

## The Analysis of Transitions in Economic Performance Using Covariate Dependent Markov Model

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**Abstract:** *The GDP or GNP as a measure of economic performance of a country changes continuously. We can identify the factors that precede its ups and downs. For such forecasting, the use of Markov models are not new, but in this paper, an attempt is made to propose a covariate-dependent Markov model to identify the factors that contribute to the estimation of transition probabilities. The proposed model is employed to estimate the transition probabilities, the factors that contribute to transition in economic performance, and other relevant characteristics. The cross-country data have been employed for the period 1980-2000 for fitting the model. This can provide a useful model for forecasting the economic performance in both developing and developed countries.*

### Introduction

The measure of GDP depends on several components such as private consumption, investment, government consumption, changes in inventories, total exports and total imports. It is observed by Swamy and Fikkert (2002) that the determinants of economic growth which rely on cross-country growth regressions may be affected by bias from two sources: (i) omitted variables, and simultaneity. The first one is attributable to country characteristics that affect growth but omitted by the econometricians. The second one is due to the fact that the determinants of growth of GDP, such as investment in physical capital, may also be affected by this growth. Currently, there is evidence of relationship between human capital accumulation and economic growth. Asteriou and Agiomirgianakis (2001) demonstrated such relationship between education variables and GDP as well as the causal direction between them for Greece. Similar relationship was observed in Iran as well (Yousefi, 1995). The transition from agrarian to predominantly industrial economy had been successful in raising the pace of economic growth in some Asian countries during the recent past.

In this paper, the growth of GDP has been analyzed in order to identify the factors that contribute to the change in the economic performance. For the purpose of this study, we have employed the cross-country data. Due to missing observations for many variables, we have used only some selected variables that are associated with the change in the economic performance in the cross-country setting. The main objective of this paper is to demonstrate the utility of Markov models in identifying the role of the selected characteristics in explaining the growth in GDP over time. The advantage of such model is that we can use repeated observations to identify the factors that attribute to the change in economic performance.

### Data and Methods

We have used the cross-country data for the period 1980-2000. To demonstrate a distinct trend over time, we have taken into consideration data after every five years from the World Development Reports for the years 1980, 1985, 1990, 1995, and 2000. The gross domestic product (GDP) is considered as the outcome variable, which measures the economic performance of a country. The selected variables are: growth of industry (indgr), population growth (popgr), labor force growth (lfgr), use of energy (eneruse)

We have used both linear regression as well as logistic regression models in this study for different time periods. Then a covariate dependent Markov model is used to examine the change in performance in economic growth over time.

## Linear Regression Model for Economic Growth

The linear regression model for economic growth is presented here. The outcome variable is GDP and the explanatory variables are: growth of industry (indgr), population growth (popgr), labor force growth (lfgr), use of energy (eneruse). We have employed two different sets of models here, one for each selected year and the other sets of models include the lag variables. The models are shown below for the  $i$ th country in the  $j$ th year:

Model 1a:

$$GDP_{ij} = \beta_{0j} + \beta_{1j} \text{indgr}_{ij} + \beta_{2j} \text{popgr}_{ij} + \beta_{3j} \text{lfgr}_{ij} + \beta_{4j} \text{eneruse}_{ij} + \varepsilon_{ij}$$

Model 1b:

$$GDP_{ij} = \beta_{0j} + \beta_{1j} \text{indgr}_{j-5} + \beta_{2j} \text{popgr}_{j-5} + \beta_{3j} \text{lfgr}_{j-5} + \beta_{4j} \text{eneruse}_{j-5} + \varepsilon_{ij}$$

The first model (Model 1a) considers both the outcome and explanatory variables during the same year for the selected countries. However the second model (Model 1b) employs explanatory variables observed five years before the outcome variable. This model is expected to take account of the time-lag in explaining the outcome variable, GDP.

