UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2004/2005

March 2005

MAT 516 – CURVE AND SURFACE METHODS FOR CAGD
[KAEDAH LENGKUNG DAN PERMUKAAN UNTUK RGBK]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SEVEN [7]** pages of printed material before begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH [7]** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Answer **all FIVE [5]** questions.

*Jawab **semua LIMA [5]** soalan.*

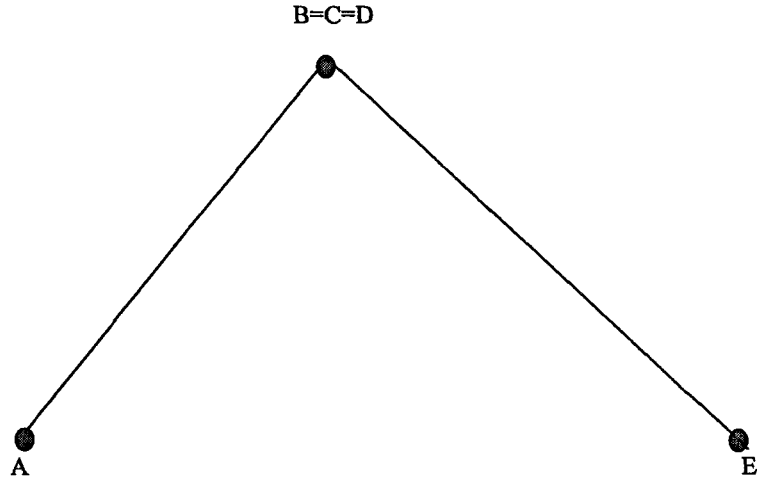
...2/-

1. (a) If $B_i^n(t) = \frac{n!}{(n-i)!i!}(1-t)^{n-i}t^i$ with $B_0^0(t) = 1, 0 \le t \le 1$, show that

(i) $(1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t) = B_i^n(t)$

(ii) $\frac{dB_i^n(t)}{dt} = n(B_{i-1}^{n-1}(t) - B_i^{n-1}(t))$

(b) Assume that ABE is a control polygon for a quartic Bezier curve. If the points B, C and D are the same, by using des Casteljau algorithm, show the location of the Bezier point when t =0.5.



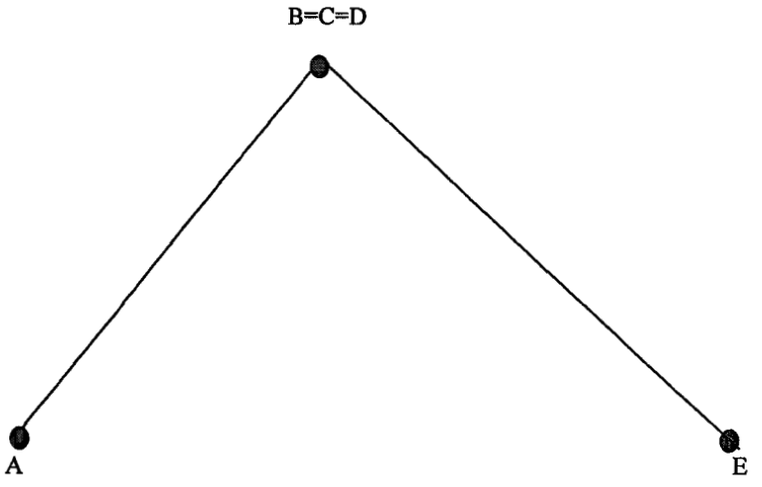
[15 marks]

1. (a) Jika $B_i^n(t) = \frac{n!}{(n-i)!i!}(1-t)^{n-i}t^i$ *dengan* $B_0^0(t) = 1, 0 \le t \le 1$, *tunjukkan*

(i) $(1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t) = B_i^n(t)$

(ii) $\frac{dB_i^n(t)}{dt} = n(B_{i-1}^{n-1}(t) - B_i^{n-1}(t))$

(b) Andaikan ABE adalah poligon kawalan untuk suatu lengkung Bezier kuartik. Jika B,C dan D adalah titik-titik yang sama, dengan algoritma des Casteljau tandakan titik Bezier apabila $t = 0.5$.



[15 markah]

2. (a) A rational quartic Bezier is defined by,

$$r(t)=\frac{\sum_{i=0}^{4} B_i^4(t)w_i P_i}{\sum_{i=0}^{4} B_i^4(t)w_i},$$

where $B_i^4(t)={}^4C_i(1-t)^{4-i}t^i$, $t\in[0,1]$, P_i $i=0,1,2,3,4$ the control points and w_i $i=0,1,2,3,4$ the corresponding weights. If $r'(0)=T_0$, and $r'(1)=T_1$, show that

$$P_1=P_0+\frac{T_0 w_0}{4w_1} \text{ and } P_3=P_4-\frac{T_1 w_4}{4w_3}$$

(b) A line $r(t)$ joining the points P and Q is given by

$$r(t)=\frac{(1-t)\alpha P+t\beta Q}{(1-t)\alpha+t\beta},\quad 0\le t\le 1,$$

Give an example the values of α and β such that r(0.5) divides the segment PQ in the ratio of 3:1. If α and β both tend to ∞, determine the location of point r(0.5).

