

Optimized Arbitrary Size Networks¹

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Abstract

A rearrangeable nonblocking network of Benes network is well known for its realization for any arbitrary permutation. Later, the introduction of Arbitrary Size (AS) Benes network which improving Benes network in term of the size of network, hence giving a better performance by performing a Benes network to arbitrary size. By reducing the number of switching elements in the network, the performance and utilization of switching elements (SEs) can be increased and optimized. In this paper, we introduce an Optimized AS-Benes (OAS-Benes) that comes with an enhancement by reducing the number of SEs based on the previous model of AS-Benes.

Keyword: Interconnection Network, Rearrangeable Nonblocking Network, and Benes Network.

1. Introduction

A nonblocking network is a switching network that can connect an unused input by a path through unused edges to any unused output, regardless of which inputs and outputs have been already connected. In addition, it is rearrangeable if, given any set of disjoint paths between some inputs and outputs; every unused input can be connected by a path through unused edges to any unused output with possible rearrangements of the previously established paths [3].

The Benes network is rearrangeable nonblocking network which can realize any arbitrary permutation. The Benes network of dimension n is shown to be strictly nonblocking if only a suitable chosen fraction of $1/n$ of inputs and outputs is used. In Benes network, the r -dimensional Benes network connects 2^r inputs to 2^r outputs through $2r-1$ levels of 2×2 switches. Here, each level of switches consists of 2^{r-1} switches, and therefore the size of the network has to be a power of two.

This paper is based on rearrangeable nonblocking networks. In order to provide a better computational performance especially for tasks that require real-time response, a high performance switching networks is needed. This can be achieving through a good utilization of processors and a good control algorithms. Here, we proposed a rearrangeable nonblocking network with reduced switching elements. The reduction of the SEs is achieved by eliminating SEs which is not going to be used by the routing algorithm.

2. Arbitrary Size (AS) Benes Network

AS - Benes network is an extended of Benes model with arbitrary size. It is constructed from 2×2 switches with the internal state being determined by a binary control signal $b = \{0,1\}$ [2]. Each element of switches can be set by a control line into a direct-connection state or a crossed-connection state as shown in Figure 1, thus realizing all permutations from two inputs to two outputs [3].

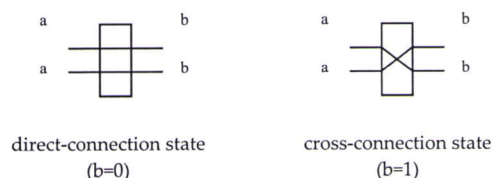


Figure 1. Two State of 2×2 Switch

Here, by introducing a 3×3 AS-Benes network, it can be generalized to recursively construct a network of any size. To construct a 3×3 permutation, it consist three 2×2 switches. By considering a simple wire to be a network that can realize any 1×1 permutation, we can view the 3×3 network in Figure 2 as being built from a 2×2 network and 1×1 network [1].

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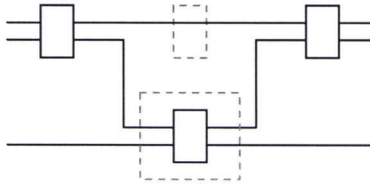


Figure 2. A 3 x 3 AS-Benes Network

Specifically, an AS-Benes of size n is constructed recursively from an upper sub-network AS-Benes of size $\lfloor n/2 \rfloor$ and lower sub-network AS-Benes of size $\lceil n/2 \rceil$. When n is even the construction is similar to that of the Benes network. Meanwhile, for odd size, the first $n-1$ output are connected to upper sub-network of size $\lfloor n/2 \rfloor$ switches and each switch is connected to the two smaller $(\lfloor n/4 \rfloor \times \lfloor n/4 \rfloor)$ AS-Benes networks. The last input and the last output are connected directly to the lower sub-network AS-Benes of size $\lceil n/2 \rceil$.

2.1 Routing Algorithm

A connection between outputs and inputs may be established through either the upper AS-Benes sub-network (of size $\lfloor n/2 \rfloor$) or the lower AS-Benes sub-network (of size $\lceil n/2 \rceil$). Given that each switch at the first and last levels in an AS-Benes has precisely one connection to each of the upper and lower sub-networks, the realization of any given permutation, Π , in an AS-Benes should satisfy the property that paths sharing any switch at the first or last levels must go to different sub-networks. Using this property, it can be shown that, given any one-to-one mapping, Π , of n inputs to n outputs, there is a set of edge disjoint paths from the inputs of a size n AS-Benes to its outputs connecting input i to output $\Pi(i)$ for $0 \leq i \leq n-1$ [1].

