

UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2004/2005

Oktober 2004

MSG 366 – ANALISIS MULTIVARIAT

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH EMPAT [24]** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **semua empat** soalan.

...2/-

1. (a) Katakan $\mathbf{X}' = (X_1, X_2, X_3, X_4)$ tertabur $N_4(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ di mana

$$\boldsymbol{\mu}' = [2, -3, 1, -2] \quad \text{dan} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 3 & 2 \\ 1 & 3 & 2 & 4 \\ 1 & 2 & 4 & 5 \end{bmatrix}.$$

Cari taburan bagi $5X_1 - 4X_2 + 3X_3 - 2X_4$.

- (b) Biarkan \mathbf{X} tertabur $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ dengan $\boldsymbol{\mu}' = [2, -1, 1]$ dan

$$\boldsymbol{\Sigma} = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}.$$

Bagi setiap pembolehubah rawak berikut, tentukan sama ada ianya bersandar atau tak bersandar? Huraikan.

- (i) X_1 dan X_2
 - (ii) X_1 dan X_3
 - (iii) X_2 dan X_3
 - (iv) (X_1, X_3) dan X_2
 - (v) X_1 dan $X_1 + 2X_2 - 3X_3$
- (c) Merujuk kepada bahagian (b), dapatkan taburan berikut:
- (i) Taburan bersyarat X_1 diberikan $X_3 = x_3$
 - (ii) Taburan bersyarat X_1 diberikan $X_2 = x_2$ dan $X_3 = x_3$
- (d) Biarkan matriks

$$\mathbf{A} = \begin{bmatrix} 3 & -2 \\ -2 & 6 \end{bmatrix}.$$

- (i) Adakah \mathbf{A} bersimetri?
- (ii) Adakah \mathbf{A} tentu positif?
- (iii) Dapatkan pasangan nilai eigen – vektor eigen \mathbf{A} .

[100 markah]

2. (a) Pertimbangkan matriks data

$$\mathbf{X} = \begin{bmatrix} 9 & 1 \\ 5 & 3 \\ 1 & 2 \end{bmatrix}$$

Terdapat $n = 3$ cerapan pada $p = 2$ pembolehubah X_1 dan X_2 . Bentukkan gabungan-gabungan linear

$$\mathbf{c}'\mathbf{X} = [-1 \quad 2] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -X_1 + 2X_2$$

$$\mathbf{b}'\mathbf{X} = [2 \quad 3] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 2X_1 + 3X_2$$

- (i) Dapatkan min-min sampel, varians dan kovarians bagi $\mathbf{b}'\mathbf{X}$ dan $\mathbf{c}'\mathbf{X}$ dari prinsip-prinsip asas. (Petunjuk: kira nilai-nilai yang dicerapi bagi $\mathbf{b}'\mathbf{X}$ dan $\mathbf{c}'\mathbf{X}$, dan kemudian gunakan rumus-rumus min sampel, varians dan kovarians.)
- (ii) Dapatkan min-min sampel, varians dan kovarians bagi $\mathbf{b}'\mathbf{X}$ dan $\mathbf{c}'\mathbf{X}$ dengan menggunakan $\mathbf{b}'\bar{\mathbf{x}}$, $\mathbf{c}'\bar{\mathbf{x}}$, $\mathbf{b}'\mathbf{S}\mathbf{b}$, $\mathbf{c}'\mathbf{S}\mathbf{c}$, dan $\mathbf{b}'\mathbf{S}\mathbf{c}$, masing-masing. Bandingkan keputusan-keputusan dalam (i) dan (ii).
- (b) Cerapan-cerapan pada dua balasan, X_1 dan X_2 , dikutip bagi tiga rawatan. Vektor-vektor cerapan $\mathbf{x}' = (x_1 \quad x_2)$.

$$\text{Rawatan 1 : } [7 \quad 6], [9 \quad 5]$$

$$\text{Rawatan 2 : } [3 \quad 3], [6 \quad 1]$$

$$\text{Rawatan 3 : } [3 \quad 2], [1 \quad 5]$$

- (i) Binakan suatu jadual MANOVA satu hala bagi data ini.
- (ii) Nilaikan Lambda Wilks dan ujikan bagi kesan rawatan. Gunakan $\alpha = 0.01$.
- (iii) Ulangkan ujian dengan menggunakan penghampiran khi-kuasa dua dengan pembetulan Bartlett. Bandingkan keputusan-keputusan.

[100 markah]

3. (a) (i) Dengan menggunakan data

$$\mathbf{X} = \begin{bmatrix} 9 & 6 \\ 6 & 10 \\ 3 & 8 \end{bmatrix}$$

nilaikan T^2 Hotelling untuk menguji $H_0: \mu' = (5, 9)$.

- (ii) Nyatakan taburan T^2 bagi keadaan dalam (i) di atas. Nyata andaian-andaian yang telah anda gunakan.
- (iii) Dengan menggunakan (i) dan (ii) di atas, ujikan H_0 pada paras $\alpha = 0.01$. Apakah kesimpulan anda?

