UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2004/2005

Mac 2005

MSG 367 – ANALISIS SIRI MASA

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **ENAM BELAS [16]** halaman yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab semua **EMPAT [4]** soalan.

1\. a) Beri definasi bagi proses hangar putih dan bincangkan, mungkin dengan menggunakan beberapa contoh, mengapa hangar putih dan sifat-sifatnya kerapkali digunakan sebagai perbandingan kepada proses-proses lain dan juga sebagai hipotesis nol dalam banyak ujian-ujian siri masa.

[30 markah]

b) Huraikan dengan ringkas beberapa penyemakan diagnostic yang boleh dilakukan bagi menyelidik sama ada model siri masa yang telah disuaikan mencukupi dan signifikan.

[30 markah]

c) Tulis semula model-model berikut menggunakan pengoperasi anjak kebelakang B dan nyatakan bentuk ARKPB(p,d,q) atau Bermusim ARKPB$(p,d,q)(P,D,Q)$.

(i) $Y_t = \mu(1+\phi_1+\phi_2-\phi_3)-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}+\phi_3 Y_{t-3}+\varepsilon_t-\theta_1\varepsilon_{t-1}$

(ii) $Y_t = (1+\phi_1)Y_{t-1}+(\phi_2-\phi_1)Y_{t-2}-\phi_2 Y_{t-3}+\varepsilon_t-\theta_1\varepsilon_{t-1}-\theta_2\varepsilon_{t-2}$

(iii) $Y_t = -\phi_1 Y_{t-1}+\varepsilon_t-(\theta_1+\theta_2)\varepsilon_{t-1}+\theta_1\theta_2\varepsilon_{t-2}$

(iv) $Y_t = \phi_1 Y_{t-12}+\varepsilon_t+\theta_1\varepsilon_{t-1}$

[20 markah]

d) Terangkan

(i) logiknya melakukan pembezaan biasa dan bermusim. Bila pembezaan bermusim dilakukan?

(ii) bagaimana fak bagi ralat berbeza daripada fak teranggar yang dikira daripada siri masa pegun.

[20 markah]

1. a) Give the definition of white noise process and discuss, perhaps through examples, why the white noise and its properties is often used as comparison to other processes and also as a null hypothesis in many time series tests.

[30 marks]

b) Briefly explain a few diagnostic checking that can be carried out to investigate the adequacy and significance of a fitted time series model.

[30 marks]

c) Rewrite each of the models below using the backward operator B and state the form of ARIMA(p,d,q) *or* SARIMA$(p,d,q)(P,D,Q)$

(i) $Y_t = \mu(1+\phi_1+\phi_2-\phi_3)-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}+\phi_3 Y_{t-3}+\varepsilon_t-\theta_1\varepsilon_{t-1}$

(ii) $Y_t = (1+\phi_1)Y_{t-1}+(\phi_2-\phi_1)Y_{t-2}-\phi_2 Y_{t-3}+\varepsilon_t-\theta_1\varepsilon_{t-1}-\theta_2\varepsilon_{t-2}$

(iii) $Y_t = -\phi_1 Y_{t-1}+\varepsilon_t-(\theta_1+\theta_2)\varepsilon_{t-1}+\theta_1\theta_2\varepsilon_{t-2}$

(iv) $Y_t = \phi_1 Y_{t-12}+\varepsilon_t+\theta_1\varepsilon_{t-1}$

[20 marks]

d) *Explain*

(i) *the logic behind normal and seasonal differencing. When is seasonal differencing performed?*

(ii) *how does a residual acf differ from an estimated acf calculated from a stationary time series*

[20 marks]

2\. a) (i) Dapatkan selang bagi α supaya proses AR(2)

$$Y_t = Y_{t-1} + \frac{\alpha}{2} Y_{t-2} + \varepsilon_t$$

adalah pegun.

(ii) Diberi bahawa Y_t dihasilkan daripada proses purata bergerak

$$Y_t = \varepsilon_t + E\varepsilon_{t-1} + \ldots + E\varepsilon_1, \text{ untuk } t \geq 1$$

dengan E adalah pemalar. Cari min dan kovarians bagi Y_t. Adakah ia proses pegun?

[20 markah]

b) Dapatkan suatu rumus am fungsi autokovarians, fungsi autokorelasi dan fungsi autokorelasi separa bagi proses ARPB(3,2) seperti diberi di bawah

$$\left(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3\right) Y_t = \left(1 - \theta_1 B - \theta_2 B^2\right) \varepsilon_t$$

Satu sample siri masa sebanyak 530 cerapan telah diperoleh dan sedang dipertimbangkan bagi model ARPB(3,2) dengan koefisien-koefisien $\phi_1 = 0.7$, $\phi_2 = 0.42$, $\phi_3 = -0.216$, $\theta_1 = -0.10$, $\theta_2 = 0.2475$. Hitung autokorelasi untuk susulan k = 1, 2, 3, 4, 5 dan autokorelasi separa untuk susulan k = 1 dan 2. Komen corak yang diperoleh. Adakah fungsi autokorelasi dan fungsi autokorelasi separa mencadangkan satu model ARPB(p,q)? [Diberi nilai fungsi autokorelasi pada susulan 6 hingga 8 masing-masing 0.501, 0.412 dan 0.363, dan nilai autokorelasi separa susulan 3 hingga 5 masing-masing 0.03, -0.02 dan 0.04].

