UNIVERSITI SAINS MALAYSIA

KSCP Examination
Academic Session 2007/2008

June 2008

ZCT 211/2 – Vector Analysis
[Analisis Vektor]

Duration: 2 hours
[Masa : 2 jam]

Please ensure that this examination paper contains **FIVE** printed pages before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer **ALL FIVE (5)** questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**Arahan**: Jawab **SEMUA LIMA (5)** yang diberikan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

.../2-

1. (a) $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$ and $\mathbf{B} = B_1\mathbf{i} - B_2\mathbf{j} + B_3\mathbf{k}$, prove that $\mathbf{A.B} = A_1B_1 + A_2B_2 + A_3B_3$.
[$\boldsymbol{A} = A_1\boldsymbol{i} + A_2\boldsymbol{j} + A_3\boldsymbol{k}$ and $\boldsymbol{B} = B_1\boldsymbol{i} - B_2\boldsymbol{j} + B_3\boldsymbol{k}$, buktikan bahawa $\boldsymbol{A.B} = A_1B_1 + A_2B_2 + A_3B_3$.]

(20/100)

(b) Find the angle between $\mathbf{A} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$
[Dapatkan sudut di antara $\boldsymbol{A} = 2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{B} = 6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$]

(20/100)

(c) If $\mathbf{A.B} = 0$ and if A and B are not zero, show that **A** is perpendicular to **B**.
*[Jika $\boldsymbol{A.B} = 0$ dan jika A dan B bukan sifar, tunjukkan yang **A** adalah tegak lurus kepada **B**.]*

(30/100)

(d) Determine the value of a so that $\mathbf{A} = 2\mathbf{i} + a\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ are perpendicular.
[Tentukan nilai bagi a supaya $\boldsymbol{A} = 2\boldsymbol{i} + a\boldsymbol{j} + \boldsymbol{k}$ dan $\boldsymbol{B} = 4\boldsymbol{i} - 2\boldsymbol{j} - 2\boldsymbol{k}$ adalah tegak lurus.]

(30/100)

2. (a) If $\mathbf{A} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{C} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, show that
$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \neq (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$.
[Jika $\boldsymbol{A} = \boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{B} = 2\boldsymbol{i} - 5\boldsymbol{j} - \boldsymbol{k}$ dan $\boldsymbol{C} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, tunjukkan $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) \neq (\boldsymbol{A} \times \boldsymbol{B}) \times \boldsymbol{C}$.]

(30/100)

(b) If $\mathbf{A} = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{B} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and that $\mathbf{C} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ determine $\mathbf{A}.(\mathbf{B} \times \mathbf{C})$
[Jika $\boldsymbol{A} = 5\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ dan $\boldsymbol{B} = 3\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ dan $\boldsymbol{C} = \boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$ hitungkan $\boldsymbol{A}.(\boldsymbol{B} \times \boldsymbol{C})$]

(30/100)

(c) If $\mathbf{A} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{B} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ **and** $\mathbf{C} = \mathbf{i} + p\mathbf{j} + 4\mathbf{k}$ are coplanar find the value of p.
[Jika $\boldsymbol{A} = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ *dan* $\boldsymbol{B} = 3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ *dan* $\boldsymbol{C} = \boldsymbol{i} + p\boldsymbol{j} + 4\boldsymbol{k}$ *adalah koplanar hitung nilai p.]*

(40/100)

3. (a) If $\mathbf{A} = (u + 3)\,\mathbf{i} - (2 + u^2)\,\mathbf{j} + 2u^3\mathbf{k}$ determine
[Jika $\mathbf{A} = (u + 3)\,\mathbf{i} - (2 + u^2)\,\mathbf{j} + 2u^3\mathbf{k}$ *tentukan]*

(i) $\dfrac{dA}{du}$

(ii) $\dfrac{d^2A}{du^2}$

(iii) $\left|\dfrac{dA}{du}\right|$

(iv) $\left|\dfrac{d^2A}{du^2}\right|$

(40/100)