- (b) Tuliskan nota pendek tentang tajuk-tajuk di bawah:

- (i) MANOVA
 (ii) Analisis faktor
 (iii) Analisis pembezaan
 (iv) Analisis kelompok

[100 markah]

4. Satu soalselidik dikendalikan atas pelanggan-pelanggan syarikat 'Hair, Anderson dan Tatham Company' (HATCO), sebuah pembekal industri (*industrial supplier*). Sebuah firma penyelidikan pemasaran mengutip data soalselidik HATCO. Pangkalan datanya terdiri daripada 100 cerapan bagi setiap 14 pembolehubah. Tiga jenis maklumat dikutip. Jenis yang pertama ialah **persepsi pelanggan terhadap HATCO** ke atas tujuh atribut yang dicamkan dalam kajian-kajian lepas sebagai atribut yang sangat mempengaruhi pilihan pembekal. Responden-responden, iaitu pengurus-pengurus pembelian (*purchasing managers*) bagi firma-firma yang membeli dari HATCO, menilai HATCO ke atas setiap atribut. Maklumat jenis yang kedua berkenaan dengan **keputusan pembelian** yang sebenar, sama ada penilaian kepuasan dengan HATCO bagi setiap responden atau peratusan pembelian produk responden dari HATCO. Maklumat jenis yang ketiga mengandungi **ciri am syarikat pembelian** (misalnya, saiz firma, jenis industri).

Data yang dikutip harus memberi HATCO suatu pemahaman yang baik tentang ciri-ciri pelanggannya dan perhubungan antara persepsi pelanggan terhadap HATCO dan tindakan pelanggan terhadap HATCO (pembelian dan kepuasan). Takrif bagi setiap pembolehubah dan penerangan kodnya diberi dalam bahagian-bahagian yang berikut.

I. Persepsi Pelanggan

Setiap pembolehubah disukat atas satu skala grafik, yang mana satu garis 10 sentimeter dilukis antara titik dilabel 'Lemah' dan titik dilabel 'Cemerlang'. Responden menunjukkan persepsinya dengan membuat satu tanda pada mana-mana garis itu. Tanda disukat dan jarak dari 0 (dalam sentimeter) direkod. Keputusan adalah satu skala dari 0 hingga 10, dibundarkan kepada satu tempat perpuhuan. Tujuh atribut HATCO yang dinilai oleh setiap responden adalah seperti berikut:

- X_1 : Kelajuan penghantaran (*Delivery speed*) – amaun masa yang diambil untuk menghantar produk setelah permintaan disahkan.
- X_2 : Paras harga (*Price level*) – persepsi paras harga yang ditetapkan oleh pembekal-pembekal produk.
- X_3 : Fleksibiliti harga (*Price flexibility*) – persepsi kesediaan wakil-wakil HATCO untuk merunding harga pada semua jenis pembelian.
- X_4 : Imej pengeluar (*Manufacturer's image*) – imej keseluruhan pengeluar atau pembekal.
- X_5 : Perkhidmatan keseluruhan (*Overall service*) – paras keseluruhan perkhidmatan yang diperlukan untuk menjaga perhubungan yang memuaskan antara pembekal dan pembeli.
- X_6 : Imej pasukan jualan (*Salesforce image*) – imej keseluruhan bagi pasukan jualan pengeluar.
- X_7 : Kualiti produk (*Product quality*) – persepsi paras kualiti bagi suatu produk (misalnya, prestasi atau hasil).

II. Keputusan Pembelian

- X_8 : Parasgunaan (*Usage level*) – berapa banyak jumlah produk dibeli dari HATCO, disukat pada satu skala peratusan 100-titik, berjulat dari 0 hingga 100 peratus.
- X_{10} : Paras kepuasan (*Satisfaction level*) – betapa puasnya pembeli dengan pembelian lalu dari HATCO, disukat pada skala penilaian grafik yang sama seperti persepsi X_1 hingga X_7 .

III. Ciri-ciri Pembeli

Lima ciri bagi firma-firma yang digunakan dalam kajian ini adalah seperti berikut:

- X_8 : Saiz firma (*Size of firm*) – saiz firma relatif kepada firma lain dalam pasaran ini. Pembolehubah ini mempunyai dua kategori: 1 = besar, 0 = kecil.
- X_{11} : Pembelian spesifikasi (*Specification buying*) – sejauh manakah seorang pembeli menilai setiap pembelian secara berasingan (analisis jumlah nilai) melawan penggunaan pembelian spesifikasi, yang menerangkan dengan terperinci ciri-ciri produk yang diinginkan. Pembolehubah ini mempunyai dua kategori: 1 = menggunakan pendekatan analisis jumlah nilai, menilai setiap pembelian secara berasingan; 0 = menggunakan pembelian spesifikasi.
- X_{12} : Struktur perolehan (*Structure of procurement*) – kaedah memperoleh atau membeli produk dalam sebuah syarikat. Pembolehubah ini mempunyai dua kategori: 1 = perolehan memusat, 0 = perolehan tidak memusat.
- X_{13} : Jenis industri (*Type of industry*) – klasifikasi industri bagi seorang pembeli produk. Pembolehubah ini mempunyai dua kategori: 1 = industri A, 0 = industri lain.
- X_{14} : Jenis situasi membeli (*Type of buying situation*) – jenis situasi yang dihadapi pembeli. Pembolehubah ini mempunyai tiga kategori: 1 = tugas baru, 2 = pembelian semula terubahsuai, 3 = pembelian semula secara terus.

