UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 1992/93

April 1993

CSI 501 - Logic and Inference Systems

Masa: [3 jam]

ARAHAN KEPADA CALON:

- Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Answer **ALL** questions.

1. (a) Describe briefly what is meant by interpretation of formulas ; valid, satisfiable, and unsatisfiable formulas in predicate calculus. For each of the following formula, state whether it is valid or not. If it is not, state whether it is satisfiable or unsatisfiable. Give some justifications to your answers.

(i) $\forall x\, P(x) \rightarrow \exists x\, P(x)$

(ii) $\exists y \forall x\, Q(x,y) \rightarrow \forall x\, \exists y\, Q(x,y)$

(iii) $\exists x\, P(x) \rightarrow \forall x\, P(x)$

(7/20)

(b) Show informally that in predicate calculus a well-formed formula A is valid if and only if ~A is unsatisfiable.

(2/20)

(c) Describe what is meant by a clause form of a well-formed formula.

(2/20)

(d) Put the following predicate-calculus formulas into clause form:

(i) $\forall x\, \forall y\, [\, (P(x) \wedge Q(y)) \rightarrow \exists z\, R(x,y,z)\,]$

(ii) $\exists x\, \forall y\, \exists z\, [P(x) \rightarrow (Q(y) \rightarrow R(z))]$

(4/20)

(e) For each of the following sets of literals, find a most general unifier or determine that the set is not unifiable by giving a valid reason for it.

(i) { P(f(y), w, g(z)) , P(u,u,v) }

(ii) { P(f(y), w, g(z)) , P(v,u,v) }

(iii) { P(a,x,f(g(y))) , P(z, h(z,w),f(w)) }

[5/20]

2. Consider the following paragraph of statements :

"Ahmad, Abdul, and Vincent belong to the Alpha Club. Every member of the Alpha Club is either a footballer or a mountain climber or both. No mountain climber likes rain, and all footballers like forest. Abdul dislikes whatever Ahmad likes and likes whatever Ahmad dislikes. Ahmad likes rain and forest."

(a) Translate the sentences in the above paragraph into formulas in predicate logic. You may use the following four predicates :

* AC(x) : x belongs to the Alpha club.
* FT(x) : x is a footballer.
* MC(x) : x is a mountain climber.
* L(x,y) : x likes y.

(3/14)

(b) Convert the formulas in part (a) into clause form. (2/14)

(c) Use resolution method to find (if any) a member of the Alpha Club who is a mountain climber but not a footballer. (6/14)

(d) Verify your answer to the question in part (c) by giving a line of reasoning expressed in English. (3/14)

3. Consider the following formulas which represents some properties of certain predicate P of the predicate calculus :

P1. $\forall x \forall y [P(x,y) \rightarrow P(y,x)]$

P2. $\forall x \forall y \forall z [P(x,y) \wedge P(y,z) \rightarrow P(x,z)]$

P3. $\forall x \exists y P(x,y)$

(a) Prove using resolution method that another property $\forall x P(x,x)$ is a logical consequence of P1,P2,P3. (6/10)

(b) Express the above four properties in English language. (4/10)

4. Consider the following set S of clauses :

C1. P(x,y,f(x,y))
C2. ~P(x,y,u) ~P(y,z,v) ~P(x,v,w) P(u,z,w)
C3. ~P(x,y,u) ~P(y,z,v) ~P(u,z,w) P(x,v,w)
C4. P(b,y,y)
C5. P(g(y),y,b)
C6. ~P(x,h(x),h(x)) ~P(k(x),z,x)

where only b is a skolem constant.

(a) Find a linear resolution-based refutation of S.

(12/20)

(b) Find another refutation of S using Set-of-Support strategy with T = { C2, C3 } as the initial support.

(8/20)

5. (a) Let S = { ~P(x) ∨ Q(y) , P(f(y)) , ~Q(f(y)) } be a set of clauses. Describe the Herbrand universe of S, Herbrand base of S and give one Herbrand interpretation of S which is not a model. You must justify your answer.

(4/18)

(b) Consider the following three theorems regarding predicate calculus :

Theorem 1: Every well-formed formula W can be transformed into a well-formed formula W' in clause form such that W is satisfiable if and only if W' is satisfiable.

Theorem 2 : A well-formed formula W in clause form is unsatisfiable if and only if it yields the value false under all Herbrand interpretations of W.

Theorem 3 (Herbrand) **:** A well-formed formula W in clause form is unsatisfiable if and only if there is a finite conjunction of ground intances of its clauses which is unsatisfiable.

(i) Explain clearly the meaning of Theorem 2 and Herbrand's Theorem above.

(ii) Let S = { P(x) ∨ Q(y) , ~P(a) , ~Q(b) }be a set of clauses. By making use of any of the above theorems but without making use of the resolution method, show that the set of clauses S is unsatisfiable.

(iii) Give a brief description of a method which relies on the above Herbrand's Theorem and which can be used to proof the unsatisfiability of any given unsatisfiable well-formed formula. Give some justifications on why the method works and can also be used to show that any valid well-formed formula is indeed valid.

(10/18)

(c) State whether the validity problem for propositional calculus and predicate calculus are decidable or not. If decidable, give some justifications. Otherwise, briefly discuss the practical implications of such a limitation and why any one of the known automated reasoning method cannot be used for this purpose.

(4/18)

6. Consider the following knowledge base :

$\forall x \, \forall y$ [cat(x) $\wedge$ fish(y) -> eat(x,y)]

$\forall x$ [calico(x) -> cat(x)]

$\forall x$ [tuna(x) -> fish(x)]

tuna(Pascal)

tuna(Fortran)

calico(Cobol)

(a) Convert the above knowledge base into a PROLOG program.

(4/18)

(b) Write a PROLOG query corresponding to the question "What does the animal Cobol like to eat ?" and describe how the program finds all answers to this question.

(7/18)

(c) Convert the knowledge base into Horn clauses and simulate the behavior of your PROLOG program in answering the above question by using SLD-refutation.

(7/18)

- oooOooo -