[50 markah]

c) Diandaikan bahawa model ARPB(3,2) di atas mempunyai lebihan parameter, suatu model yang lebih ringkas sedang dipertimbangkan dan terutamanya oleh sebab interpretasi yang lebih mudah, suatu model AR diberi kelebihan berbanding model PB. Tunjukkan bahawa model ARPB(3,2) di atas boleh ditulis sebagai

$$\left(1 - 0.35B - 0.5325B^2 - 0.0236B^3 - 0.0107B^4 \ldots\right) Y_t = \left(1 + 0.55B\right) \varepsilon_t$$

atau

$$\left(1 - 0.9B - 0.0375B^2 - \ldots\right) Y_t = \varepsilon_t$$

yang masing-masing lebih hampir kepada model ARPB(2,1) dan AR(1). Bincang dengan memberikan beberapa alasan model yang manakah yang lebih sesuai bagi siri masa yang dicerap.

[30 markah]

2. a) (i) *Find the range of* α *such that the* AR(2) *process*

$$Y_t = Y_{t-1} + \frac{\alpha}{2} Y_{t-2} + \varepsilon_t$$

is stationary.

(ii) *Suppose that* Y_t *is generated according to moving average process*

$$Y_t = \varepsilon_t + E\varepsilon_{t-1} + \ldots + E\varepsilon_1, \text{ for } t \geq 1,$$

where E is a constant. Find the mean and covariance for Y_t. *Is the process stationary?*

[20 marks]

b) *Find a general formula for autocovariance, autocorrelation and partial autocorrelation functions for an* ARMA(3,2) *process as given below*

$$\left(1-\phi_1 B-\phi_2 B^2-\phi_3 B^3\right) Y_t=\left(1-\theta_1 B-\theta_2 B^2\right) \varepsilon_t$$

A time series sample of 530 *observations has been collected and is being considered for an* ARMA(3,2) *model with the following coefficients* $\phi_1 = 0.7$, $\phi_2 = 0.42$, $\phi_3 = -0.216$, $\theta_1 = -0.10$, $\theta_2 = 0.2475$. *Calculate the autocorrelation for* k = 1, 2, 3, 4, 5, *and partial autocorrelation for* k = 1 *and* 2. *Comment on the pattern observed. Does the acf and pacf suggest an* ARMA*(p,q) model?. [Given the values of acf at lag* 6 *through to* 8 *are* 0.501, 0.412 *and* 0.363 *respectively, and pacf at lag* 3 *through to* 5 *are* 0.03, -0.02 *and* 0.04 *respectively].*

[50 marks]

c) *Assuming an* ARMA(3,2) *model above is overparameterized, a simpler model is being considered and in particular due to its simpler interpretation, an* AR *model has a higher priority than a* MA *model. Show that the* ARMA(3,2) *model above can be rewritten as*

$$\left(1-0.35B-0.5325B^2-0.0236B^3-0.0107B^4\ldots\right)Y_t=\left(1+0.55B\right)\varepsilon_t$$

or

$$\left(1-0.9B-0.0375B^2-\ldots\right)Y_t=\varepsilon_t$$

which can be approximated as an ARMA(2,1) *and an* AR(1) *model respectively. Discuss with reasons which of these two alternative models is more appropriate for the time series collected.*

[30 marks]

3. a) Diberi suatu proses AR(2)

$$Y_t - \mu = \mu(\phi_1 - \phi_2) + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

dengan ε_t adalah proses hingar putih yang tidak bersandar dan secaman dengan min sifar dan varians σ_ε^2. Tunjukkan bahawa penganggar kuasa dua terkecil bagi ϕ_1 dan ϕ_2 adalah seperti berikut

$$\hat{\phi}_1 = \frac{\hat{\rho}_1(1-\hat{\rho}_2)}{1-\hat{\rho}_1^2} \qquad \hat{\phi}_2 = \frac{\hat{\rho}_2(1-\hat{\rho}_1)}{1-\hat{\rho}_1^2}$$

[30 markah]

b) (i) Diketahui bahawa penganggar bagi koefisien yang diperoleh di bahagian (a) adalah sama bagi suatu siri masa dengan min sifar yang mengikuti proses AR(2)

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

Tunjukkan bahawa varians bagi penganggar koefisien-koefisien mempunyai taburan asimptot yang diberi oleh

$$Var\begin{bmatrix}\hat{\phi}_1 \\ \hat{\phi}_2\end{bmatrix} \;\tilde{a}\; \frac{1}{N}\begin{bmatrix}1-\phi_2^2 & -\phi_1(1+\phi_2) \\ -\phi_1(1+\phi_2) & 1-\phi_2^2\end{bmatrix}$$

(ii) Suatu siri masa 400 cerapan dengan min sifar menghasilkan sampel autokovarians seperti berikut

$$\hat{\gamma}_0 = 1382.2 \quad \hat{\gamma}_1 = 1114.4 \quad \hat{\gamma}_2 = 591.72 \quad \hat{\gamma}_3 = 96.215$$

Dapatkan penganggar kuasa dua terkecil bagi koefisien-koefisien model AR(2) yang telah disuaikan kepada siri masa tersebut bersama dengan variansnya. Dapatkan selang keyakinan 95% bagi koefisien-koefisien. Juga beri komen terhadap kestabilan penganggar koefisien-koefisien tersebut.

