definitions of some important terms used in this study were given in section 1.8. This chapter ends with section 1.9 which gives a summarized outline for the whole chapter. The next chapter will present and discuss the review of related literatures pertaining to the theoretical framework, variables and methodology of this study.

CHAPTER 2

LITERATURE REVIEW

2.1 Chapter Overview

The literature review for this study as presented in this chapter serves three purposes: (a) to construct a theoretical framework for the study, (b) to review related literature pertaining to the four variables of this study – solution strategy, mode of representation, mathematical justification and pre-algebraic thinking, and (c) to review related literature on the methodology to be used in this study.

2.2 Constructing a Theoretical Framework for the Study

The theoretical framework of this study was constructed based on research studies about transition from arithmetic to algebra. This transition of the cognitive gap between arithmetic and algebra is called 'pre-algebraic thinking' (Warren, 2000a). This theoretical framework then guided the research activities by its reliance on a theory (Lester, 2005) – the Piaget's stage theory of cognitive development, which is one often-quoted theory used in the study of children's learning. The other sub-sections present and discuss the important components in the theoretical framework of this study, namely algebra, algebraic thinking, pre-algebra, the relation of operation sense and symbol sense to pre-algebraic thinking as well as development of algebraic thinking and pre-algebraic thinking.

2.2.1 Transition from Arithmetic to Algebra

In mathematics, there are two hierarchical levels of thought – arithmetic and algebraic (Esty & Teppo, 1996). Arithmetic deals with straight-forward calculations with known numbers. In other words, arithmetic proceeds directly from the known to the unknown using known computations. On the other hand, algebra requires reasoning about unknown or variable quantities when it proceeds from the unknown, via the known, to the equations.

Thus the difference between arithmetic and algebra is that the former deals with a specific situation while the later deals with a general solution (Van Amerom, 2003).

This point of view seemed to be in agreement with the opinion that the difference between arithmetic and algebra depend on the ways questions are couched (Usiskin, 1997). For instance, when students were asked to find n when 5 x 7 = n and being asked ‘what is the answer?’ then the question was treated as arithmetic. However, if the question ‘what number can I replace n and make this a true statement?’ was asked, the statement was treated as algebra. Therefore, a conceptual change is needed to occur in students’ thinking as they move from arithmetic to algebra. The focus of thought must shift from number to operations on numbers and relationships between numbers. This follows that ‘operations on numbers’ and ‘relationships between numbers’ formed the arithmetic structures which are required for the understanding of algebra structure (Warren & English, 2000). They classified the knowledge of arithmetic structures as

a) relationships between quantities (equivalence and inequality)
b) properties of quantitative relationships (transitivity of equality)
c) properties of operations (associativity and commutativity)
d) relationships between operations (distributivity)

In addition, arithmetic understanding of (a) order of operations, (b) properties of numbers, (c) structure of obtaining solutions, (d) possible range of solutions, and (e) false generalization, can lead to algebraic understanding (Warren, 2000b). So, it appeared that concepts underlying the arithmetic would be difficult to describe without algebra, particularly in generalizations and relationships. For instance, algebraic descriptions like ‘0 + n = n’ is used to generalize the mathematical fact that zero added to any number is the number itself. Thus, two important conclusions about algebra (Usiskin, 1997) are:

a) algebra is the most appropriate language for writing down general properties in arithmetic; and

b) algebra should support arithmetic, not separated from it.

The second conclusion above seemed to be related to the idea that "historically, algebra grew out of arithmetic" (Lee & Wheeler, 1989, p. 41). Algebra can thus be viewed as a natural extension of arithmetic and be defined as a process of organizing the arithmetic needed to find an answer to a question involving quantities that are not yet known (Choike, 2000). Therefore, algebra can be related to the study of operations (Saul, 2001). Irwin and Britt (2005) who proposed "algebra in arithmetic" viewed arithmetic as a precursor to algebra. These views seemed to suggest that there is a transition from arithmetic to algebra.