[15 marks]

2. (a) *Suatu lengkung nisbah Bezier kuartik ditakrifkan oleh,,*

$$r(t)=\frac{\sum_{i=0}^{4} B_i^4(t)w_i P_i}{\sum_{i=0}^{4} B_i^4(t)w_i},$$

dengan $B_i^4(t)={}^4C_i(1-t)^{4-i}t^i$, $t\in[0,1]$, P_i $i=0,1,2,3,4$ *titik-titik kawalan* w_i $i=0,1,2,3,4$ *pemberat yang sepadan. Jika* $r'(0)=T_0$, *dan* $r'(1)=T_1$, *tunjukkan*

$$P_1=P_0+\frac{T_0 w_0}{4w_1} \text{ and } P_3=P_4-\frac{T_1 w_4}{4w_3}$$

(b) *Suatu garis r(t) yang menghubungkan P dan Q diberi sebagai*

$$r(t)=\frac{(1-t)\alpha P+t\beta Q}{(1-t)\alpha+t\beta},\quad 0\le t\le 1,$$

Beri suatu contoh nilai-nilai α *dan* β *supaya r(0.5) membahagikan segmen PQ pada nisbah 3:1. Jika* α *dan* β *menumpu ke infiniti, tentukan kedudukan titik r(0.5).*

[15 markah]

3. (a) A line joining (0,0) and (3,0) intersects a cubic Bezier curve $r(t)=\sum_{i=0}^{3} B_i^3(t)P_i$; where $P_0=(0,0)$, $P_1=(1,2)$, $P_2=(0,-2)$, *and* $P_3=(3,0)$. Find the intersection points.

(b) Write the function $f(t)=t^3+2t-5$; $0\le t\le 1$ as
$f(t)=(1-t)^2 f_0+2(1-t)^2 tf_1+2(1-t)t^2 f_2+t^2 f_3$
where f_0, f_1, f_2 *and* f_3 are real values.

[15 marks]

3. (a) *Suatu garis menghubungkan (0,0) dan (3,0) bersilang dengan suatu lengkung Bezier kubil*

$r(t)=\sum_{i=0}^{3} B_i^3(t)P_i$ *; dengan* $P_0=(0,0)$, $P_1=(1,2)$, $P_2=(0,-2)$, *dan* $P_3=(3,0)$. *Cari titik-titik persilangan.*

(b) *Tulis fungsi* $f(t) = t^3 + 2t - 5; \quad 0 \le t \le 1$ *sebagai*
$f(t) = (1-t)^2 f_0 + 2(1-t)^2 t f_1 + 2(1-t)t^2 f_2 + t^2 f_3$
dengan f_0, f_1, f_2 *dan* f_3 *nombor nyata.*

[15 markah]

4. (a) A surface S with parameters u and v is given by
$S(u,v) = (u + 2v^2, uv, u - v^2); \quad 0 \le u, v \le 1.$
State the equation of a plane in Cartesian form which is tangential to the surface $S(u,v)$ at $u = 0.5$ and $v = 0.5$.

(b) A curve r(t) is given by
$r(t) = (t^3 + t - 1, -t^2 - t + 3); \quad 0 \le t \le 3$
Determine the centre of curvature of $r(t)$ at $t = 1$.

(c) Show that a rational cubic Bezier

$$\boldsymbol{r}(t) = \frac{\sum_{i=0}^{3} B_i^3(t) w_i \boldsymbol{P}_i}{\sum_{i=0}^{3} B_i^3(t) w_i}$$

with weights $w_0 = 2, w_1 = 4, w_2 = 8,$ and $w_3 = 16$ is equivalent to a cubic Bezier curve

$$\boldsymbol{p}(t) = \sum_{i=0}^{3} B_i^3(t) \boldsymbol{P}_i$$

[25 marks]

4. *(a) Suatu permukaan S dengan parameter u dan v diberi sebagai*
$S(u,v) = (u + 2v^2, uv, u - v^2); \quad 0 \le u, v \le 1.$
Berikan dalam bentuk Cartesian persamaan satah yang menyentuh permukaan S(u,v) pad u = 0.5 dan v = 0.5.

(b) Suatu lengkung r(t) diberikan sebagai
$r(t) = (t^3 + t - 1, -t^2 - t + 3); \quad 0 \le t \le 3$
Tentukan pusat kelengkungan r(t) pada t = 1.

