PREDICTING BRIDGE PIER SCOUR DEPTH USING LTS REGRESSION

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Abstract. *In this paper, we discuss seven equations that have been suggested by various authors to predict scour depth. The seven equations are (1) Inglis-Poona scour equation, (2) Laursen-Toch curve (3) Railway Practice after Lacey and Inglis (4) Mushtaq Ahmad's equation (5) Blench's equation and (6) Aminuddin and Nalluri's equation. Almost all observed data must consist of one or more extreme observations which are called outliers. In the presence of outliers ordinary least squares (OLS) method used in multiple linear regression are not appropriate. Thus we use the least trimmed squares (LTS) method to obtain the model for predicting pier scour depth. The result obtained from the analysis show that Aminuddin and Nalluri's equation produced the best model.*

1 Introduction

Local scour may be defined as the lowering of bed level in the vicinity of an obstruction such as a bridge pier placed in a flowing stream. This is caused by the abrupt change in the direction of the approach flow. Sediment movement in the approach flow determines whether the local scour is clear (no sediment transport) or live bed (with sediment discharge).

Extensive experimental studies (Simons and Senturk, 1992) have been conducted on the subject resulting in many empirical equations. Kafi and Alam (1995) used field data to further improve the accuracy of the laboratory based equations. These equations depend on many variables such as flow characteristics (depth of flow, velocity of flow, vortex velocity), geometry of the piers (width of obstruction or pier), bed material size (sediment size, mean diameter of bed material, silt factor) and fluid properties.

In this paper, we compare seven equations that have been suggested by various authors to predict scour depth. The seven equations are (1) Inglis-Poona scour equation, (2) Laursen-Toch curve (3) Railway Practice after Lacey and Inglis (4) Mushtaq Ahmad's equation (5) Blench's equation and (6) two equations from Aminuddin and Nalluri. Almost all observed data must consist of one or more extreme observations which are called outliers. In the presence of outliers, ordinary least squares (OLS) method is not suitable. Thus we use the least trimmed squares (LTS) method to obtain the model for predicting pier scour depth. The models obtained from LTS method was compared with those obtained using OLS.

2 The LTS method

The multiple linear regression model has the form

$$y_i = x_i^T \beta + \varepsilon_i, \quad i = 1, 2, \ldots, n$$

where y_i is the scalar dependent variable associated with the ith observation, x_i is a p-dimensional vector of independent variables, $\beta = \left(\beta_0, \beta_1, \ldots, \beta_p\right)$ represents the coefficients and the ε_i are errors.

The LTS estimate $\hat{\beta}_{LTS}$ for the coefficients in the above multiple linear regression model minimizes the following objective function:

$$\sum_{i=1}^{q} r_i^2 \beta$$

where $r_i\beta$ is the ith ordered residual. The value of q is often set to be slightly larger than half of n, the number of observations in the model. However, the ordinary least squares estimate $\hat{\beta}_{LS}$ for the regression coefficients minimizes the sum of all squared residuals:

$$\sum_{i=1}^{n} r_i^2\beta .$$

Thus, LTS is equivalent to ordering the residuals from a least squares fit, trimming the observations that correspond to the largest residuals, and then computing a least squares regression model for the remaining observations. The ordinary least squares estimator lacks robustness because a single observation can cause $\hat{\beta}_{LS}$ to take on any value.

3 The data

The data was obtained at random from field data in the literature (Kafi and Alam (1995), Kothyari et al. (1992)). The independent variables given in the data are width of scour (b), mean diameter of bed material (d), force of gravity (g), discharge quantity (q), constant depending on pier shape (K), flow depth (Y) and maximum flow discharge (Q) while the variable of interest is the scour depth at piers (D_s). The sample size is 40.

4 The model

The seven general equations considered in this paper are given below:

1. Inglis-Poona Equation $\frac{D_s}{b} = \beta_0\left(\frac{q}{b}\right)^{\beta_1}$

2. Laursen-Toch Curve $\frac{D_s}{b} = \beta_0 K\left(\frac{Y}{b}\right)^{\beta_1}$

3. Railway Practise $D_s = \beta_0\left(d_{LQ}\right)^{\beta_1}$ with $d_{LQ} = 1.33\left(\frac{q^2}{f}\right)^{1/3}$ and $f = 1.76\sqrt{d}$

4. Mushtaq Ahmad $D_s = \beta_0 K q^{\beta_1}$

5. Blench $\frac{D_s}{y_t} = \beta_0\left(\frac{b}{y_t}\right)^{\beta_1}$ with $y_t = \left(\frac{q^2}{1.9d}\right)^{1/3}$

6. Aminuddin and Nalluri (a) $\frac{D_s}{b} = \beta_0\left(\frac{Y}{b}\right)^{\beta_1}\left(\frac{b}{d}\right)^{\beta_2}\left(\frac{Q}{bY\sqrt{gY}}\right)^{\beta_3}$

7. Aminuddin and Nalluri (b) $\frac{D_s}{d} = \beta_0\left(\frac{Y}{b}\right)^{\beta_1}\left(\frac{b}{d}\right)^{\beta_2}\left(\frac{Q}{bY\sqrt{gY}}\right)^{\beta_3}$

5 Results

The linear regression models obtained using ordinary least squares (OLS) is presented in Table 1 below while Table 2 gives the regression model obtained by using the LTS method.

