

UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2004/2005

Mac 2005

MGM 551 – OPERATIONS RESEARCH
[Penyelidikan Operasi]

Duration : 3 hours

[Masa : 3 jam]

Please check that this examination paper consists of **EIGHT [8]** pages of printed material before you begin the examination.

Answer **all six** questions.

Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN [8]** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **semua enam** soalan.

...2/-

1. Given a linear programming formulation:

$$\begin{array}{llll} \text{Minimize} & Z = 5x_1 + 2x_2 & & \\ \\ \text{Subject to} & 5x_1 + 8x_2 \geq 200 & \text{constraint} & 1 \\ & -2x_1 + 4x_2 \leq 40 & \text{constraint} & 2 \\ & 5x_1 - 4x_2 \geq 60 & \text{constraint} & 3 \\ & x_1 \leq 70 & \text{constraint} & 4 \\ \\ \text{and} & x_1 \geq 0, \quad x_2 \geq 0 & & \end{array}$$

- (i) Using graph, determine the feasible region, the optimum solution and its value.
 (ii) Find the range of values for the coefficient of variable x_1 of the objective function so that the optimum solution found in (i) remains.
 (iii) Determine the range of values for the right hand side value of the third constraint so that the optimum solution found in (i) remains.

[15 marks]

1. *Diberi satu rumus pengaturcaraan linear,*

$$\begin{array}{llll} \text{Minimumkan} & Z = 5x_1 + 2x_2 & & \\ \\ \text{terhadap} & 5x_1 + 8x_2 \geq 200 & \text{kekangan} & 1 \\ & -2x_1 + 4x_2 \leq 40 & \text{kekangan} & 2 \\ & 5x_1 - 4x_2 \geq 60 & \text{kekangan} & 3 \\ & x_1 \leq 70 & \text{kekangan} & 4 \\ \\ \text{dan} & x_1 \geq 0, \quad x_2 \geq 0 & & \end{array}$$

- (i) *Dengan menggunakan graf, tentukan ruang tersaur dan penyelesaian optimum serta nilainya.*
 (ii) *Dapatkan julat nilai bagi pekali fungsi matlamat bagi pemboluhubah x_1 fungsi matlamat supaya penyelesaian yang didapati dalam (i) kekal optimum.*
 (iii) *Tentukan julat bagi nilai sisi kanan bagi kekangan ketiga supaya penyelesaian yang didapati dalam (i) kekal optimum.*

[15 markah]

2. Given the activities, precedence activities and duration in days of a project.

| Activities | Precedence Activities | Duration (days) |
|------------|-----------------------|-----------------|
| A | - | 5 |
| B | - | 7 |
| C | - | 3 |
| D | - | 4 |
| E | C | 3 |
| F | D,E | 4 |
| G | B | 3 |
| H | A,B | 5 |
| I | F,G | 2 |
| J | H,I | 3 |

- (i) Draw the arrow diagram and show the free float (FF) and the total float (TF) times on the diagram.
- (ii) Determine the critical path of the project and the shortest possible time of completion.

[15 marks]

2. *Diberikan jadual kegiatan dan kegiatan pendahulu serta jangka masanya di dalam hari.*

| Kegiatan | Kegiatan Pendahulu | Jangka masa (hari) |
|----------|--------------------|--------------------|
| A | - | 5 |
| B | - | 7 |
| C | - | 3 |
| D | - | 4 |
| E | C | 3 |
| F | D,E | 4 |
| G | B | 3 |
| H | A,B | 5 |
| I | F,G | 2 |
| J | H,I | 3 |

- (i) *Lakarkan gambarajah anak panah dan tunjukkan di situ apungan bebas (FF) dan jumlah apungan (TF).*
- (ii) *Tentukan lintasan genting dan masa terpendek yang mungkin menyiapkan projek tersebut.*

[15 markah]

- 3 (a) Determine the saddle point and the value for the following matrix game:

$$\begin{pmatrix} 5 & -4 & -5 & 8 \\ -3 & -7 & -8 & 2 \\ 6 & 8 & -9 & -7 \\ 7 & 3 & -7 & 5 \end{pmatrix}$$

- (b) Find the probability of strategies and the corresponding value of the game below:

$$\begin{array}{cc} & y_1 & y_2 \\ x_1 & \begin{pmatrix} 10 & 11 \end{pmatrix} \\ x_2 & \begin{pmatrix} 12 & 9 \end{pmatrix} \end{array}$$

- (c) Player A has an RM1 note and a RM 20 note; while player B has an RM5 note and RM10 note. Each player will pick a note from the other without knowing what has been picked.
If the total value of RM selected is odd, player A gets both notes, but if it is even, player B gets both the bills.
- Develop a payoff matrix for this game.
 - What are the best strategies for each player?
 - What is the value of the game?