[40 markah]

c) 500 cerapan telah diperoleh daripada Indek Komposit Kuala Lumpur bermula 19hb Mac 1997. Plot siri masa indek tersebut dan juga peratusan pulangan adalah seperti di Lampiran A. Lampiran B menunjukkan fungsi autokorelasi (fak) dan fungsi autokorelasi separa (faks) bagi siri indeks dan siri pulangan. Seorang pelajar telah cuba menyuaikan satu model siri masa bagi siri pulangan dan telah menggunakan model PB(1). Output bagi model PB(1) yang tersuai, analisis ralat menggunakan fak dan faks, penghasilan model baru dan analisis ralat bagi model baru tersebut adalah seperti di Lampiran C. pelajar tersebut berpuashati dengan model terakhir yang diperoleh iaitu, ARPB(0,4)-GARCH(1,1) seperti di Lampiran D. Menggunakan maklumat yang diberi dalam Lampiran A hingga D, bincangkan setiap langkah yang telah diambil oleh pelajar tersebut, terutamanya beri alasan bagi memilih model GARCH berbanding model PB(4).

[30 markah]

3. a) *Given an* AR(2) *process*

$$Y_t - \mu = \mu(\phi_1 - \phi_2) + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

with ε_t *is an independent and identically distributed noise process with mean zero and variance* σ_ε^2. *Show that the least squares estimates of* ϕ_1 *and* ϕ_2 *are given by*

$$\hat{\phi}_1 = \frac{\hat{\rho}_1(1-\hat{\rho}_2)}{1-\hat{\rho}_1^2} \qquad \hat{\phi}_2 = \frac{\hat{\rho}_2(1-\hat{\rho}_1)}{1-\hat{\rho}_1^2}$$

[30 marks]

b) (i) *Note that the estimates for the coefficients found in (a) remain the same for a zero-mean time series that follows an* AR(2) *process*

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

Show that the variance of the estimated coefficients has an asymptotic distribution as given by

$$Var\begin{bmatrix}\hat{\phi}_1 \\ \hat{\phi}_2\end{bmatrix} \; ã \; \frac{1}{N}\begin{bmatrix}1-\phi_2^2 & -\phi_1(1+\phi_2) \\ -\phi_1(1+\phi_2) & 1-\phi_2^2\end{bmatrix}$$

(ii) *A particular zero-mean time series of* 400 *observations produce the following sample autocovariances*

$$\hat{\gamma}_0 = 1382.2 \quad \hat{\gamma}_1 = 1114.4 \quad \hat{\gamma}_2 = 591.72 \quad \hat{\gamma}_3 = 96.215$$

Determine the least squares estimates for the coefficients of an AR(2) *model fitted to the series together with the corresponding variance. Find the* 95% *confidence interval for the coefficients. Also comment on the stability of the estimated coefficients.*

[40 marks]

c) 500 *observations of the Kuala Lumpur Composite Index starting from* 19th *March* 1997 *has been collected. Time series plots of the index and its percentage returns are shown in Appendix A. Appendix B shows the acf and pacf of the index and returns series. A student have attempted to fit a ime series model to the return series and has done so with a* MA(1) *model. Output of the estimated* MA(1) *model, residuals analysis using acf and pacf, derivation of a new model and its residual analysis are shown is Appendix C. The student is satisfied with the final model for the return series, that is* ARMA(0,4)-GARCH(1,1) *as shown in Appendix D. Using the information provided in Appendix A through to D, discuss each of the steps taken by the students, in particular give reasons for preferences of the* GARCH *model over the* MA(4) *model.*

[30 marks]

4. a) 200 cerapan siri masa dengan min bukan sifar telah disuaikan dengan model ARPB(1,3)

$$\left(1-\phi_1 B\right)\left(Y_t-\mu\right)=\left(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3\right)\varepsilon_t$$

$\{\varepsilon_t\}$ adalah suatu proses hangar putih dengan min 0 dan varians σ_ε^2.

Tunjukkan bahawa telahan 1-langkah kehadapan yang dilakukan pada $t = N$ diberi oleh

$$\hat{Y}_N(1)=\mu\left(1-\phi_1\right)+\phi_1 Y_N-\theta_1\varepsilon_N-\theta_2\varepsilon_{N-1}-\theta_3\varepsilon_{N-2}$$

dan tunjukkan juga bahawa telahan m-langkah kehadapan diberi oleh rumus

$$\hat{Y}_t(m)=\mu\left(1-\phi_1\right)+\phi_1\hat{Y}_t(m-1) \quad \text{untuk } m\geq 4$$

[20 markah]

b) (i) Jika nilai-nilai anggaran bagi koefisien-koefisien adalah $\hat{\phi}_1=0.8$, $\theta_1=0.5$, $\theta_2=0.29$, $\theta_2=-0.105$, $\hat{\mu}=200$, $s_\varepsilon^2=9$ dengan $Y_{200}=192$, $Y_{199}=199$, $\varepsilon_{199}=4$, $\varepsilon_{198}=-8$ dan $\varepsilon_{197}=16$, dapatkan nilai $\hat{Y}_{200}(m)$ bagi m = 1, 2, ..., 6. Bina selang telahan 95% bagi Y_{201}, ..., Y_{204}. Komen 6 nilai ramalan yang diperoleh. Apakah nilai berkemungkinan bagi nilai telahan pada t = 300 dan nilai sepadan selang telahan 95%, beri penjelasan.

[40 markah]

(ii) Sekarang diketahui pula bahawa $Y_{201}=221$, hitung nilai ramalan kemaskini bagi $Y_{202},\ldots Y_{206}$. Bandingkan nilai ramalan terkini dengan nilai di bahagian (i) dan beri penjelasan. Pada t = 207, telah diperoleh cerapan dari t = 201 hingga t = 206 masing-masing adalah 221, 203, 189, 191, 218 dan 189. Bincangkan keupayaan model ARPB(1,3) ini dalam menghasilkan telahan untuk nilai-nilai akan datang.