Berbagai teknik multivariat digunakan untuk menganalisa data kajian ini. Lampiran 4.1 memaparkan sebahagian data yang dikutip. Berikut adalah output-output dari perisian statistik (Sila rujuk kepada Lampiran 4.2, 4.3 dan 4.4.). Huraikan tujuan penggunaan teknik multivariat dalam setiap output yang dipaparkan dan tafsirkan keputusan-keputusan yang diperoleh. Apakah kesimpulan-kesimpulan anda?

[100 markah]

LAMPIRAN 4.1

	x1	x2	x3	x4	x5	x6	x7
1	4.1	.6	6.9	4.7	2.4	2.3	5.2
2	1.8	3.0	6.3	6.6	2.5	4.0	8.4
3	3.4	5.2	5.7	6.0	4.3	2.7	8.2
4	2.7	1.0	7.1	5.9	1.8	2.3	7.8
5	6.0	.9	9.6	7.8	3.4	4.6	4.5
6	1.9	3.3	7.9	4.8	2.6	1.9	9.7
7	4.6	2.4	9.5	6.6	3.5	4.5	7.6
8	1.3	4.2	6.2	5.1	2.8	2.2	6.9
9	5.5	1.6	9.4	4.7	3.5	3.0	7.6
10	4.0	3.5	6.5	6.0	3.7	3.2	8.7
11	2.4	1.6	8.8	4.8	2.0	2.8	5.8
12	3.9	2.2	9.1	4.6	3.0	2.5	8.3
13	2.8	1.4	8.1	3.8	2.1	1.4	6.6
14	3.7	1.5	8.6	5.7	2.7	3.7	6.7
15	4.7	1.3	9.9	6.7	3.0	2.6	6.8
16	3.4	2.0	9.7	4.7	2.7	1.7	4.8
17	3.2	4.1	5.7	5.1	3.6	2.9	6.2
18	4.9	1.8	7.7	4.3	3.4	1.5	5.9
19	5.3	1.4	9.7	6.1	3.3	3.9	6.8
20	4.7	1.3	9.9	6.7	3.0	2.6	6.8
21	3.3	.9	8.6	4.0	2.1	1.8	6.3
22	3.4	.4	8.3	2.5	1.2	1.7	5.2
23	3.0	4.0	9.1	7.1	3.5	3.4	8.4
24	2.4	1.5	6.7	4.8	1.9	2.5	7.2
25	5.1	1.4	8.7	4.8	3.3	2.6	3.8
26	4.6	2.1	7.9	5.8	3.4	2.8	4.7
27	2.4	1.5	6.6	4.8	1.9	2.5	7.2
28	5.2	1.3	9.7	6.1	3.2	3.9	6.7
29	3.5	2.8	9.9	3.5	3.1	1.7	5.4
30	4.1	3.7	5.9	5.5	3.9	3.0	8.4
31	3.0	3.2	6.0	5.3	3.1	3.0	8.0
32	2.8	3.8	8.9	6.9	3.3	3.2	8.2
33	5.2	2.0	9.3	5.9	3.7	2.4	4.6
34	3.4	3.7	6.4	5.7	3.5	3.4	8.4
35	2.4	1.0	7.7	3.4	1.7	1.1	6.2
36	1.8	3.3	7.5	4.5	2.5	2.4	7.6
37	3.6	4.0	5.8	5.8	3.7	2.5	9.3
38	4.0	.9	9.1	5.4	2.4	2.6	7.3
39	.0	2.1	6.9	5.4	1.1	2.6	8.9

	x8	x9	x10	x11	x12	x13	x14
1	0	32.0	4.2	1	0	1	1
2	1	43.0	4.3	0	1	0	1
3	1	48.0	5.2	0	1	1	2
4	1	32.0	3.9	0	1	1	1
5	0	58.0	6.8	1	0	1	3
6	1	45.0	4.4	0	1	1	2
7	0	46.0	5.8	1	0	1	1
8	1	44.0	4.3	0	1	0	2
9	0	63.0	5.4	1	0	1	3
10	1	54.0	5.4	0	1	0	2
11	0	32.0	4.3	1	0	0	1
12	0	47.0	5.0	1	0	1	2
13	1	39.0	4.4	0	1	0	1
14	0	38.0	5.0	1	0	1	1
15	0	54.0	5.9	1	0	0	3
16	0	49.0	4.7	1	0	0	3
17	0	38.0	4.4	1	1	1	2
18	0	40.0	5.6	1	0	0	2
19	0	54.0	5.9	1	0	1	3
20	0	55.0	6.0	1	0	0	3
21	0	41.0	4.5	1	0	0	2
22	0	35.0	3.3	1	0	0	1
23	0	55.0	5.2	1	1	0	3
24	1	36.0	3.7	0	1	0	1
25	0	49.0	4.9	1	0	0	2
26	0	49.0	5.9	1	0	1	3
27	1	36.0	3.7	0	1	0	1
28	0	54.0	5.8	1	0	1	3
29	0	49.0	5.4	1	0	1	3
30	1	46.0	5.1	0	1	0	2
31	1	43.0	3.3	0	1	0	1
32	0	53.0	5.0	1	1	0	3
33	0	60.0	6.1	1	0	0	3
34	1	47.3	3.8	0	1	0	1
35	1	35.0	4.1	0	1	0	1
36	1	39.0	3.6	0	1	1	1
37	1	44.0	4.8	0	1	1	2
38	0	46.0	5.1	1	0	1	3
39	1	29.0	3.9	0	1	1	1

