UNIVERSITI SAINS MALAYSIA

KSCP Semester Examination
Academic Session 2007/2008

June 2008

ZCT 304/3 – Electricity and Magnetism
[Keelektrikan dan Kemagnetan]

Duration: 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **SEVEN** printed pages before you begin the examination.
*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer all **FIVE** questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**Arahan**: Jawab semua **LIMA** soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

1. A long dielectric cylinder, with radius a, contains free charges of density $\rho_f = r/\alpha$ where α is a constant. Consider the cylinder of length l. Obtain the electric field in the regions $r \le a$ and $r \ge a$.
[Satu silinder dielektrik yang panjang berjejari a mengandungi ketumpatan cas bebas $\rho_f = r/\alpha$ di mana α adalah pemalar. Pertimbangkan silinder tersebut sepanjang l. Dapatkan medan elektrik di kawasan $r \le a$ dan $r \ge a$.]
(20/100)

2. A spherical dielectric of radius R is polarized with polarization vector $\vec{P} = \left(K/\ r^{\alpha}\right)\hat{r}$, $\hat{r}$ is the radial unit vector and α is a constant.
[Suatu sfera dielektrik telah terkutub dengan vektor pengkutuban $\vec{P} = \left(K/\ r^{\alpha}\right)\hat{r}$, $\hat{r}$ merupakan vektor unit jejarian dan α adalah pemalar.]

(a) Calculate the density of volume bound charges, ρ_b, and the density of surface bound charges, σ_b. Show that the total amount of bound charges is zero.
[Hitung ketumpatan isipadu cas terikat, ρ_b, dan ketumpatan permukaan cas terikat, σ_b. Tunjukkan bahawa jumlah cas terikat adalah sifar.]

(b) If χ_0 is the electric susceptibility of the dielectric sphere, obtain the electric filed **E** produced in the sphere. Then, using the Gauss' law for dielectric, calculate the density of volume free charges, ρ_f, in the sphere.
*[Jika χ_0 adalah kerentanan elektrik bagi sfera dielektrik tersebut, dapatkan medan elektrik **E** di dalam sfera. Kemudian dengan menggunakan hukum Gauss bagi dielektrik, hitung ketumpatan isipadu cas bebas, ρ_f, di dalam sfera.]*
(20/100)

3. (i) Prove that the surface free charge density on the surface of a conductor is $\sigma = \varepsilon_0 E_{\perp}$ where $E_{\perp}$ is the component of electric field normal to the conductor surface.
Buktikan bahawa ketumpatan permukaan cas bebas di permukaan konduktor adalah $\sigma = \varepsilon_0 E_{\perp}$ di mana $E_{\perp}$ adalah komponen medan elektrik yang tegak lurus dengan permukaan konduktor.

(ii) A capacitor in a shape of a sphere contains two concentric spherical shells with radius r_a and r_b ($r_a < r_b$). The inner shell is earthed and the outer shell has potential V_0. The space between the shell is filled with charges such that the density of the charges is $\rho = \rho_0 r^2$ (ρ_0 is a constant).

[Satu kapasitor berbentuk sfera mengandungi dua petala sfera konduktor yang sepusat berjejari r_a dan r_b ($r_a < r_b$). Petala bahagian dalam di bumikan dan petala bahagian luar mempunyai keupayaan elektrik V_0. Ruang di antara kedua petala telah diisi dengan cas di mana ketumpatan casnya adalah $\rho = \rho_0 r^2$ (ρ_0 adalah pemalar).]

(a) Using the Poisson's equation, calculate the electric potential, V, produced in the space between the two conductors.
[Dengan menggunakan persamaan Poisson, hitung keupayaan elektrik, V, yang terhasil di ruang antara kedua konduktor.]

(b) What is the surface charge density on each conductor?
[Apakah ketumpatan permukaan cas bebas di tiap-tiap konduktor?]

(20/100)

4. Using the Bio-Savart's law, calculate the magnetic filed **B** at point P due to the flow of current I in the wire with configuration as shown in Figure 1 below.
*[Dengan menggunakan hukum Biot-Savart, hitung medan magnet **B** di titik P akibat dari pengaliran arus I pada dawai yang berkonfigurasi seperti di Rajah 1 di bawah.]*

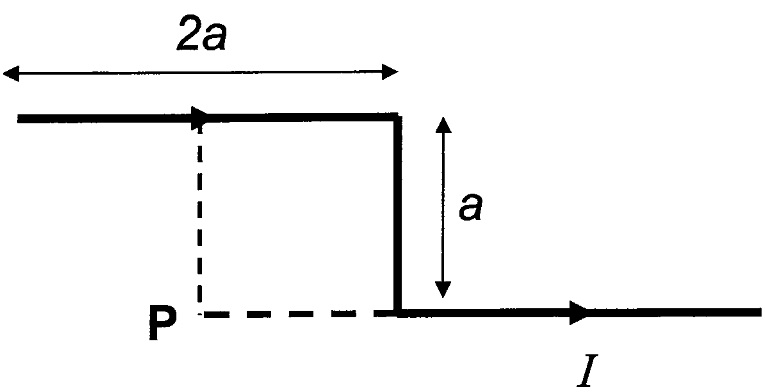

Figure 1 *[Rajah 1]*

(20/100)

5. (a) Prove that a magnetic field produced by a perfect solenoid is $B_z = \mu_0 nI$ where n is the number of turns per meter.
[Buktikan bahawa medan magnet yang dihasilkan oleh satu solenoid unggul adalah $B_z = \mu_0 nI$ di mana n adalah bilangan lilitan per meter.]