[20 markah]

c) Bincang secara ringkas sebab-sebab bagi model siri masa di bawah disuaikan kepada siri masa tertentu

(i) Bermusim ARKPB$(p,0,0)(0,1,Q)_{12}$

(ii) ARIMA(p,1,q)-GARCH(1,1)

Pertimbangkan model (i) di atas dengan $p = Q = 1$. Cari rumus umum bagi telahan m-langkah kehadapan.

[20 markah]

4. a) 200 observations time series with non-zero mean has been modeled as an ARMA(1,3) *model*

$$(1-\phi_1 B)(Y_t-\mu)=(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)\varepsilon_t$$

$\{\varepsilon_t\}$ *is a white noise process with mean 0 and variance* σ_ε^2

Show that the 1*-step ahead forecast made at time* $t = N$ *is given by*

$$\hat{Y}_N(1)=\mu(1-\phi_1)+\phi_1 Y_N-\theta_1\varepsilon_N-\theta_2\varepsilon_{N-1}-\theta_3\varepsilon_{N-2}$$

and also show that the m-step-ahead forecast is given by

$$\hat{Y}_t(m)=\mu(1-\phi_1)+\phi_1\hat{Y}_t(m-1) \quad \text{for } m\ge 4$$

[20 marks]

b) (i) If estimated values for the coefficients are $\hat{\phi}_1=0.8$, $\theta_1=0.5$, $\theta_2=0.29$, $\theta_3=-0.105$, $\hat{\mu}=200$, $s_\varepsilon^2=9$ *with* $Y_{200}=192$, $Y_{199}=199$, $\varepsilon_{199}=4$, $\varepsilon_{198}=-8$ *and* $\varepsilon_{197}=16$, *obtain value of* $\hat{Y}_{200}(m)$ *for m* = 1, 2, ..., 6. *Construct a* 95% *forecast interval for* Y_{201}, ..., Y_{204}. *Comment on the* 6 *forecast values obtained above. What is the likely value for the forecast at time t* = 300 *and its corresponding* 95% *forecast interval, give explanation.*

[40 marks]

(ii) It is now observed that $Y_{201}=221$, *calculate the updated values for* $Y_{202},\ldots Y_{206}$. *Compare these new forecasts with those calculated in (i) and discuss. At time* $t = 207$, *it was gathered that the observations from time t* = 201 *through to t* = 206 *are* 221, 203, 189, 191, 218 *and* 189 *respectively. Discuss on the ability of this* ARMA(1,3) *model in making forecast of future values.*

[20 marks]

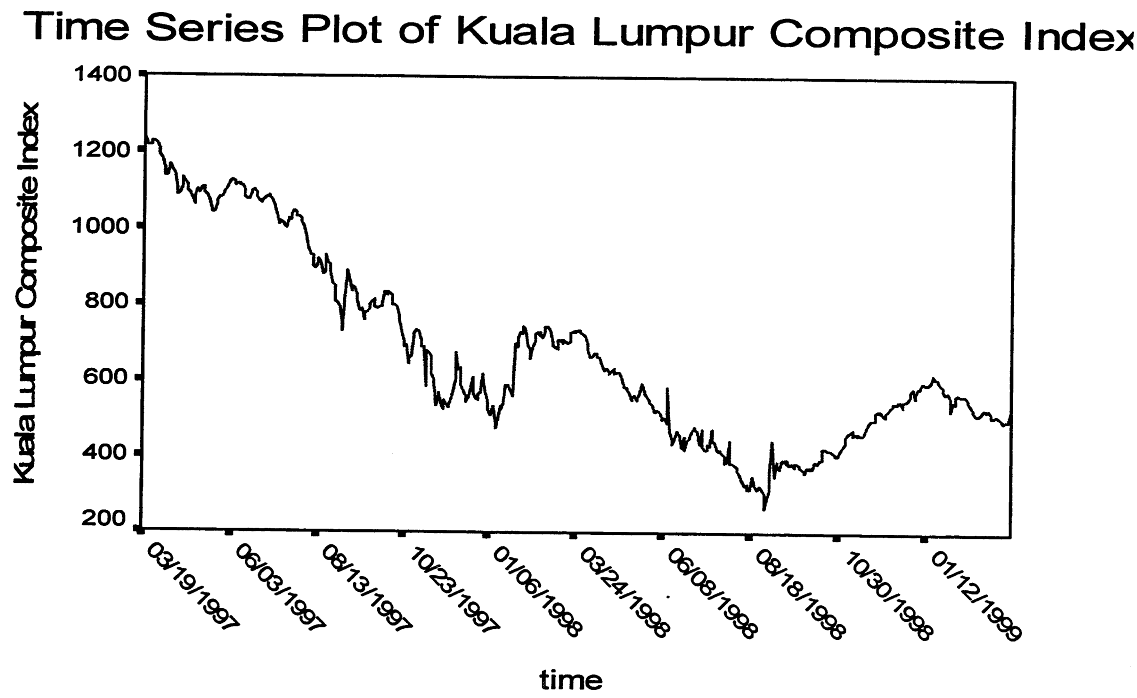
c) *Discuss briefly the circumstances for the following type of time series models to be fitted to a certain time series*