LAMPIRAN 4.2

Factor Analysis

Correlation Matrix

		Delivery Speed	Price Level	Price Flexibility	Manufacturer Image	Service	Salesforce Image	Product Quality
Correlation	Delivery Speed	1.000	-.349	.509	.050	.612	.077	-.483
	Price Level	-.349	1.000	-.487	.272	.513	.186	.470
	Price Flexibility	.509	-.487	1.000	-.116	.067	-.034	-.448
	Manufacturer Image	.050	.272	-.116	1.000	.299	.788	.200
	Service	.612	.513	.067	.299	1.000	.241	-.055
	Salesforce Image	.077	.186	-.034	.788	.241	1.000	.177
	Product Quality	-.483	.470	-.448	.200	-.055	.177	1.000
Sig. (1-tailed)	Delivery Speed		.000	.000	.309	.000	.223	.000
	Price Level	.000		.000	.003	.000	.032	.000
	Price Flexibility	.000	.000		.125	.255	.367	.000
	Manufacturer Image	.309	.003	.125		.001	.000	.023
	Service	.000	.000	.255	.001		.008	.293
	Salesforce Image	.223	.032	.367	.000	.008		.039
	Product Quality	.000	.000	.000	.023	.293	.039	

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.446
Bartlett's Test of Sphericity	Approx. Chi-Square	567.541
	df	21
	Sig.	.000

Communalities

	Initial	Extraction
Delivery Speed	1.000	.884
Price Level	1.000	.895
Price Flexibility	1.000	.649
Manufacturer Image	1.000	.885
Service	1.000	.995
Salesforce Image	1.000	.901
Product Quality	1.000	.618

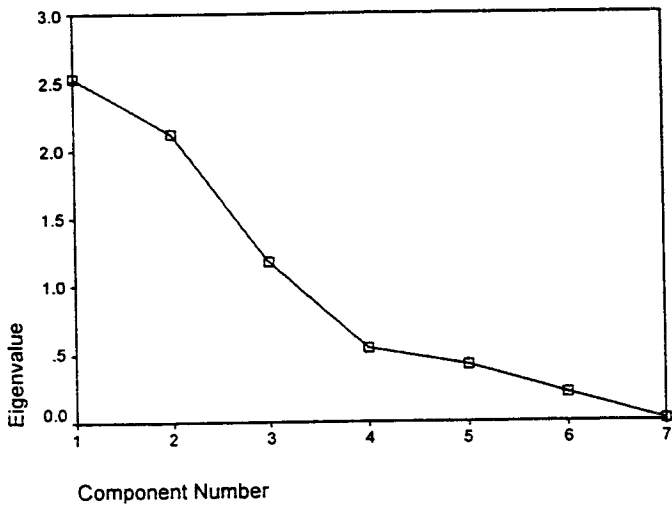
Extraction Method: Principal Component Analysis.

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squar	
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance
1	2.526	36.082	36.082	2.526	36.082	36.082	2.379	33.984
2	2.120	30.291	66.374	2.120	30.291	66.374	1.827	26.098
3	1.181	16.873	83.246	1.181	16.873	83.246	1.622	23.165
4	.541	7.731	90.977					
5	.418	5.972	96.949					
6	.204	2.920	99.869					
7	9.161E-03	.131	100.000					

Extraction Method: Principal Component Analysis.

Scree Plot



Component Matrix^a

	Component		
	1	2	3
Delivery Speed	-.528	.752	.202
Price Level	.792	9.306E-02	.508
Price Flexibility	-.692	.374	-.173
Manufacturer Image	.564	.602	-.452
Service	.186	.779	.595
Salesforce Image	.492	.604	-.542
Product Quality	.739	-.270	-5.43E-03

Extraction Method: Principal Component Analysis.

a. 3 components extracted.

Rotated Component Matrix

	Component		
	1	2	3
Delivery Speed	-.752	7.112E-02	.560
Price Level	.754	.108	.561
Price Flexibility	-.806	6.298E-03	9.531E-03
Manufacturer Image	.117	.921	.153
Service	-6.20E-02	.176	.980
Salesforce Image	3.413E-02	.945	7.656E-02
Product Quality	.760	.193	-6.44E-02

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 5 iterations.