As an example, we illustrate the paths in Figure 3 for the mapping

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 8 & 6 & 2 & 1 & 0 & 3 & 5 \end{bmatrix}$$

in a 9 X 9 AS-Benes network. The bold paths represent the first loop which starts at input $n-1$ and terminate output $n-1$. After this loop, there are only two pairs of input/output left which form a second loop. In this way, all paths can be assigned to the upper or lower sub-networks without any conflict.

2.2 Applications of Benes Networks

The Benes network has been proposed for use in telephone networks [4]. It and other closely related networks were also used as interconnection networks for parallel computers. It has also been suggested for use in ATM switches [5]

3 Optimized AS-Benes Networks

From the looping algorithm introduced before, a connection between inputs and outputs can be constructed through either the upper or lower AS-Benes network. This can happen since, to construct any size $n \times n$ AS-Benes network, recursively a network of size 3 can be used for an odd size while for an even size, it's based on an original Benes network.

Here the left-most of the network can be eliminated by taking an advantage of the straight state of the SEs rather than the cross-state. This method can be accepted since the network is constructed from two sub network (upper and lower) and each sub network is based on its own routing algorithm. From the routing algorithm perspective, each sub-network has an independent algorithm and did not affect each other; therefore, the upper leftmost switch of even sub-network can be eliminated. This approach will continue until we reach the smallest size of AS-Benes, AS-Benes of size 3×3 or AS-Benes of size 2×2 . Figure 4 shows a 9 X 9 Optimized AS-Benes similar to Figure 3, except the leftmost switches depicted by circles are eliminated.

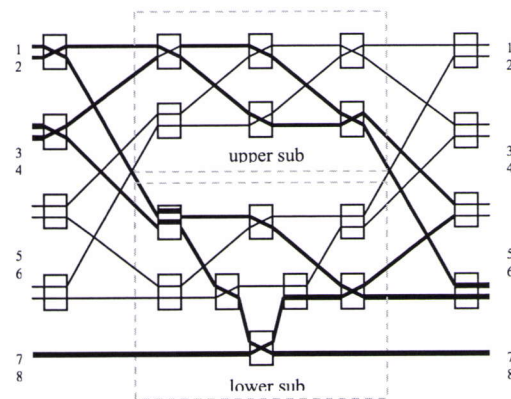


Figure 3. Two Loops in the Realization of a Permutation in a 9 x 9 AS-Benes

Figure 5 shows the OAS-Benes for all upper leftmost switches being eliminated. Circles depicted in the Figure 5 indicate the location of the switches that are being eliminated. The numbers of switches being eliminated in OAS-Benes of size 17 are 5 switches.

The recursive function was developed to calculate the number of switches that will be used for each node after the elimination in each sub-network. This recursive function can be used to compare the number of switches needed in an $n \times n$ AS-Benes. This comparison is shown in Figure 6 for n up to 32.