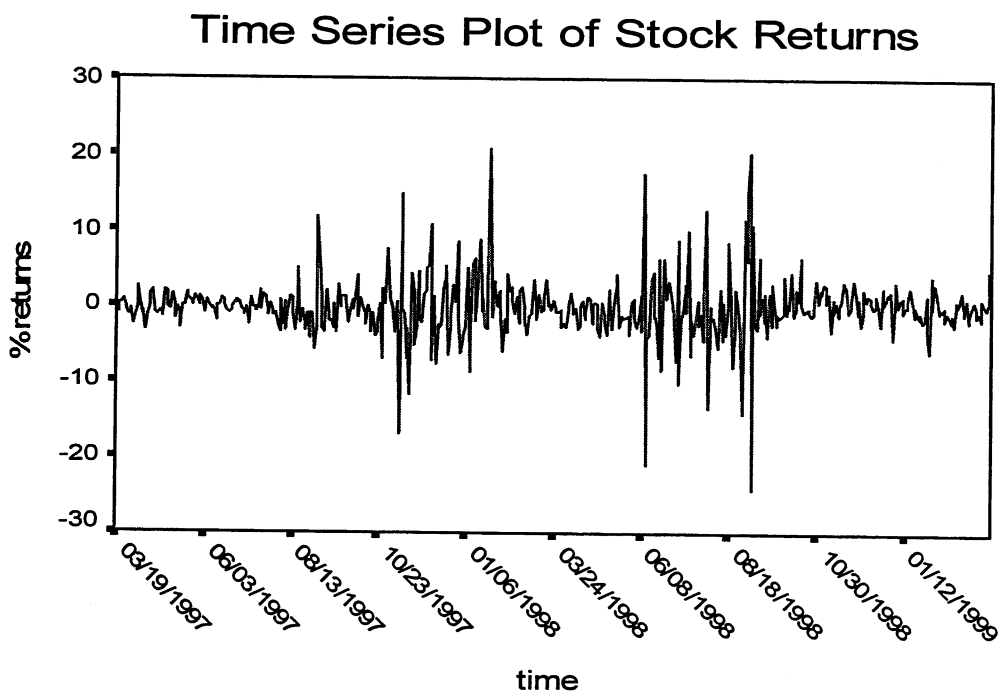
(i) SARIMA$(p,0,0)(0,1,Q)_{12}$
(ii) ARIMA$(p,1,q)$-GARCH$(1,1)$

Consider a specific model (i) above with $p = Q = 1$. *Find the general formula for m-step ahead forecast.*

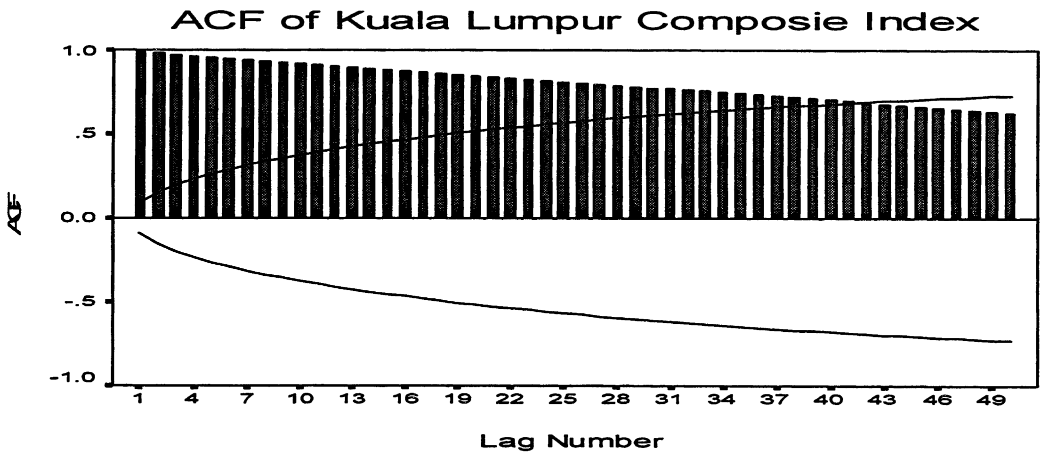
[20 marks]