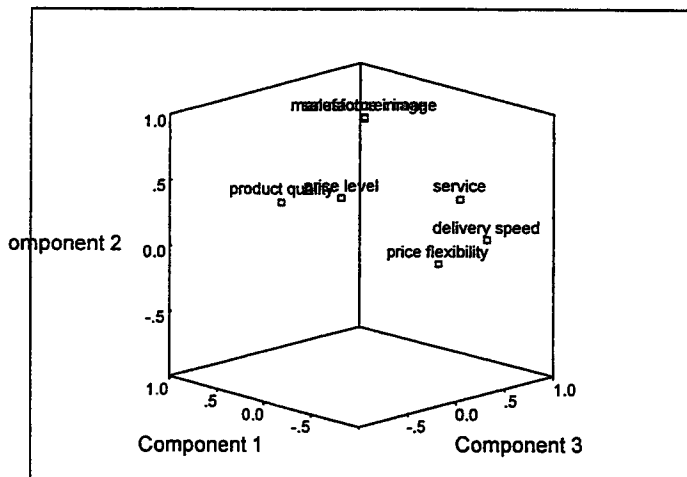
Component Transformation Matrix

Component	1	2	3
1	.865	.477	.159
2	-.452	.602	.658
3	.218	-.641	.736

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

Component Plot in Rotated Space



LAMPIRAN 4.3

Discriminant

Analysis Case Processing Summary

Unweighted Cases		N	Percent
Valid		100	100.0
Excluded	Missing or out-of-range group codes	0	.0
	At least one missing discriminating variable	0	.0
	Both missing or out-of-range group codes and at least one missing discriminating variable	0	.0
	Total	0	.0
Total		100	100.0

Group Statistics

		Valid N (listwise)	
		Unweighted	Weighted
Articulation Of Needs			
Specification Buying	Delivery Speed	40	40.000
	Price Level	40	40.000
	Price Flexibility	40	40.000
	Manufacturer Image	40	40.000
	Service	40	40.000
	Salesforce Image	40	40.000
	Product Quality	40	40.000
Total Value Analysis	Delivery Speed	60	60.000
	Price Level	60	60.000
	Price Flexibility	60	60.000
	Manufacturer Image	60	60.000
	Service	60	60.000
	Salesforce Image	60	60.000
	Product Quality	60	60.000
Total	Delivery Speed	100	100.000
	Price Level	100	100.000
	Price Flexibility	100	100.000
	Manufacturer Image	100	100.000
	Service	100	100.000
	Salesforce Image	100	100.000
	Product Quality	100	100.000

Tests of Equality of Group Means

	Wilks' Lambda	F	df1	df2	Sig.
Delivery Speed	.602	64.716	1	98	.000
Price Level	.817	21.968	1	98	.000
Price Flexibility	.583	70.191	1	98	.000
Manufacturer Image	.999	.140	1	98	.709
Service	.952	4.963	1	98	.028
Salesforce Image	.998	.178	1	98	.674
Product Quality	.532	86.200	1	98	.000

Analysis 1

Box's Test of Equality of Covariance Matrices

Log Determinants

Articulation Of Needs	Rank	Log Determinant
Specification Buying	7	-7.940
Total Value Analysis	7	-4.835
Pooled within-groups	7	-5.323

The ranks and natural logarithms of determinants printed are those of the group covariance matrices.

Test Results

Box's M		73.276
F	Approx.	2.406
	df1	28
	df2	24453.161
	Sig.	.000

Tests null hypothesis of equal population covariance matrices.

Summary of Canonical Discriminant Functions

Eigenvalues

Function	Eigenvalue	% of Variance	Cumulative %	Canonical Correlation
1	2.050 ^a	100.0	100.0	.820

a. First 1 canonical discriminant functions were used in the analysis.

Wilks' Lambda

Test of Function(s)	Wilks' Lambda	Chi-square	df	Sig.
1	.328	105.380	7	.000

Standardized Canonical Discriminant Function Coefficients

	Function
	1
Delivery Speed	.627
Price Level	.259
Price Flexibility	.539
Manufacturer Image	-.064
Service	-.316
Salesforce Image	.295
Product Quality	-.688

Structure Matrix

	Function
	1
Product Quality	-.655
Price Flexibility	.591
Delivery Speed	.568
Price Level	-.331
Service	.157
Salesforce Image	.030
Manufacturer Image	-.026

Pooled within-groups correlations between discriminating variables and standardized canonical discriminant functions
Variables ordered by absolute size of correlation within function.

Functions at Group Centroids

	Function
	1
Articulation Of Needs	1
Specification Buying	-1.736
Total Value Analysis	1.157

Unstandardized canonical discriminant functions evaluated at group means

Classification Statistics**Classification Processing Summary**

Processed		100
Excluded	Missing or out-of-range group codes	0
	At least one missing discriminating variable	0
Used in Output		100

Prior Probabilities for Groups

Articulation Of Needs	Prior	Cases Used in Analysis	
		Unweighted	Weighted
Specification Buying	.500	40	40.000
Total Value Analysis	.500	60	60.000
Total	1.000	100	100.000

Classification Function Coefficients

	Articulation Of Needs	
	Specification Buying	Total Value Analysis
Delivery Speed	10.077	11.837
Price Level	11.178	11.867
Price Flexibility	7.154	8.619
Manufacturer Image	5.084	4.920
Service	-16.775	-18.017
Salesforce Image	-3.531	-2.428
Product Quality	5.114	3.402
(Constant)	-61.586	-66.657

Fisher's linear discriminant functions

Classification Results^a

			Predicted Group Membership		Total
			Specification Buying	Total Value Analysis	
Original	Count	Specification Buying	38	2	40
		Total Value Analysis	9	51	60
	%	Specification Buying	95.0	5.0	100.0
		Total Value Analysis	15.0	85.0	100.0

a. 89.0% of original grouped cases correctly classified.

