UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

MST 564 – Statistical Reliability
[Kebolehpercayaan Statistik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

1. (a) Consider the following probability density function (p.d.f.) of lifetime T:

$$f(t) = \lambda\beta(\lambda t)^{\beta-1}e^{-(\lambda t)^{\beta}} \quad , \quad t \geq 0$$

If the lifetime of a particular product follows this distribution with $\lambda = 1$ and $\beta = 2$, then the p.d.f. is

$$f(t) = 2te^{-t^2} \quad , \quad t \geq 0$$

Find:

(i) the cumulative distribution function, $F(t)$,
(ii) the survival function, $S(t)$, and
(iii) the hazard function, $h(t)$. Does this distribution has an IFR or a DFR?

(b) Construct the likelihood function for the probability density function in part (a) and obtain the equations for estimating the parameters under Type II censoring.

(c) If it is known that the lifetime of a product follows the Weibull distribution with $\lambda = 0.002$ and $\beta = 2$, then find

(i) the probability that the product will operate for 300 hours,
(ii) the probability that the product which has operated for 100 hours without failure will operate for another 300 hours.

[100 marks]

2. (a) Twelve components are placed on a life test that is discontinued after the eighth failure. The failure times, in hours, are

9, 13, 18, 23, 31, 32, 34, 48

(i) Assuming that the population is exponentially distributed, find a point estimate and a 95% confidence interval estimate for the probability of survival to 20 hours.

(ii) Using nonparametric methods, find a point estimate and a 95% confidence interval estimate for the probability of survival to 20 hours.

1. (a) *Pertimbangkan fungsi ketumpatan kebarangkalian (f.k.k.) berikut bagi masa hayat T:*

$$f(t) = \lambda\beta(\lambda t)^{\beta-1} e^{-(\lambda t)^{\beta}} \quad , \quad t \geq 0$$

Jika masa hayat suatu pengeluaran tertentu mengikuti taburan di atas dengan $\lambda = 1$ *and* $\beta = 2$*, jadi f.k.k.nya adalah*

$$f(t) = 2te^{-t^2} \quad , \quad t \geq 0$$

Dapatkan:

(i) *fungsi taburan longgokan,* $F(t)$,

(ii) *fungsi survival,* $S(t)$, *dan*

(iii) *fungsi bahaya,* $h(t)$. *Adakah taburan ini mempunyai suatu IFR atau DFR?*

(b) *Bina fungsi kebolehjadian bagi fungsi ketumpatan kebarangkalian di bahagian (a) dan dapatkan persamaan-persamaan untuk menganggar parameter-parameter di bawah penapisan Jenis II.*

(c) *Jika diketahui bahawa masa hayat suatu produk mengikuti taburan Weibull dengan* $\lambda = 0.002$ *dan* $\beta = 2$*, maka dapatkan*

(i) *kebarangkalian bahawa produk tersebut akan berfungsi selama 300 jam,*

(ii) *kebarangkalian bahawa produk yang telah berfungsi selama 100 jam tanpa henti akan terus berfungsi selama 300 jam lagi.*

[100 markah]

2. (a) *Dua belas komponen telah diletakkan atas sebuah alat ujian hayat yang dihentikan selepas kegagalan kelapan terjadi. Masa kegagalan, dalam jam, adalah*

9, 13, 18, 23, 31, 32, 34, 48

(i) *Dengan anggapan bahawa populasi ini tertabur secara eksponen, dapatkan satu anggaran titik dan satu anggaran selang keyakinan 95% bagi kebarangkalian survival sehingga 20 jam.*

(ii) *Dengan menggunakan kaedah tak berparameter, dapatkan satu anggaran titik dan satu anggaran selang keyakinan 95% bagi kebarangkalian survival sehingga 20 jam.*

(b) Show that the Kaplan-Meier product–limit estimates reduces to the empirical estimate of the survivor function if the data set is complete and when the failure times are distinct.

(c) Consider a life testing situation for which a manufacturer claims that his product has an exponential time to failure with a mean of 10,000 hours. A consumer questions this claim and will tolerate a consumer's risk of no more than $\beta = 0.10$ when the population mean is 3500 hours. Find the smallest number of failures, r, to satisfy both the producer and consumer. Assume there is Type II censoring.

[100 marks]

3. (a) Give the definition for the proportional hazards model and the accelerated failure time model

(b) Write down the Cox Proportional Hazards model and give two reasons why it is widely used for the analysis of censored data.

(c) Consider the baseline hazard function

$$h_o(t) = \begin{cases} 1 & 0 \le t < 1, \\ t & t \ge 1. \end{cases}$$

In a proportional hazards model, find the probability that an item with covariates $\mathbf{x}$ and link function $g(\mathbf{x})$ will survive to time t.

(d) If survival time T follows a Weibull distribution, show that the proportional hazards model and the accelerated time model coincide. That is, show that if

$$S_o(t) = e^{-(\lambda t)^{\beta}},$$

then

$$(S_o(t))^{g_1(x)} = S_o(tg_2(x))$$

for specific functions $g_1(x)$ and $g_2(x)$.

[100 marks]

4. (a) What is system reliability? What are the reliability block diagrams for series and parallel systems for two components? Illustrate with examples.