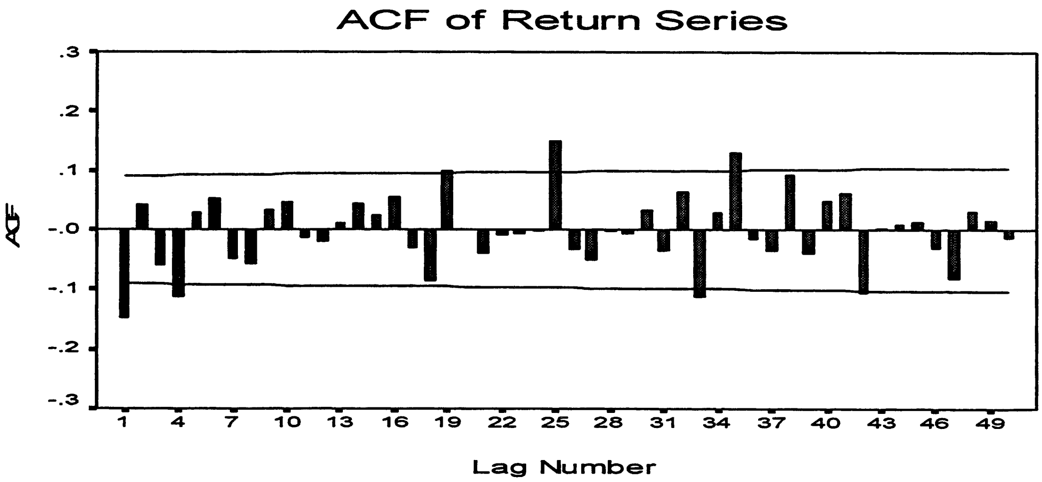
APPENDIX/LAMPIRAN A

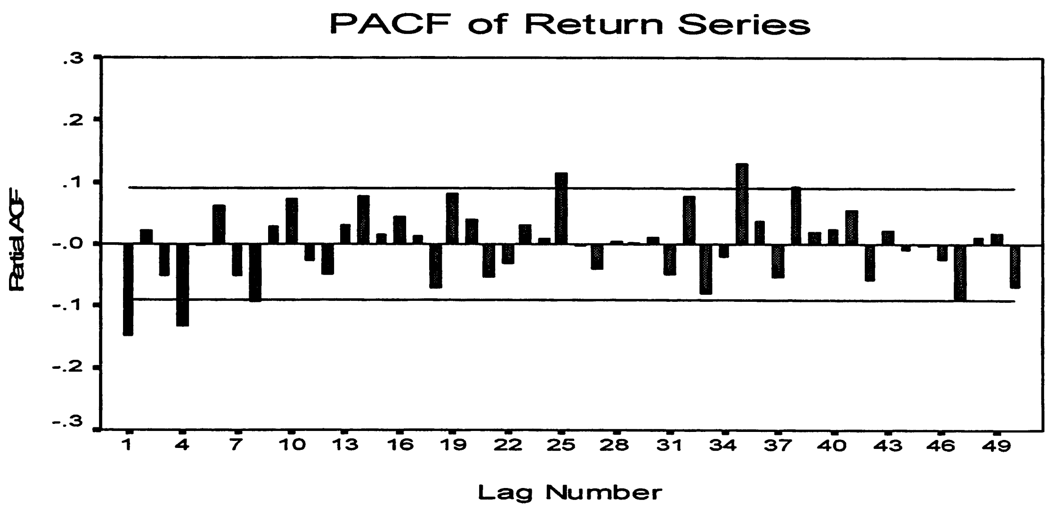




APPENDIX/LAMPIRAN B



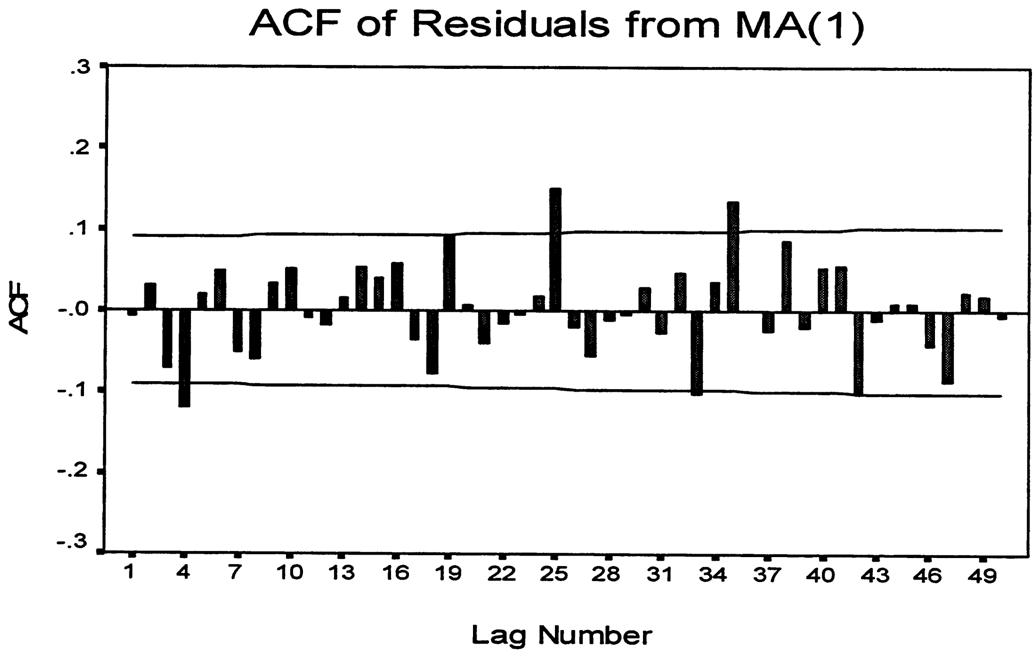


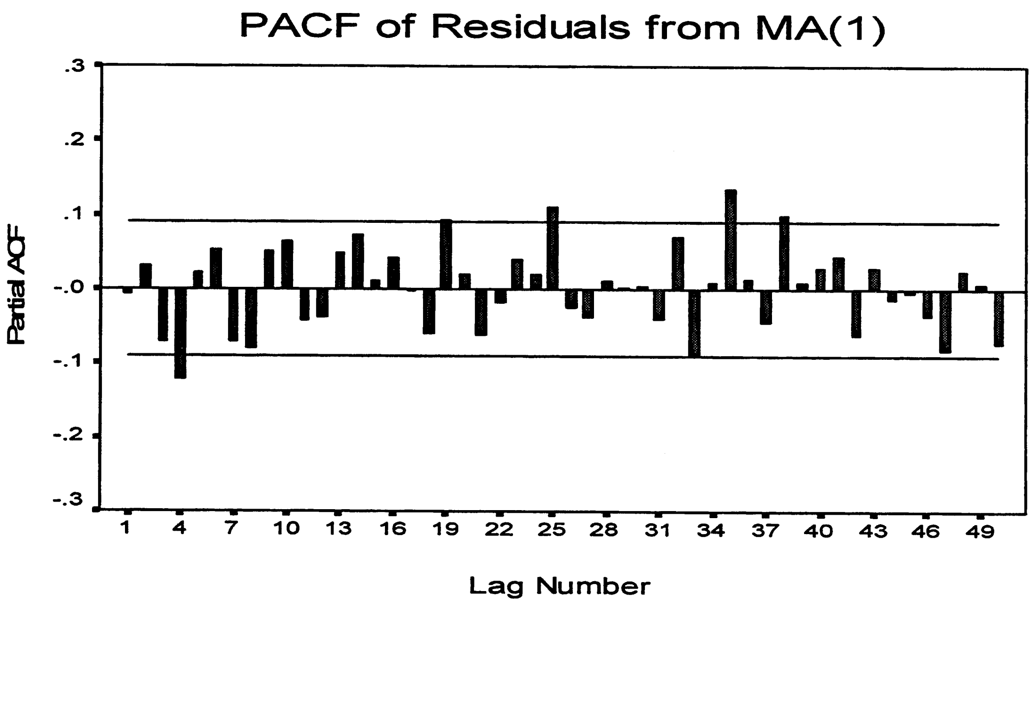


APPENDIX/LAMPIRAN C

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.1404	0.0444	-3.16	0.0017
R-squared	0.0183	Mean dependent var		-0.1654
Adjusted R-squared	0.0183	S.D. dependent var		3.8787
S.E. of regression	3.8429	Akaike info criterion		5.5323
Sum squared resid	7369.3	Schwarz criterion		5.5408
Log likelihood	-1382.1	Durbin-Watson stat		2.0049
Q-stats	16.883	30.215	61.904	80.489
Prob	0.111	0.143	0.003	0.002

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	-0.1453	0.0444	-3.28	0.0011
R-squared	0.0193	Mean dependent var		-0.1643
Adjusted R-squared	0.0193	S.D. dependent var		3.8825
S.E. of regression	3.8448	Akaike info criterion		5.5333
Sum squared resid	7361.6	Schwarz criterion		5.5417