LAMPIRAN 4.4

General Linear Model

Between-Subjects Factors

	Value Label	N
Type of Buying Situation	1 New Task	34
	2 Modified Rebuy	32
	3 Straight Rebuy	34

Descriptive Statistics

	Type of Buying Situation	Mean	Std. Deviation	N
Usage Level	New Task	36.912	5.0595	34
	Modified Rebuy	46.531	5.3036	32
	Straight Rebuy	54.882	4.8727	34
	Total	46.100	8.9888	100
Satisfaction Level	New Task	3.929	.5312	34
	Modified Rebuy	5.003	.4869	32
	Straight Rebuy	5.394	.7135	34
	Total	4.771	.8556	100

Box's Test of Equality of Covariance Matrices

Box's M	9.796
F	1.584
df1	6
df2	229276.4
Sig.	.147

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.

a. Design: Intercept+X14

Bartlett's Test of Sphericity

Likelihood Ratio	.000
Approx. Chi-Square	293.124
df	2
Sig.	.000

Tests the null hypothesis that the residual covariance matrix is proportional to an identity matrix.

a. Design: Intercept+X14

Multivariate Tests^d

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Intercept	Pillai's Trace	.992	5634.870 ^b	2.000	96.000	.000	.992
	Wilks' Lambda	.008	5634.870 ^b	2.000	96.000	.000	.992
	Hotelling's Trace	117.393	5634.870 ^b	2.000	96.000	.000	.992
	Roy's Largest Root	117.393	5634.870 ^b	2.000	96.000	.000	.992
X14	Pillai's Trace	.771	30.419	4.000	194.000	.000	.385
	Wilks' Lambda	.264	45.411 ^b	4.000	192.000	.000	.486
	Hotelling's Trace	2.655	63.052	4.000	190.000	.000	.570
	Roy's Largest Root	2.604	126.293 ^c	2.000	97.000	.000	.723

a. Computed using alpha = .05

b. Exact statistic

c. The statistic is an upper bound on F that yields a lower bound on the significance level.

d. Design: Intercept+X14

Levene's Test of Equality of Error Variances^s

	F	df1	df2	Sig.
Usage Level	.056	2	97	.945
Satisfaction Level	3.302	2	97	.041

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+X14

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	Usage Level	5498.767 ^b	2	2749.383	106.666	.000
	Satisfaction Level	39.007 ^c	2	19.503	56.542	.000
Intercept	Usage Level	212425.420	1	212425.420	8241.337	.000
	Satisfaction Level	2278.727	1	2278.727	6606.172	.000
X14	Usage Level	5498.767	2	2749.383	106.666	.000
	Satisfaction Level	39.007	2	19.503	56.542	.000
Error	Usage Level	2500.233	97	25.776		
	Satisfaction Level	33.459	97	.345		
Total	Usage Level	220520.000	100			
	Satisfaction Level	2348.710	100			
Corrected Total	Usage Level	7999.000	99			
	Satisfaction Level	72.466	99			

a. Computed using alpha = .05

b. R Squared = .687 (Adjusted R Squared = .681)

c. R Squared = .538 (Adjusted R Squared = .529)

Parameter Estimates

Dependent Variable	Parameter	B	Std. Error	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Usage Level	Intercept	54.882	.871	63.033	.000	53.154	56.610
	[X14=1]	-17.971	1.231	-14.594	.000	-20.414	-15.527
	[X14=2]	-8.351	1.250	-6.679	.000	-10.833	-5.869
	[X14=3]	0 ^b
Satisfaction Level	Intercept	5.394	.101	53.554	.000	5.194	5.594
	[X14=1]	-1.465	.142	-10.283	.000	-1.747	-1.182
	[X14=2]	-.391	.145	-2.703	.008	-.678	-.104
	[X14=3]	0 ^b

a. Computed using alpha = .05

b. This parameter is set to zero because it is redundant.

Between-Subjects SSCP Matrix

			Usage Level	Satisfaction Level
Hypothesis	Intercept	Usage Level	212425.420	22001.355
		Satisfaction Level	22001.355	2278.727
	X14	Usage Level	5498.767	452.178
		Satisfaction Level	452.178	39.007
Error		Usage Level	2500.233	88.912
		Satisfaction Level	88.912	33.459

Based on Type III Sum of Squares

Residual SSCP Matrix

		Usage Level	Satisfaction Level
Sum-of-Squares and Cross-Products	Usage Level	2500.233	88.912
	Satisfaction Level	88.912	33.459
Covariance	Usage Level	25.776	.917
	Satisfaction Level	.917	.345
Correlation	Usage Level	1.000	.307
	Satisfaction Level	.307	1.000

Based on Type III Sum of Squares

Tatatanda adalah seperti di dalam kuliah.