(b) *Tunjukkan bahawa penganggar had-hasil darab Kaplan-Meier adalah sama dengan anggaran empirik bagi suatu fungsi survivor jika data adalah set data lengkap dan sekiranya tiada ulangan masa kegagalan.*

(c) *Pertimbangkan satu situasi ujian hayat dimana seorang pengeluar mendakwa bahawa hasil pengeluarannya mempunyai masa kegagalan eksponen dengan min 10,000 jam. Seorang konsumer mempersoalkan dakwaan ini dan dia akan menerima satu risiko konsumer yang tidak lebih daripada* $\beta = 0.10$ *apabila min populasi adalah 3500 jam. Dapatkan bilangan kegagalan yang terkecil, r untuk memuaskan kedua-dua pembekal dan pembeli. Andaikan bahawa terdapat penapisan data Jenis II.*

[100 markah]

3. (a) *Beri definasi bagi model bahaya berkadaran dan model masa kegagalan terpecut.*

(b) *Tuliskan model bahaya berkadaran Cox dan berikan dua alasan kenapa model ini digunakan secara meluas bagi analisis data tertapis.*

(c) *Pertimbangkan fungsi bahaya garis asas berikut:*

$$h_o(t) = \begin{cases} 1 & 0 \le t < 1, \\ t & t \ge 1. \end{cases}$$

Dalam satu model bahaya berkadaran, dapatkan kebarangkalian bahawa satu butiran dengan kovariat $\mathbf{x}$ *dan fungsi paut* $g(\mathbf{x})$ *akan hidup sehingga masa t.*

(d) *Jika masa survival T mengikuti satu taburan Weibull, tunjukkan bahawa model bahaya berkadaran dan model masa kegagalan terpecut adalah selari. Iaitu, tunjukkan bahawa jika*

$$S_o(t) = e^{-(\lambda t)^\beta},$$

maka

$$(S_o(t))^{g_1(x)} = S_o(tg_2(x))$$

bagi fungsi-fungsi tertentu $g_1(x)$ *dan* $g_2(x)$.

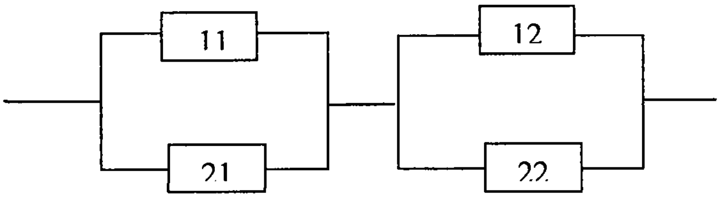
[100 markah]

4. (a) *Apakah reliabiliti sistem? Apakah gambarajah blok reliabiliti bagi sistem bersiri dan sistem selari untuk dua komponen? Tunjukkan dengan mengguna contoh-contoh.*

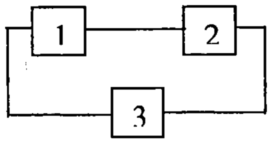
(b) Find the following probabilities:

(i) $F_T(t)$ for a parallel structure with s components, and

(ii) $F_T(t)$ for the series-parallel system structure with component level redundancy as follows:



(c) Consider a system of three components with exponential lifetimes arranged as shown by the block diagram below. If the first component lifetime is exponential with mean $1/\lambda_1$, the second component lifetime is exponential with mean $1/\lambda_2$ and the third component lifetime is exponential with mean $1/\lambda_3$, find the system survivor function and the expected system lifetime.



(d) Suppose that n components have independent lifetimes $T_1, T_2, \ldots, T_n$. Suppose that $S_i(t)$, the survival function for the ith component, is given by

$$S_i(t) = e^{-(\lambda t)^2}.$$

(i) If the components are linked in series, determine the survival function and hazard function for the system.

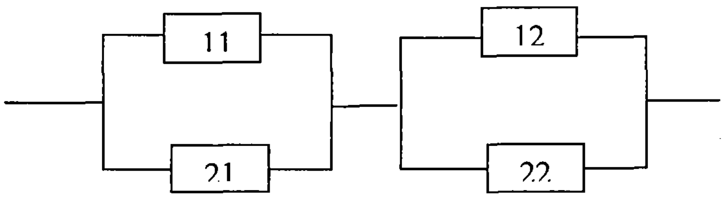
(ii) Give the probability density function for the lifetime of the series system in (i). Identify the parameters of the distribution.

[100 marks]

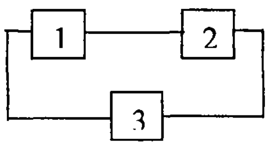
(b) *Dapatkan kebarangkalian berikut:*

(i) $F_T(t)$ *bagi suatu struktur selari dengan s komponen, dan*

(ii) $F_T(t)$ *bagi struktur sistem bersiri-selari dengan aras komponen limpahan seperti berikut:*



(c) *Pertimbangkan satu sistem tiga komponen dengan masa hayat tertabur secara eksponen disusun seperti yang ditunjukkan oleh gambarajah blok di bawah. Jika masa hayat komponen pertama adalah eksponen dengan min* $1/\lambda_1$, *masa hayat kedua adalah eksponen dengan min* $1/\lambda_2$ *dan masa hayat ketiga adalah eksponen dengan min* $1/\lambda_3$, *dapatkan fungsi survivor sistem dan jangkaan masa hayat sistem.*



(d) *Andaikan bahawa n komponen mempunyai masa hayat tak bersandar* $T_1, T_2, \ldots, T_n$. *Andaikan bahawa* $S_i(t)$, *fungsi survival bagi komponen ke-i, diberikan oleh*

$$S_i(t) = e^{-(\lambda t)^2}.$$

(i) *Jika komponen dihubung secara bersiri, tentukan fungsi survival dan fungsi bahaya bagi sistem tersebut.*

(ii) *Dapatkan fungsi ketumpatan kebarangkalian masa hayat sistem bersiri dalam bahagian (i). Camkan parameter taburan tersebut.*

[100 markah]

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