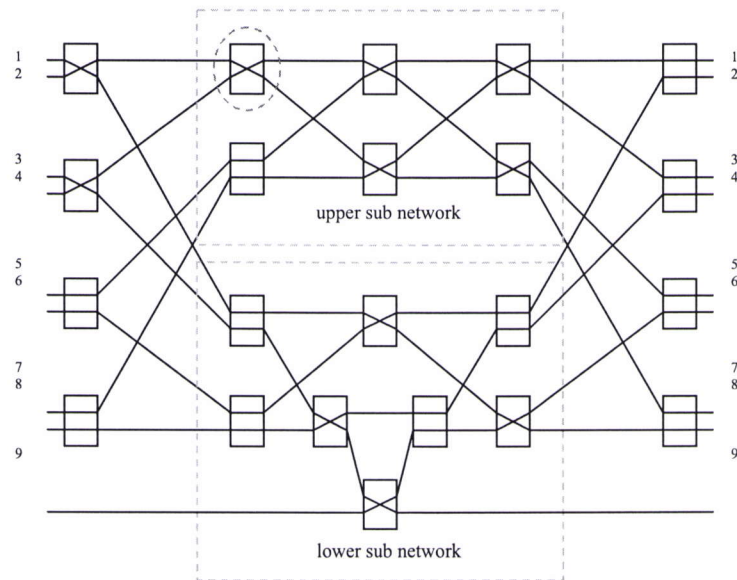


Figure 4. A 9 x 9 Optimized AS-Benes

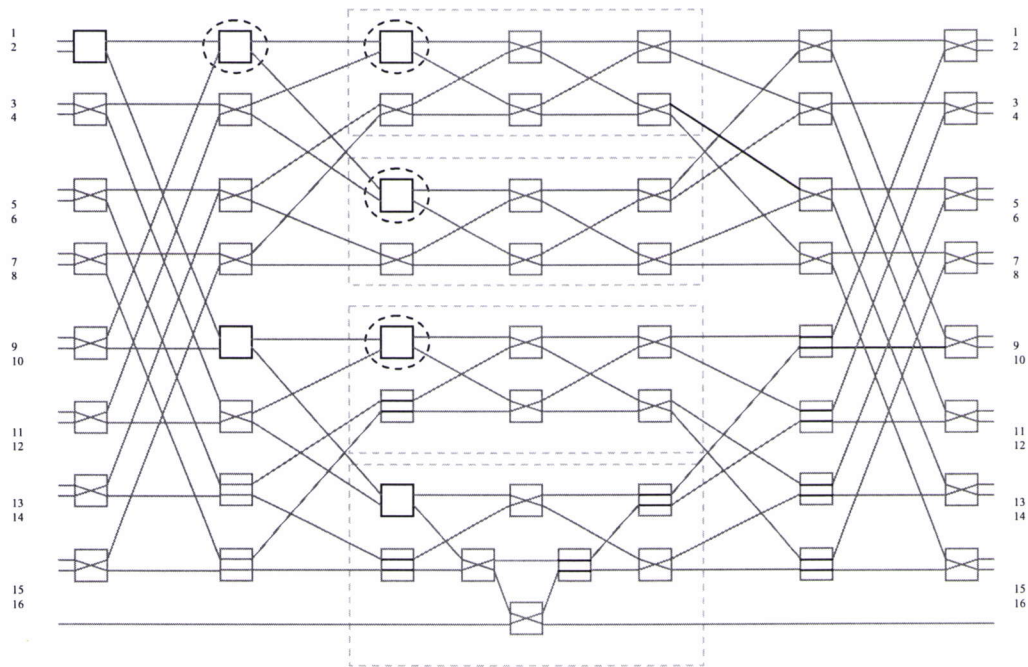


Figure 5. A 17 x 17 Optimized AS-Benes

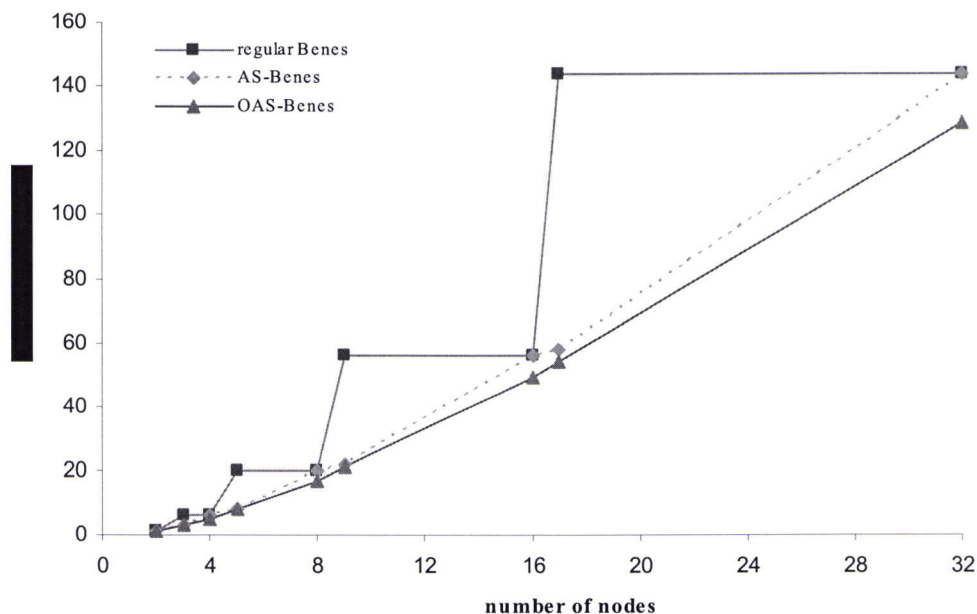


Figure 6. Comparison of Permutation Networks

4 Conclusions

By reducing the number of Switching Elements used in this design, the cost of switches can be cut and performance can be increase simultaneously. In term of a large networks or a huge usage of Switching Elements, the number of reduced Switching Elements can be significant.

5 References

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