1. Penguraian spektrum bagi suatu matriks simetrik $k \times k$, A diberikan oleh

$$A = \lambda_1 e_1 e_1' + \lambda_2 e_2 e_2' + \dots + \lambda_k e_k e_k'$$

di mana $\lambda_1, \lambda_2, \dots, \lambda_k$ adalah nilai-nilai eigen A dan e_1, e_2, \dots, e_k adalah vektor-vektor eigen terpiawai yang berkaitan.

2. Katakan X mempunyai $E(X) = \mu$ dan $\text{Kov}(X) = \Sigma$. Maka $c'X$ mempunyai min, $c'\mu$ dan varians, $c'\Sigma c$.

3. f.k.k. normal bivariat:

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{\sigma_{11}\sigma_{22}(1-\rho_{12}^2)}} \times \exp \left\{ -\frac{1}{2(1-\rho_{12}^2)} \left[\left(\frac{x_1 - \mu_1}{\sqrt{\sigma_{11}}} \right)^2 + \left(\frac{x_2 - \mu_2}{\sqrt{\sigma_{22}}} \right)^2 - 2\rho_{12} \left(\frac{x_1 - \mu_1}{\sqrt{\sigma_{11}}} \right) \left(\frac{x_2 - \mu_2}{\sqrt{\sigma_{22}}} \right) \right] \right\}$$

4. f.k.k. normal multivariat:

$$f(x) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(x - \mu)' \Sigma^{-1} (x - \mu)}$$

5. Jika $X \sim N_p(\mu, \Sigma)$, maka $AX \sim N_q(A\mu, A\Sigma A')$.

6. Satu sampel:

$$(a) \quad T^2 = n (\bar{X} - \mu)' S^{-1} (\bar{X} - \mu)$$

$$\bar{X} = \frac{1}{n} \sum_{j=1}^n X_j, \quad S = \frac{1}{n-1} \sum_{j=1}^n (X_j - \bar{X})(X_j - \bar{X})'$$

$$T^2 \sim \frac{(n-1)p}{n-p} F_{p, n-p}$$

$$(b) \quad \text{Lambda Wilks } \Lambda^{2/n} = \frac{|\hat{\Sigma}|}{|\hat{\Sigma}_0|} = \left[1 + \frac{T^2}{(n-1)} \right]^{-1}$$

(c) Selang keyakinan serentak $100(1-\alpha)\%$ bagi $\ell' \mu$:

$$\ell' \bar{X} \pm \sqrt{\frac{p(n-1)}{n(n-p)} F_{p, n-p}(\alpha) \ell' S \ell}$$

(d) Selang keyakinan serentak Bonferroni $100(1-\alpha)\%$ bagi

$$\mu_i, \quad i = 1, \dots, p:$$

$$\bar{X}_i \pm t_{n-1} \left[\frac{\alpha}{2p} \right] \sqrt{\frac{S_{ii}}{n}}$$

7. Dua sampel tak bersandar:

$$(a) \quad T^2 = \left[\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2) \right]' \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) S_p \right]^{-1} \left[\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2) \right]$$

$$T^2 \sim \frac{\left[\frac{n_1 + n_2 - 2}{n_1 + n_2 - p - 1} \right] p}{F_{p, n_1 + n_2 - p - 1}}$$

$$(c) h_i^2 = \ell_{11}^2 + \ell_{12}^2 + \dots + \ell_{im}^2, \quad i = 1, 2, \dots, p.$$

$$\sigma_{1i}^2 = h_i^2 + \psi_i, \quad i = 1, 2, \dots, p.$$

(d) Kriteria varimax: Pilih transformasi ortogon T yang menjadikan

$$V = \frac{1}{p} \sum_{j=1}^m \left[\sum_{i=1}^p \tilde{\ell}_{ij}^{*4} - \frac{\left(\sum_{i=1}^p \tilde{\ell}_{ij}^{*2} \right)^2}{p} \right]$$

sebesar yang mungkin.

13. Analisis Pembezaian

$$(a) Y = \underline{\ell}' X = (\underline{\mu}_1 - \underline{\mu}_2)' \underline{\Sigma}^{-1} X$$

$$m = \frac{1}{2} (\underline{\mu}_1 - \underline{\mu}_2)' \underline{\Sigma}^{-1} (\underline{\mu}_1 + \underline{\mu}_2)$$

$$(b) y = \hat{\underline{\ell}}' \underline{x} = \left[\bar{x}_1 - \bar{x}_2 \right]' S_p^{-1} \underline{x}$$

$$\hat{m} = \frac{1}{2} \left[\bar{x}_1 - \bar{x}_2 \right]' S_p^{-1} \left[\bar{x}_1 + \bar{x}_2 \right]$$

(c) Petua peruntukan:

$$\text{Untukkan } x_o \text{ kepada } \begin{cases} \pi_1 & \text{jika } y_o \geq \hat{m} \\ \pi_2 & \text{jika } y_o < \hat{m} \end{cases}$$

