UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2004/2005

Mei 2005

MGM 511 – ALJABAR LINEAR DAN TEORI PENGKODAN
[LINEAR ALGEBRA AND CODING THEORY]

Masa : 3 jam
[Duration : 3 hours]

Sila pastikan bahawa kertas peperiksaan ini mengandungi **EMPAT [4]** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

*[Please check that this examination paper consists of **FOUR [4]** pages of printed material before you begin the examination.]*

Jawab **semua LIMA [5]** soalan.

*[Answer **all FIVE [5]** questions.]*

1. Let $u = (1+7i,\ 2-6i)$ and $v = (5-2i,\ 3-4i)$

   Find :

   (a) $2iu + (4+7i)v$

   (b) $u \cdot v$

   (c) $\|u\|$ and $\|v\|$

   [20 marks]

1. *Biar $u = (1+7i,\ 2-6i)$ dan $v = (5-2i,\ 3-4i)$*

   Cari :

   (a) $2iu + (4+7i)v$

   (b) $u \cdot v$

   (c) $\|u\|$ *dan* $\|v\|$

   [20 markah]

2. Find a 3×3 orthogonal matrix whose first two rows are multiples of $(1,2,3)$ and $(0,-2,3)$.

   [10 marks]

2. *Cari suatu matriks berortogon 3×3 yang dua baris pertamanya merupakan gandaan $(1,2,3)$ dan $(0,-2,3)$.*

   [10 markah]

3. Given the system of homogeneous equations as follows :

   $$x + 2y - 3z + 4w = 0$$
   $$2x - 3y + 5z - 7w = 0$$
   $$5x + 6y - 9z + 8w = 0$$

   (a) Without solving the system above, discuss the existence of nonzero solutions for it.

   (b) Obtain the set of solutions for the system.

   (c) Prove that the set of solutions of any system of homogeneous equations forms a vector space.

   (d) Obtain a basis for the solution space found in (b) and state its dimension.

   [30 marks]

3. *Diberikan sistem persamaan homogen seperti berikut :*

$$x + 2y - 3z + 4w = 0$$
$$2x - 3y + 5z - 7w = 0$$
$$5x + 6y - 9z + 8w = 0$$

(a) *Tanpa menyelesaikan sistem di atas, bincangkan kewujudan penyelesaian tak sifar baginya.*

(b) *Dapatkan set penyelesaian bagi sistem itu.*

(c) *Buktikan bahawa set penyelesaian bagi sebarang sistem persamaan homogen membentuk suatu ruang vektor.*

(d) *Dapatkan suatu asas bagi ruang penyelesaian yang diperoleh di (b) dan nyatakan dimensinya.*

[30 markah]

4. Let $F : \mathbb{R}^3 \to \mathbb{R}^2$ and $G : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $F(x,y,z) = (2x, y+z)$ and $G(x,y) = (y,x)$.

(a) Derive formulas defining the mappings $G \circ F$ and $F \circ G$ if possible.

(b) Find a basis and the dimension of each of the kernel and the image of F and G.

[20 marks]

4. *Biar $F : \mathbb{R}^3 \to \mathbb{R}^2$ dan $G : \mathbb{R}^2 \to \mathbb{R}^2$ ditakrifkan oleh $F(x,y,z) = (2x, y+z)$ dan $G(x,y) = (y,x)$.*

(a) *Dapatkan, jika mungkin, rumus bagi pemetaan $G \circ F$ dan $F \circ G$.*

(b) *Cari suatu asas dan dimensi bagi inti dan imej bagi F dan G.*

[20 markah]

5. Suppose the x- and y- axes in the plane $\mathbb{R}^2$ are rotated counter clockwise 45^0 so that the new x'- and y'- axes are along the lines $y = x$ and $y = -x$, respectively.

(a) Find the change-of-basis matrix for the rotation.

(b) Find the coordinates of the point (5,6) under the rotation.

[20 marks]

5.   *Andaikan paksi-paksi $-x$ dan $-y$ dalam satah $\mathbb{R}^2$ diputarkan lawan arah jam sebanyak 45^0 supaya paksi-paksi baru $-x'$ dan $-y'$ berada pada garis-garis $y = x$ dan $y = -x$ masing-masing.*

(a)   *Cari matriks penukaran asas bagi putaran ini.*

(b)   *Cari koordinat-koordinat bagi titik (5,6) selepas melalui putaran ini.*

[20 markah]

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