(c) Tunjukkan suatu lengkung nisbah kubik Bezier

$$\boldsymbol{r}(t) = \frac{\sum_{i=0}^{3} B_i^3(t) w_i \boldsymbol{P}_i}{\sum_{i=0}^{3} B_i^3(t) w_i}$$

dengan pemberat $w_0 = 2, w_1 = 4, w_2 = 8,$ *dan* $w_3 = 16$ *adalah setara dengan*

$$\boldsymbol{p}(t) = \sum_{i=0}^{3} B_i^3(t)\boldsymbol{P}_i$$

[25 markah]

5. (a) A B-spline curve of degree five with the knot vector $\{t_0, t_1, t_2, ..., t_{25}, t_{26}\}$ is being controlled by the set of points $\{P_0, P_1, P_2, ..., P_{19}, P_{20}\}$. List the control points of a curve segment defined by the knots $\{t_7, t_8\}$. How to assign the knots in order for the curve to interpolate the first and the final control points.

(b) Write an n'th degree Bezier curve $\boldsymbol{p}(t) = \sum_{i=0}^{n} B_i^n(t)\boldsymbol{P}_i$, $0 \le t \le 1$, in an arbitrary parameter interval $t_0 \le t \le t_1$, such that $\boldsymbol{p}(t_0) = \boldsymbol{P}_0$, and $\boldsymbol{p}(t_1) = \boldsymbol{P}_n$.

(c) A B-spline quadratic curve with a knot vector {1,1,1,3,3,3} and control points $\{P_0, P_1, P_2\}$ is given by

$$\boldsymbol{p}(t) = \sum_{i=0}^{2} N_{i,2}(t)\boldsymbol{P}_i$$

where $N_i d(t)$ is defined by

$$N_{i,0}(t) = \begin{cases} 1 & \text{if } t \in [t_i, t_{i+1}) \\ 0 & \text{elsewhere} \end{cases}$$

$$N_{i,d}(t) = \frac{t - t_i}{t_{i+d} - t_i} N_{i,d-1}(t) + \frac{t_{i+d+1} - t}{t_{i+d+1} - t_{i+1}} N_{i+1,d-1}(t)$$

Find the coefficients of $\boldsymbol{P}_i$, *i = 0, 1, and 2* at t = 2.

(d) Given a helix *r(u) = (a cos(u), a sin(u), bu);* where *u* is a positive real number, find the first derivative of *r(u).* Also find the length of helix from $u = 0$ to $u = 2\pi$. If $b = 0$, what is the torsion of the helix.

[30 marks]

5. *(a) Suatu lengkung splin-B berdarjah lima dengan vektor knot* $\{t_0, t_1, t_2, ..., t_{25}, t_{26}\}$ *adalah dikawal oleh set titik* $\{P_0, P_1, P_2, ..., P_{19}, P_{20}\}$. *Senaraikan titik kawalan suatu segmen yang ditakrif pada knot* $\{t_7, t_8\}$. *Bagaimana knot diumpikkan supaya lengkung akan menginterpolasi titik kawalan awal dan titik kawalan akhir.*

(b) *Tulis fungsi* $\boldsymbol{p}(t)=\sum_{i=0}^{n} B_i^n(t)\boldsymbol{P}_i$, $0 \le t \le 1$, *pada sebarang selang parameter* $t_0 \le t \le t_1$, *supaya* $\boldsymbol{p}(t_0)=\boldsymbol{P}_0$, *dan* $\boldsymbol{p}(t_1)=\boldsymbol{P}_n$.

(c) *Suatu splin-B kuadratik denga vektor knot* {1,1,1,3,3,3} *dan titik kawalan* $\{P_0, P_1, P_2\}$ *diberi sebagai*

$$\boldsymbol{p}(t)=\sum_{i=0}^{2} N_{i,2}(t)\boldsymbol{P}_i$$

dengan $N_i d(t)$ ditakrifkan sebagai

$$N_{i,0}(t)=\begin{cases}1 & \textit{jika } t \in [t_i, t_{i+1}) \\ 0 & \textit{sebaliknya}\end{cases}$$

$$N_{i,d}(t)=\frac{t-t_i}{t_{i+d}-t_i}N_{i,d-1}(t)+\frac{t_{i+d+1}-t}{t_{i+d+1}-t_{i+1}}N_{i+1,d-1}(t)$$

cari pekali $\boldsymbol{P}_i$, $i = 0, 1,$ *dan 2 pada* $t = 2$.

(d) *Diberi suatu heliks r(u) = (a cos(u),a sin(u), bu); dengan u suatu nombor nyata positif, cari terbitan pertama r(u). Juga cari panjang heliks dari* $u = 0$ *sehingga* $u = 2\pi$. *Jika* $b = 0$, *dapatkan nilai kilasan heliks.*

[30 markah]

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