- (d) Solve the matrix game below graphically

$$\begin{array}{cc} & \text{Player Y} \\ \text{Player X} & \begin{pmatrix} -5 & -10 \\ 12 & 8 \\ 4 & 12 \\ -40 & -5 \end{pmatrix} \end{array}$$

[20 marks]

- 3 (a) Tentukan titik pelana dan nilai permainan matriks berikut:

$$\begin{pmatrix} 5 & -4 & -5 & 8 \\ -3 & -7 & -8 & 2 \\ 6 & 8 & -9 & -7 \\ 7 & 3 & -7 & 5 \end{pmatrix}$$

(b) Dapatkan kebarangkalian bagi strategi dan juga nilai permainan berpadanan untuk permainan berikut:

$$\begin{array}{cc} & y_1 & y_2 \\ x_1 & (10 & 11) \\ x_2 & (12 & 9) \end{array}$$

(c) Pemain A mempunyai wang kertas RM1 dan RM 20; sedangkan pemain B mempunyai RM5 dan RM10. Setiap pemain akan memilih satu helai wang daripada lawannya, masing-masing tidak mengetahui nilai yang dipilihnya. Kemudian mereka jumlahkan nilai wang yang dipilih, jika jumlah RM yang diambil ialah ganjil, pemain A dapat kedua-dua wang tersebut, tetapi jika ia genap, pemain B dapat kedua-duanya.

- (i) Bina satu matriks pembayaran bagi permainan ini.
- (ii) Apakah strategi yang terbaik bagi setiap pemain ini?
- (iii) Berapakah nilai permainan ini?

(d) Selesaikan permainan matriks berikut secara bergraf.

$$\begin{array}{cc} & \text{Pemain Y} \\ \text{Pemain X} & \begin{pmatrix} -5 & -10 \\ 12 & 8 \\ 4 & 12 \\ -40 & -5 \end{pmatrix} \end{array}$$

[20 markah]

4 (a) Given the following simplex tableau,

| Basis | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | Solution |
|-------|-------|-------|-------|-------|-------|-------|----------|
| Z | -3 | 2 | 0 | 0 | -1 | 0 | 4 |
| ? | 2 | 0 | 1 | 0 | 0 | 0 | 13 |
| ? | -2 | 1 | 0 | 1 | -7 | 0 | 7 |
| ? | 4 | 2 | 0 | 0 | 2 | 1 | 2 |

- (i) What is the special property that the tableau exhibits?
- (ii) What is the current basic feasible solution and the value of the objective function?

(b) A paper company sells rolls of paper of fixed length in four standard widths of 5, 8, 12, and 15 feet. Its manufacturing process produces a standard 25 foot-wide-rolls so all orders must be cut from stock of this size. The demands for the 5, 8, 12 and 15 foot rolls are 40, 35, 30 and 25 rolls, respectively. The problem is to cut the manufactured rolls in a fashion that minimizes the total number of standard rolls required.

Formulate this problem as a linear programming problem.

[15 marks]

...6/-

4 (a) Diberikan tablo simpleks berikut,

| Asas | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | Penyelesaian |
|------|-------|-------|-------|-------|-------|-------|--------------|
| Z | -3 | 2 | 0 | 0 | -1 | 0 | 4 |
| ? | 2 | 0 | 1 | 0 | 0 | 0 | 13 |
| ? | -2 | 1 | 0 | 1 | -7 | 0 | 7 |
| ? | 4 | 2 | 0 | 0 | 2 | 1 | 2 |

- (i) Apakah ciri istimewa yang ditunjukkan oleh tablo diatas?
 (ii) Apakah penyelesaian asas tersaur kini dan berapakah nilai fungsi matlamatnya?

(b) Sebuah syarikat kertas menjual gulungan kertas yang panjangnya tetap di dalam empat bidang (lebar) berbeza iaitu 5, 8, 12, dan 15 kaki. Kilang ini menghasilkan kertas yang lebarnya 25 kaki, maka semua pesanan perlu dipenuhi dengan memotong kertas kepada kelebaran yang dikehendaki. Permintaan untuk kelebaran 5, 8, 12, dan 15 kaki ialah 40, 35, 30 and 25 gulung masing-masing. Masalahnya sekarang ialah untuk memotong kertas daripada lebar piawai kepada kelebaran yang dikehendakki dengan bilangan gulungan piawai yang minimum.

Rumuskan sebagai satu masalah pengaturcaraan linear.

[15 markah]

5. Given a linear programming formulation:

$$\begin{aligned} \text{Minimize} \quad & Z = -3x_1 + 2x_2 \\ \text{Subject to} \quad & 6x_1 + 2x_2 \geq 4 \quad \text{Resource 1} \\ & 2x_1 + x_2 \leq 4 \quad \text{Resource 2} \\ \text{and} \quad & x_1 \geq 0, \quad x_2 \geq 0 \end{aligned}$$

The optimum tableau with x_3 and x_4 as slack variables for the problem is,

| Basis | x_1 | x_2 | x_3 | x_4 | Solution |
|-------|-------|-------|-------|-------|----------|
| Z | 7 | 0 | 0 | 2 | 8 |
| x_2 | 2 | 1 | 0 | 1 | 4 |
| x_3 | -2 | 0 | 1 | 2 | 4 |

- (i) What is the optimum solution and the value of the objective function?
 (ii) Which constraint(s) is/are binding?
 (iii) Which resource(s) is/are scarce or surplus?
 (iv) If the right-hand side value of resource 1 and resource 2 is 6 and 2 respectively, reoptimize the solution found.
 (v) If the objective coefficient of x_2 is now 4, what happens to the current optimum solution in (i).
 (vi) If another constraint $x_2 \leq 1$ is added, reoptimize using dual simplex method.

