
UNIVERSITI SAINS MALAYSIA

KSCP Examination
Academic Session 2007/2008

June 2008

ZCA 110/4 – Calculus and Linear Algebra
[Kalkulus dan Aljabar Linear]

Duration: 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **FOUR** printed pages before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instruction: Answer **ALL** questions in Section A and Section B. Students are allowed to answer all questions in Bahasa Malaysia or in English.

Arahan: Jawab **SEMUA** soalan dalam Seksyen A dan Seksyen B. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]

SECTION A

1.

(a)(i). Find the asymptotes of the graph of $f(x) = \frac{x^2-3}{2x-4}$.

[Dapatkan asimtot-asimtot graf $f(x) = \frac{x^2-3}{2x-4}$.]

(a)(ii). Sketch the curve of $f(x)$, indicating clearly the asymptotes on your sketch.

[Lakarkan lengkung $f(x)$. Tandakan dengan jelas asimtot-asimtot pada lakaran anda.]

(b). Find the area of the region bounded by the curve $y = xe^{-1}$ and the x -axis from $x = 0$ to $x = 4$.

[Hitungkan luas rantau yang dibatasi lengkungan $y = xe^{-1}$ dan paksi- x dari $x = 0$ ke $x = 4$.]

(17 Markah)

2.

(a). A particle moves along the x -axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1} \sqrt{t}$. What is the velocity of the particle when $t=16$? (Hint:

$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$, where u is a differentiable function of x).

[Suatu zarah bergerak sepanjang paksi- x supaya kedudukannya pada bila-bila masa $t \geq 0$ diberikan oleh $x(t) = \tan^{-1} \sqrt{t}$. Apakah kelajuan zarah bila $t = 16$? Diberi bahawa $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$, dengan u fungsi terbezakan x .]

(b). Sketch the curves of $y = \operatorname{sech}^{-1}x$ and $y = \operatorname{sech} x$ on the same graph. Indicating clearly the ranges and domains of each curve on your sketch.

[Lakarkan lengkungan-lengkungan $y = \operatorname{sech}^{-1}x$ dan $y = \operatorname{sech} x$ pada graf yang sama. Tandakan dengan jelas julat and domain setiap lengkungan pada lakaran

anda.]

(17 Markah)

3.

(a). Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 8$.

[Dapatkan $\frac{d^2y}{dx^2}$ jika $2x^3 - 3y^2 = 8$.]

(b). Evaluate [Nilaikan] (i) $\int \frac{dx}{x(x^2+1)^2}$, (ii) $\int \frac{6x+7}{(x+2)^2} dx$.

(16 Markah)

SECTION B

4.

X_Z and X_W are the coordinates relative to the following pair of bases;

[X_Z dan X_W adalah koordinat-koordinat terhadap sepasang basis seperti berikut;]

$$Z_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad Z_3 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \quad W_3 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix},$$

Relative to the e -basis, $X = [2, 5, 7]^T$.

[Relatif kepada basis- e , $X = [2, 5, 7]^T$.]

A linear transformation on X_Z , relative to the Z -basis is

[Suatu transformasi linear ke atas X_Z , relatif ke basis- Z ialah]

$$Y_Z = AX_Z = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} X_Z,$$

Find:

[Cari:]

(a) the adjoint of the matrices Z and W ,

[adjoint bagi matriks Z dan W ,]

(b) the determinant of the matrices Z and W ,

[determinan bagi matriks Z dan W ,]

(c) the inverse of the matrices Z and W ,

[songsang bagi matriks Z dan W ,]

(d) the coordinates of the matrices Z and W ,

[koordinat-koordinat X_Z dan X_W ,]

(e) the matrix P such that $X_W = PX_Z$,

[matriks P supaya $X_W = PX_Z$,]

(f) the image Y_Z of X_Z under the above linear transformation $Y_W = BX_W$ relative to the W -basis

[imej Y_Z bagi X_Z di bawah transformasi linear di atas,]

(g) the matrix B of the above same transformation

[matriks B bagi transformasi yang sama di atas $Y_W = BX_W$ relatif ke basis- W .]

(25 Markah)

5.

State the definition of Fourier Series of a function $f(x)$ over the interval $-L < x < L$ and find the Fourier Series expansion of the following functions.

[Nyatakan takrif Siri Fourier bagi suatu fungsi $f(x)$ atas selang $-L < x < L$ dan cari perkembangan Siri Fourier bagi fungsi-fungsi berikut.]

$$(a) \quad f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi. \end{cases}$$

$$(b) \quad f(x) = \begin{cases} 1, & -2 < x < 0 \\ 1+x, & 0 < x < 2. \end{cases}$$

(25 Markah)