(b) A particle moves in space so that at time t its position is stated as $x = 2t + 3$, $y = t^2 + 3t$, $z = t^3 + 2t^2$. Find the components of its velocity and acceleration in the direction of the vector $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ when $t = 1$.
[Suatu partikel bergerak dalam ruang supaya pada masa t kedudukannya dinyatakan sebagai $x = 2t + 3$, $y = t^2 + 3t$, $z = t^3 + 2t^2$. *Carikan komponen-komponen halaju dan pecutannya dalam arah vector* $2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ *bila* $t = 1$.*]*

(60/100)

4. (a) Determine the unit tangent vector for the curve $x = 3t$, $y = 2t^2$, $z = t^2 + t$ at the point (6, 8, 6).
[Tentukan vector unit tangen untuk lengkung $x = 3t$, $y = 2t^2$, $z = t^2 + t$ pada titik (6, 8, 6).]

(40/100)

(b) If $\mathbf{F} = \mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ where $\mathbf{A} = 3t^2\,\mathbf{i} + (2t - 3)\,\mathbf{j} + 4t\,\mathbf{k}$

$\mathbf{B} = 2\mathbf{i} + 4t\mathbf{j} + 3(1-t)\,\mathbf{k}$ and $\mathbf{C} = 2t\mathbf{i} + (3t^2\mathbf{j} - 2t\mathbf{k}$ determine $\int_0^1 \bar{F}dt$

[Jika $\boldsymbol{F} = \boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C})$ di mana $\boldsymbol{A} = 3t^2\boldsymbol{i} + (2t - 3)\,\boldsymbol{j} + 4t\,\boldsymbol{k}$

$\boldsymbol{B} = 2\boldsymbol{i} + 4t\boldsymbol{j} + 3(1-t)\,\boldsymbol{k}$ dan $\boldsymbol{C} = 2t\boldsymbol{i} + (3t^2\boldsymbol{j} - 2t\boldsymbol{k}$ tentukan $\int_0^1 \bar{F}dt$]

(60/100)

5. (a) If $\varphi = x^2y^2 + x^3yz - yz^2$ and $\mathbf{F} = xy^2\mathbf{i} - 2yz\mathbf{j} + xyz\mathbf{k}$ determine for the point P(1, -1, 2),
[Jika $\varphi = x^2y^2 + x^3yz - yz^2$ dan $\boldsymbol{F} = xy^2\mathbf{i} - 2yz\mathbf{j} + xyz\mathbf{k}$ tentukan untuk titik P(1, -1, 2),]

(i) $\nabla\varphi$

(ii) unit normal *[unit tegak lurus]*

(iii) $\nabla \cdot \bar{F}$

(iv) $\nabla \times \bar{F}$

(30/100)

(b) If $\mathbf{F}= x^2y^2\mathbf{i} - y^3z\,\mathbf{j} + z^2\mathbf{k}$ evaluate $\int_c \vec{F}\cdot dr$ along the curve $x = 2u^2$, $y = 3u$, $z = u^3$ between A(2,-3,-1) and B(2,3,1).

[Jika $\mathbf{F} = x^2y^2\mathbf{i} - y^3z\,\mathbf{j} + z^2\mathbf{k}$ *hitungkan* $\int_c \vec{F}\cdot dr$ *di sepanjang lengkung* $x = 2u^2$, $y = 3u$, $z = u^3$ *di antara A(2, -3, -1) dan B(2, 3, 1).]*

(30/100)

(c) Evaluate $\int_c \vec{F}\cdot dV$ where V is the region bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y= 3$, $z = 0$, $z = 4$ and $\mathbf{F} = xy\mathbf{i} - z\,\mathbf{j} + x^2\,\mathbf{k}$

[Hitungkan $\int_c \vec{F}\cdot dV$ *di mana V adalah kawasan dikelilingi oleh satah-satah* $x = 0$, $x = 2$, $y = 0$, $y= 3$, $z = 0$, $z = 4$ *dan* $\mathbf{F} = xy\boldsymbol{i} - z\,\boldsymbol{j} + x^2\,\boldsymbol{k}$*]*

(40/100)

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