Table 1. The scour equations using OLS

	Regression Equation	
Inglis-Poona Equation	$\frac{D_s}{b}=1.667\left(\frac{q}{b}\right)^{0.638}$	R^2=0.835
Laursen-Toch Curve	$\frac{D_s}{b}=1.262K\left(\frac{Y}{b}\right)^{0.728}$	R^2=0.793
Railway Practice	$D_s=3.387\left(d_{LQ}\right)^{0.671}$	R^2=0.736
Mushtaq Ahmad	$D_s=1.634Kq^{0.593}$	R^2=0.802
Blench	$\frac{D_s}{y_t}=2.710\left(\frac{b}{y_t}\right)^{0.508}$	R^2=0.539
Aminuddin and Nalluri (a)	$\frac{D_s}{b}=1.182\left(\frac{Y}{b}\right)^{0.524}\left(\frac{b}{d}\right)^{-0.0307}\left(\frac{Q}{bY\sqrt{gY}}\right)^{0.24}$	R^2=0.622
Aminuddin and Nalluri (b)	$\frac{D_s}{d}=1.181\left(\frac{Y}{b}\right)^{0.524}\left(\frac{b}{d}\right)^{0.969}\left(\frac{Q}{bY\sqrt{gY}}\right)^{0.24}$	R^2=0.955

Table 2. The scour equations using LTS

	Regression Equation	
Inglis-Poona Equation	$\frac{D_s}{b}=1.540\left(\frac{q}{b}\right)^{0.692}$	R^2=0.8616
Laursen-Toch Curve	$\frac{D_s}{b}=1.123K\left(\frac{Y}{b}\right)^{0.8617}$	R^2=0.9148
Railway Practice	$D_s=2.246\left(d_{LQ}\right)^{0.8703}$	R^2=0.8193
Mushtaq Ahmad	$D_s=1.956Kq^{0.5273}$	R^2=0.8153
Blench	$\frac{D_s}{y_t}=2.303\left(\frac{b}{y_t}\right)^{0.1833}$	R^2=0.4203
Aminuddin and Nalluri (a)	$\frac{D_s}{b}=1.779\left(\frac{Y}{b}\right)^{0.8464}\left(\frac{b}{d}\right)^{0.0209}\left(\frac{Q}{bY\sqrt{gY}}\right)^{0.234}$	R^2=0.7979
Aminuddin and Nalluri (b)	$\frac{D_s}{d}=1.779\left(\frac{Y}{b}\right)^{0.8464}\left(\frac{b}{d}\right)^{1.0209}\left(\frac{Q}{bY\sqrt{gY}}\right)^{0.234}$	R^2=0.9576

From Table 1, it can be seen that Blench's equation has the lowest R^2 value (R^2= 0.5391) while Aminuddin and Nalluri's (b) equation has the highest value for R^2 (R^2 = 0.9553). From Table 2, Blench's equation gives the lowest R^2 value ($R^2 = 0.4203$) while the Aminuddin and Nalluri's (b) equation has the highest R^2 value ($R^2 = 0.9576$).

If the R^2 values are to be compared between the two methods, it can be seen that except for Blench's equation, the LTS method gives a higher value for R^2. The biggest difference in R^2 is obtained by using the Laursen-Toch curve.

Figure 1 below shows the comparison between the OLS method and the LTS method for the Laursen-Toch curve. Since point X is an outlier, the regression model obtained using OLS will tend towards the outlier. However, LTS would not take into account the outlier when the estimating the β's. From Figure 1, it can be seen that most points fit particularly well for the regression model using LTS when compared to the regression model obtained by using OLS.

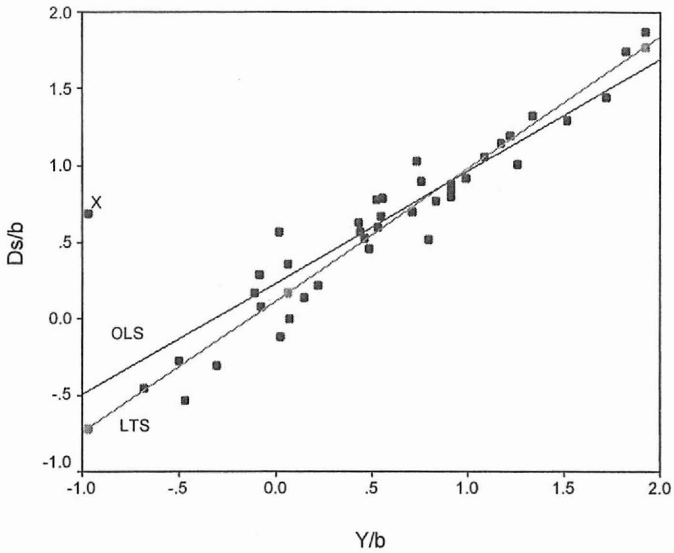

Figure 1. Comparison between OLS and LTS estimates for the Laursen-Toch curve

6 Conclusions

The OLS method for obtaining the regression models was used to obtain equations for predicting scour depth at piers. Seven equations were considered and comparisons were made via the coefficient of multiple determination, R^2. By using the OLS method, it was found that the values of R^2 were in the range of 0.5391 until 0.9553. Here higher values of R^2 indicate better predictive power. Thus Aminuddin and Nalluri's (b) equation is the best with $R^2 = 0.9553$.

The LTS method was also used for obtaining the regression models to predict scour depth at piers. This method was used because there exist outliers in the data. By using the LTS method, it was found that except for Blench's equation, the R^2 values has improved. The range of R^2 values is between 0.4203 and 0.9576. The Laursen-Toch curve gives the biggest increase from 0.7933 (using OLS) to 0.9148 (using LTS). Again Aminuddin and Nalluri's (b) equation is the best with $R^2 = 0.9576$.

7 References

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