(b) A long solenoid of length L with N number of turns and of radius a contains within it a magnetic medium with relative magnetic permeability constant μ_l. See Figure 2.
[Satu solenoid sepanjang L dengan bilangan lilitan N dan berjejari a mengandungi bahan magnet dengan pemalar relatif ketelapan magnet μ_l di bahagian dalamnya. Lihat Rajah 2.]

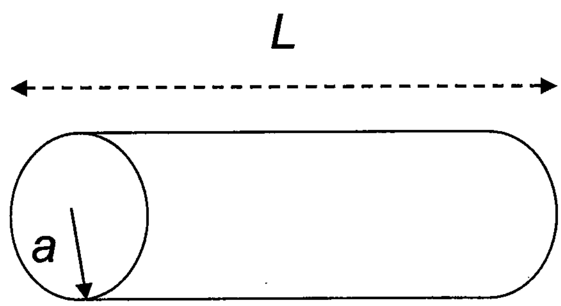

Figure 2 *[Rajah 2]*

(i) Obtain the magnetic vector potential **A** in the regions internal and external to the solenoid: $r < a$ and $r \geq a$.
*[Dapatkan vektor keupayaan magnet **A** di bahagian dalam dan luar solenoid: $r < a$ dan $r \geq a$.]*

(ii) Find **H**, and **M**, for $r < a$. Then calculate $\vec{\lambda}_e$ and $\vec{J}_e$.
*[Cari **H**, dan **M**, bagi $r < a$. Kemudian hitung $\vec{\lambda}_e$ dan $\vec{J}_e$.]*

(20/100)

Vector Derivatives

Cartesian Coordinates

$d\boldsymbol{\ell} = \hat{\mathbf{i}}\,dx + \hat{\mathbf{j}}\,dy + \hat{\mathbf{k}}\,dz\,, \qquad dV = dx\,dy\,dz$

$$\nabla f = \hat{\mathbf{i}}\frac{\partial f}{\partial x} + \hat{\mathbf{j}}\frac{\partial f}{\partial y} + \hat{\mathbf{k}}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{\mathbf{i}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{j}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{k}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinates

$d\boldsymbol{\ell} = \hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}\,r\,d\phi + \hat{\mathbf{k}}\,dz\,, \qquad dV = r\,dr\,d\phi\,dz$

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial f}{\partial \phi} + \hat{\mathbf{k}}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{k}}\left[\frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates

$d\boldsymbol{\ell} = \hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\theta}}\,r\,d\theta + \hat{\boldsymbol{\phi}}\,r\sin\theta\,d\phi\,, \qquad dV = r^2\sin\theta\,dr\,d\theta\,d\phi$

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{\hat{\mathbf{r}}}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, A_\phi) - \frac{\partial A_\theta}{\partial \phi}\right] + \frac{\hat{\boldsymbol{\theta}}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\boldsymbol{\phi}}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Vector Formulas

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) = (\mathbf{C}\times\mathbf{A})\cdot\mathbf{B} = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\,(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}\,(\mathbf{A}\cdot\mathbf{B})$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$$

Derivatives of Sums

$$\nabla(f+g) = \nabla f + \nabla g$$

$$\nabla\cdot(\mathbf{A}+\mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B}$$

$$\nabla\times(\mathbf{A}+\mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B}$$

Derivatives of Products

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$$

$$\nabla\cdot(f\mathbf{A}) = f\,(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

$$\nabla\times(f\mathbf{A}) = f\,(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,(\nabla\cdot\mathbf{B}) - \mathbf{B}\,(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$$

Second Derivatives

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\nabla\times(\nabla f) = 0$$

Integral Theorems

$$\int_V (\nabla\cdot\mathbf{A})\,dV = \oint_S \mathbf{A}\cdot\hat{\mathbf{n}}\,dS$$ Gauss's (divergence) Theorem

$$\int_S (\nabla\times\mathbf{A})\cdot\hat{\mathbf{n}}\,dS = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell}$$ Stokes's (curl) Theorem

$$\int_{\mathrm{a}}^{\mathrm{b}} (\nabla f)\cdot d\boldsymbol{\ell} = f(\mathrm{b}) - f(\mathrm{a})$$

$$\int_V \left(f\nabla^2 g - g\nabla^2 f\right) dV = \oint_S (f\nabla g - g\nabla f)\cdot\hat{\mathbf{n}}\,dS$$ Green's Theorem

Physical Constants

$c = 2.998 \times 10^8$ m/s — Speed of light

$\mu_0 = 4\pi \times 10^{-7}$ N/A^2 (or H/m) — Permeability constant in vacuum

$\epsilon_0 = \dfrac{1}{\mu_0 c^2} = 8.854 \times 10^{-12}$ C^2/Nm^2 (or F/m) — Permittivity constant in vacuum

$\dfrac{1}{4\pi\epsilon_0} = 10^{-7}c^2 = 8.988 \times 10^9$ Nm^2/C^2

$e = 1.602 \times 10^{-19}$ C — Magnitude of electron charge

$m_e = 0.9109 \times 10^{-30}$ kg — Electron mass

Useful Integrals

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x + \sqrt{a^2+x^2}\right)$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}$$

$$\int \frac{dx}{\left(a^2+x^2\right)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

Binomial Expansion

$$(1+\epsilon)^p = 1 + p\epsilon + \frac{p(p-1)}{2!}\epsilon^2 + \frac{p(p-1)(p-2)}{3!}\epsilon^3 + \ldots$$

Notation for Position Vector

$$\mathbf{x} = \hat{\mathbf{i}}\,x + \hat{\mathbf{j}}\,y + \hat{\mathbf{k}}\,z$$

$$r = |\mathbf{x}| = \sqrt{x^2+y^2+z^2} \quad \text{and} \quad \hat{\mathbf{r}} = \frac{\mathbf{x}}{r}$$

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