Log likelihood	-1379.6	Durbin-Watson stat		1.9921
Q-stats	17.33	31.17	63.639	82.538
Prob	0.098	0.119	0.002	0.001


ACF of Residuals from MA(1)
.3
.2
.1
-.0
-.1
-.2
-.3
ACF
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49
Lag Number

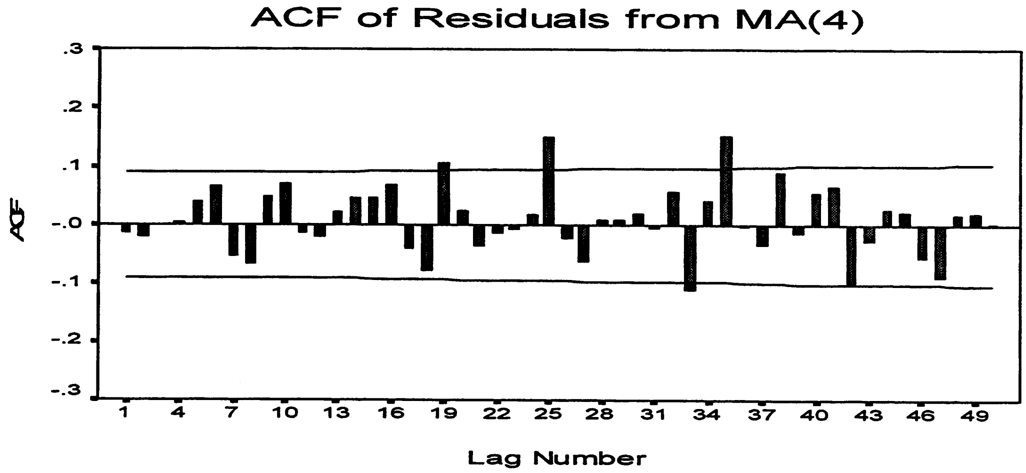

PACF of Residuals from MA(1)
.3
.2
.1
-.0
-.1
-.2
-.3
Partial ACF
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49
Lag Number

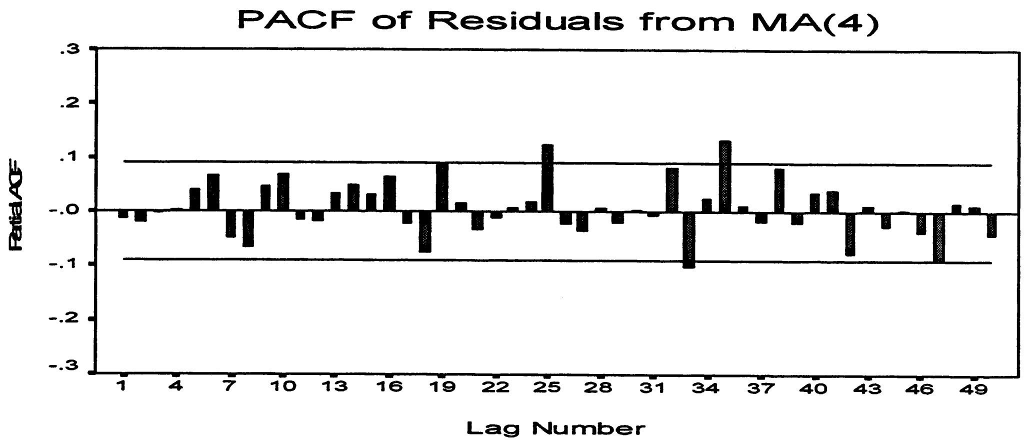
Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.1527	0.0449	-3.40	0.0007
MA(2)	0.0739	0.0449	1.64	0.1007
MA(3)	-0.0832	0.0449	-1.85	0.0645
MA(4)	-0.1527	0.0450	-3.40	0.0007
MA(5)	0.0632	0.0450	1.41	0.1603
R-squared	0.0461	Mean dependent var		-0.1654
Adjusted R-squared	0.0384	S.D. dependent var		3.8787
S.E. of regression	3.8035	Akaike info criterion		5.5197
Sum squared resid	7160.9	Schwarz criterion		5.5618
Log likelihood	-1374.9	Durbin-Watson stat		1.9896