[20 marks]

...7/-

5. Diberikan rumus pengaturcaraan linear berikut,

$$\begin{array}{lll} \text{Minimumkan} & Z = -3x_1 + 2x_2 & \\ \text{terhadap} & 6x_1 + 2x_2 \geq 4 & \text{Sumber 1} \\ & 2x_1 + x_2 \leq 4 & \text{Sumber 2} \\ \text{dan} & x_1 \geq 0, \quad x_2 \geq 0 & \end{array}$$

Tablo optimum dengan x_3 dan x_4 sebagai pembolehubah lalai ialah,

| Asas | x_1 | x_2 | x_3 | x_4 | Penyelesaian |
|-------|-------|-------|-------|-------|--------------|
| Z | 7 | 0 | 0 | 2 | 8 |
| x_2 | 2 | 1 | 0 | 1 | 4 |
| x_3 | -2 | 0 | 1 | 2 | 4 |

- (i) Berikan penyelesaian optimum dan nilai fungsi matlamatnya.
- (ii) Kekangan manakah yang terikat?
- (iii) Sumber mana yang berkurangan dan berlebihan?
- (iv) Jika nilai sisi kanan sumber 1 dan sumber 2 ialah masing-masing 6 dan 2, dapatkan penyelesaian optimum seterusnya.
- (v) Jika pekali fungsi matlamat bagi pembolehubah x_2 ialah 4, apa akan berlaku kepada penyelesaian optimum di (i)?
- (vi) Jika kekangan baru $x_2 \leq 1$ ditambah, optimumkan semula menggunakan kaedah simpleks dual.

[20 markah]

6 (a) Given the following activities of a project with nodes $i - j$ and their respective duration times in days, where a is optimistic time, m is most probable time, and b is pessimistic time.

| Activity | Nodes ($i - j$) | Duration (days) | | |
|----------|-------------------|-----------------|----|----|
| | | a | m | b |
| A | 1 - 2 | 3 | 5 | 7 |
| B | 2 - 3 | 4 | 6 | 8 |
| C | 2 - 4 | 1 | 3 | 5 |
| D | 3 - 4 | 5 | 8 | 11 |
| E | 3 - 5 | 1 | 2 | 3 |
| F | 4 - 6 | 9 | 11 | 13 |
| G | 5 - 6 | 1 | 1 | 1 |
| H | 5 - 7 | 10 | 12 | 14 |
| I | 6 - 7 | 4 | 7 | 9 |

- (i) Calculate the mean and variance for each activity.
- (ii) Draw an arrow diagram for the project.
- (iii) Determine the critical path of the project and the best possible estimated time of completion.

6 (b) Given a linear programming formulation as follows,

$$\begin{aligned} \text{Minimize } Z &= 2x_1 - 3x_2 + x_3 - 4x_4 \\ \text{Subject to } & 2x_1 - x_2 + 3x_3 - 5x_4 \geq 20 \\ & x_1 + 2x_2 - x_3 + x_4 \geq 2 \\ & x_4 \leq 20 \end{aligned}$$

and $x_i \geq 0 \quad \forall i.$

show two iterations using the dual simplex method.

[15 marks]

6 (a) Diberikan senarai tugas dengan nod $i - j$ bagi suatu projek berserta jangka masa optimis (a), masa paling bolehjadi (m) dan jangka masa pesimisnya (b) di dalam hari.

| Tugas | $i - j$ | Jangkamasa (hari) | | |
|-------|---------|-------------------|----|----|
| | | a | m | b |
| A | 1 - 2 | 3 | 5 | 7 |
| B | 2 - 3 | 4 | 6 | 8 |
| C | 2 - 4 | 1 | 3 | 5 |
| D | 3 - 4 | 5 | 8 | 11 |
| E | 3 - 5 | 1 | 2 | 3 |
| F | 4 - 6 | 9 | 11 | 13 |
| G | 5 - 6 | 1 | 1 | 1 |
| H | 5 - 7 | 10 | 12 | 14 |
| I | 6 - 7 | 4 | 7 | 9 |

- (i) Dapatkan min dan varians bagi semua kegiatan.
- (ii) Lakarkan gambarajah anak panah bagi projek ini.
- (iii) Tentukan lintasan genting projek dan jangkamasa terbaik yang mungkin bagi menyiapkannya.

(b) Diberikan rumus PL berikut:

$$\begin{aligned} \text{Minimumkan } Z &= 2x_1 - 3x_2 + x_3 - 4x_4 \\ \text{terhadap } & 2x_1 - x_2 + 3x_3 - 5x_4 \geq 20 \\ & x_1 + 2x_2 - x_3 + x_4 \geq 2 \\ & x_4 \leq 20 \\ \text{dan } & x_i \geq 0 \quad \forall i. \end{aligned}$$

Lakukan dua lelaran menggunakan simpleks dual.

[15 markah]