Transition from arithmetic to algebra was described as "evolution from arithmetical to algebraic language which corresponds to the notions and the forms of representation of the objects and operations involved in the changeover" (Filloy & Sutherland, 1996, p. 145). This transition can also be related to the use of letters as mathematical objects since children only encounter the notion of variable when they find missing addends, verbalize and generalize number patterns (Vance, 1998). They also dealt with mathematical relationships through the use of counting, pictures or chips to model the process in the early grades. The transition from arithmetic to algebra also involves a move from knowledge required to solve arithmetic equations (operating on or with numbers) to knowledge required to solve algebraic equations (operating on or with the unknown or variable) (Warren, 2000a). Warren also related this transition to students' ability with the cumulative, associative and distributive laws, inverse operations, order of operations, meanings of equal and variable. From these various views on the transition from arithmetic to algebra, it can be concluded that this transition involved conceptual and symbolic changes which marked a difference between arithmetic thinking and algebraic thinking.

There has been a growing concern about the difficulty arise in transition from arithmetic to algebra over a decade. In Lee and Wheeler's (1989) study involving Grade 10 students of 15 – 16 years old, only 10 out of 268 respondents attempted to check the 'truth' of algebraic statements with numbers. Most of the respondents seemed too remained in the world of algebra and did not think of going from algebra into arithmetic. On the other hand, three quarters of the 352 students did not use algebra at all to solve problems though they were aware that it was an algebra test. These students could not move from the arithmetic number situation into the algebraic in order to establish the arithmetic generalization. Therefore, Lee and Wheeler said that "algebra and arithmetic are two dissociated worlds" (p. 44) and concluded that "the track leading from arithmetic to algebra is littered with procedural, linguistic, conceptual and epistemological obstacles" (p. 53).

In other studies, Kieran (1992) pointed out that many students who could manipulate the algebraic symbols correctly in an equation were unable to set up such an equation given in its relationships expressed in the form of a word problem. Esty and Teppo (1996) also concluded from their studies that many of the students are not able to move beyond an arithmetic level of understanding to think algebraically about those same operations when the number is unknown.

Perhaps that is why students who enter algebra from an arithmetic-driven program often find the new content confusing and daunting due to the lack of preparation (CSBE, 2000). Another reason could be due to the fact that the language of arithmetic focuses on answers whereas language of algebra focuses on relationships (MacGregor & Stacey, 1999). To use algebra for solving a problem, the focus of attention is not getting numerical answers to each step of the solution but on the operations used. Thus, moving from arithmetic thinking to algebraic thinking involved a shift from purely numerical solutions to a consideration of method and process (Warren & English, 2000). This inappropriate cognitive load in the early algebraic experiences (Warren, 2000a) could also be where the students' difficulties stemmed from.

Indeed, learning algebra after arithmetic means developing a different way of thinking (Dettori, et. al., 2001). Arithmetic is a prerequisite for algebra in that algebraic manipulations are based on the four arithmetic operations. However, the main aim of algebra is not to perform numerical computations but to provide an operative language to represent, analyze and manipulate relations containing both numbers and letters. Therefore, style of algebra is essentially declarative while that of arithmetic is essentially procedural.

Despite the “break” due to existence of possible cognitive obstacles for students in making the links between arithmetic and algebra, there are researchers who are committed to maintaining the arithmetic connection (Lee, 1996) and look for pedagogical ways to establish the arithmetic connection (Linchevski & Livneh, 1999). There are indeed some researchers that offered their suggestions to support these two views.

One of them is Vance (1998) who suggested that by emphasizing conceptual understanding, thinking processes and mathematical connections in the early teaching of arithmetic not only makes the study of number and operations more meaningful and intellectually stimulating but prepares children for the formal study of algebra. Other researchers like MacGregor and Stacey (1999) emphasized using problems which involved recognizing the number operations, understanding important properties of numbers and describing patterns to prepare students for algebra. Warren (2000a) suggested two approaches to bridge the gap between arithmetic and algebra – generalizing the patterns in arithmetic as well as number patterns and visual patterns. However, Boulton-Lewis, Cooper, Atweh, Pillay and Willis (2000) claimed that successful transition from arithmetic thinking to algebraic thinking involved more than generalizing arithmetic. To them, contextualizing algebraic functions as well as linking and transferring between differing representations are equally important.