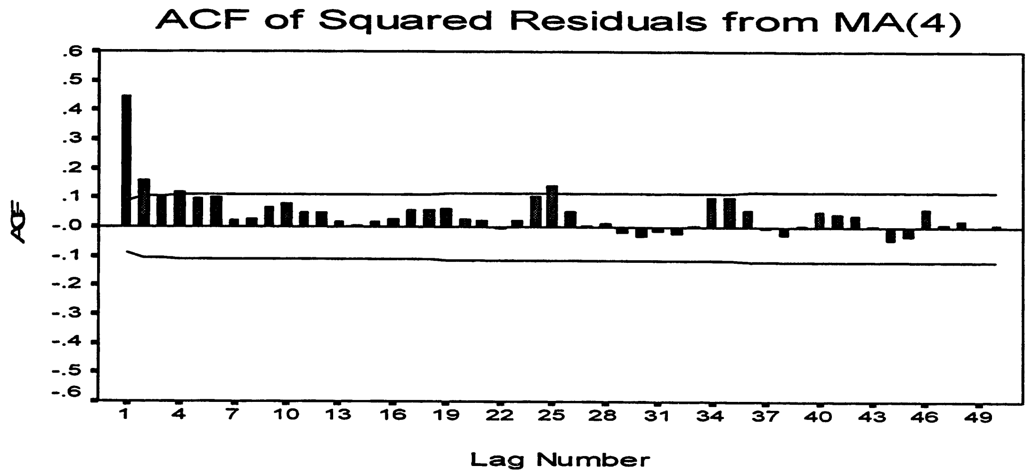
Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.1351	0.0445	-3.03	0.0025
MA(2)	0.0697	0.0448	1.55	0.1206
MA(3)	-0.0894	0.0448	-1.99	0.0466
MA(4)	-0.1357	0.0446	-3.04	0.0025
R-squared	0.0423	Mean dependent var		-0.1654
Adjusted R-squared	0.0365	S.D. dependent var		3.8787
S.E. of regression	3.8073	Akaike info criterion		5.5197
Sum squared resid	7189.7	Schwarz criterion		5.5534
Log likelihood	-1375.9	Durbin-Watson stat		2.0160
Q-stats	11.06	27.17	63.67	85.56
Prob	0.198	0.130	0.001	0.001
Q-stats(res-sq)	143.53	156.17	182.59	190.72
Prob	0.000	0.000	0.000	0.000

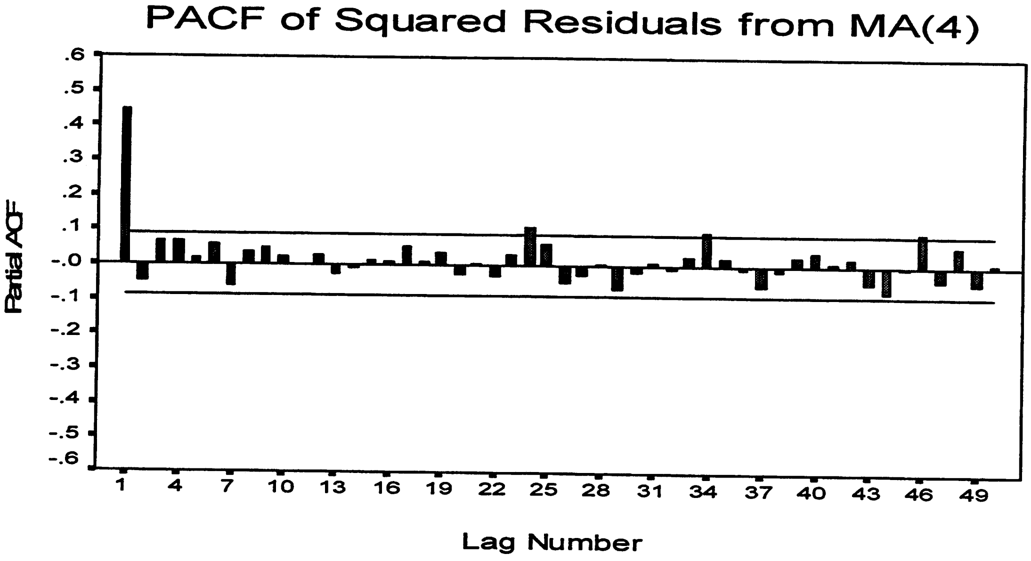
...15/-

APPENDIX/LAMPIRAN D









Variable	Coefficient	Std. Error	z-Statistic	Prob.
MA(1)	0.0563	0.0561	1.00	0.3155
MA(2)	0.0729	0.0467	1.56	0.1186
MA(3)	-0.0120	0.0499	-0.24	0.8091
MA(4)	-0.1221	0.0482	-2.53	0.0113
	Variance Equation			
C	0.1066	0.0391	2.7266	0.0064
ARCH(1)	0.2394	0.0368	6.5017	0.0000
GARCH(1)	0.8097	0.0203	39.916	0.0000
R-squared	0.0014	Mean dependent var		-0.1654
Adjusted R-squared	-0.0108	S.D. dependent var		3.8787
S.E. of regression	3.8995	Akaike info criterion		4.9763
Sum squared resid	7496.6	Schwarz criterion		5.0353
Log likelihood	-1237.1	Durbin-Watson stat		2.3822
Q-stats	6.39	27.31	40.89	59.13
Prob	0.603	0.127	0.135	0.063
Q-stats(res-sq)	6.19	8.99	18.28	23.932
Prob	0.625	0.983	0.975	0.994

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