(b) Selang keyakinan serentak $100(1-\alpha)\%$ bagi

$$\mu_1 - \mu_2$$

$$\bar{X}_1 - \bar{X}_2 \pm c \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) S_p^2}$$

$$\text{di mana } c^2 = \frac{(n_1 + n_2 - 2)p}{n_1 + n_2 - p - 1} F_{p, n_1 + n_2 - p - 1}$$

8. MANOVA satu-hala:

$$B = \sum_{\ell=1}^g n_{\ell} (\bar{x}_{\ell} - \bar{x}) (\bar{x}_{\ell} - \bar{x})'$$

$$W = \sum_{\ell=1}^g \sum_{j=1}^{n_{\ell}} (x_{\ell j} - \bar{x}_{\ell}) (x_{\ell j} - \bar{x}_{\ell})'$$

$$\Lambda^* = \frac{|W|}{|B + W|}$$

(b) Selang keyakinan serentak $100(1-\alpha)\%$ bagi $\tau_{k1} - \tau_{\ell 1}$:

$$\bar{X}_{k1} - \bar{X}_{\ell 1} \pm t_{n-g} \left[\frac{\alpha}{pg(g-1)} \right] \sqrt{\frac{W_{11}}{n-g} \left(\frac{1}{n_k} + \frac{1}{n_{\ell}} \right)}$$

$$i = 1, 2, \dots, p, \quad \ell < k = 1, 2, \dots, g$$

9. Andaikan E mempunyai d.k. m_E dan H mempunyai d.k. m_H .

$$\text{Katakan } \Lambda = \frac{|E|}{|E + H|}$$

Maka (1) Untuk $p = 1$,

$$\left[\frac{1 - \Lambda}{\Lambda} \right] \frac{m_E}{m_H} \sim F_{m_H, m_E} \text{ bagi sebarang } m_H$$

(2) Untuk $m_H = 1$,

$$\left[\frac{1 - \Lambda}{\Lambda} \right] \frac{m_E + 1 - p}{p} \sim F_{p, m_E + 1 - p} \text{ bagi sebarang } p.$$

(3) Untuk $p = 2$,

$$\left[\frac{1 - \Lambda^{1/2}}{\Lambda^{1/2}} \right] \left[\frac{m_E - 1}{m_H} \right] \sim F_{2m_H, 2(m_E - 1)}$$

untuk $m_H \geq 2$.

(4) Untuk $m_H = 2$,

$$\left[\frac{1 - \Lambda^{1/2}}{\Lambda^{1/2}} \right] \left[\frac{m_E + 1 - p}{p} \right] \sim F_{2p, 2(m_E + 1 - p)}$$

untuk $p \geq 2$.

Pembetulan Bartlett: Katakan $n_o = m_E + m_H$

Bagi m_E besar,

$$-f \log \Lambda \sim X_{pm_H}^2$$

$$\begin{aligned} \text{di mana } f &= m_E - \frac{1}{2} (p - m_H + 1) \\ &= n_o - \frac{1}{2} (p + m_H + 1) \end{aligned}$$

10. MANOVA dua-hala:

$$SSP_{\text{faktor 1}} = \sum_{\ell=1}^g bn \left[\bar{x}_{\ell} - \bar{x} \right] \left[\bar{x}_{\ell} - \bar{x} \right]'$$

$$SSP_{\text{faktor 2}} = \sum_{k=1}^b gn \left[\bar{x}_{.k} - \bar{\bar{x}} \right] \left[\bar{x}_{.k} - \bar{\bar{x}} \right]'$$

$$SSP_{\text{tindakan bersaling}} = \sum_{\ell=1}^g \sum_{k=1}^b n \left[\bar{x}_{\ell k} - \bar{\bar{x}}_{\ell.} - \bar{\bar{x}}_{.k} + \bar{\bar{x}} \right]$$

$$\left[\bar{x}_{\ell k} - \bar{\bar{x}}_{\ell.} - \bar{\bar{x}}_{.k} + \bar{\bar{x}} \right]'$$

$$SSP_{\text{residual}} = \sum_{\ell=1}^g \sum_{k=1}^b \sum_{r=1}^n \left[x_{\ell kr} - \bar{\bar{x}}_{\ell k} \right] \left[x_{\ell kr} - \bar{\bar{x}}_{\ell k} \right]'$$

11. Komponen Prinsipal

(a) $\tilde{Y}_i = \tilde{e}'_i X$, $i = 1, 2, \dots, p.$

$$\rho_{Y_i, X_k} = \frac{e_{ki} \sqrt{\lambda_i}}{\sqrt{\sigma_{kk}}} , \quad i, k = 1, 2, \dots, p.$$

(b) $\tilde{Y}_i = \tilde{e}'_i Z$

$$\rho_{Y_i, Z_k} = e_{ki} \sqrt{\lambda_i} , \quad i, k = 1, 2, \dots, p.$$

12. Analisis Faktor

(a) $X \sim \mu = L F + \epsilon$

(b) $\text{Kov}(X) = L L' + \Psi$

$\text{Kov}(X, F) = L$