The results for Model 1a are displayed in Table 1. It is evident from Table 1 that growth of industry is positively associated ( $p < 0.01$ ) during the period 1980-2000 with a steadily increasing effect on the growth of GDP. Population growth results in the growth of economy but this is significant only in 1980 ( $p < 0.10$ ) and 1990 ( $p < 0.01$ ). Growth in labor force appears to have no statistical association with growth in GDP. However, use of energy appears to have positive association with economic performance in 1990 ( $p < 0.05$ ).

Model 1b uses the 5-year lag between observed GDP and explanatory variables. Table 2 shows that only growth of industry seems to have positive association with GDP for the years 1985-90 and 1995-2000.

## Logistic Regression Models

To explore the underlying association between growth in GDP and the selected explanatory variables, two sets of logistic regression models are fitted in this section. Let us define the following dichotomous variables for the  $i$ th country in year  $j$ :

$$Y_{ij} = 0, \quad \text{if } GDP_{ij} < 3.2 \text{ percent}$$

$$Y_{ij} = 1, \quad \text{if } GDP_{ij} \geq 3.2 \text{ percent}$$

Then let us define the following models:

Model 2a:

$$g(X_{ij}) =$$

$$\beta_{0j} + \beta_{1j} \text{indgr}_{ij} + \beta_{2j} \text{popgr}_{ij} + \beta_{3j} \text{lfgr}_{ij} + \beta_{4j} \text{eneruse}_{ij} + \varepsilon_{ij}.$$

Model 2b:

$$g(X_{ij-5}) =$$

$$\beta_{0j} + \beta_{1j} \text{indgr}_{j-5} + \beta_{2j} \text{popgr}_{j-5} + \beta_{3j} \text{lfgr}_{j-5} + \beta_{4j} \text{eneruse}_{j-5} + \varepsilon_{ij}.$$

Then the logistic regression models for both 2a and 2b are:

Model 2a:

$$P(Y_{ij} = 1 / X_{ij}) = \frac{e^{g(X_{ij})}}{1 + e^{g(X_{ij})}}$$

Model 2b:

$$P(Y_{ij} = 1 / X_{ij-5}) = \frac{e^{g(X_{ij-5})}}{1 + e^{g(X_{ij-5})}}$$

Table 3 shows the estimates for Model 2a. It is clearly observed from the results that growth of industry has been positively associated with for all the years during 1980-2000 ( $p < 0.01$ ). The impact of growth on growth of GDP appears to exert the largest impacts in the years 1990 and 2000. Similarly, growth of labor force shows statistically significant positive association with growth in GDP for the years 1990 and 1995 ( $p < 0.05$ ). However, growth in population appears to have negative association in 1995 ( $p < 0.05$ ). Use of energy is associated positively with growth in GDP in 1990 ( $p < 0.05$ ).

Model 2b confirms the result that growth of industry increases the growth of GDP during the periods 1980-85, 1985-90 and 1995-2000 ( $p < 0.01$ ). However, all other selected variables do not show any statistically significant association with the outcome variable.

### Markov Model

The covariate dependent Markov model was proposed by Muenz and Rubinstein (1985) and then Islam, Chowdhury and Baharum (2003) extended the model for higher order. Let us give a brief overview of the model here from Muenz and Rubinstein and Islam, Chowdhury and Baharum. See these papers for more details.

Let us consider a two state Markov chain for a discrete time binary sequence as follows:

$$\pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix}$$

where  $\pi_{00} = 1 - \pi_{01}$  and  $\pi_{10} = 1 - \pi_{11}$ . Here, 0 and 1 are the two possible outcomes of a dependent variable, Y. Each row of the above transition probability matrix provides a model on the basis of conditional probabilities. For instance, the probability of a transition from 0 at time  $t_{j-1}$  to 1 at time  $t_j$  is  $\pi_{01} = P(Y_j = 1 / Y_{j-1} = 0)$  and similarly the probability of a transition from 1 at time  $t_{j-1}$  to 1 at time  $t_j$  is  $\pi_{11} = P(Y_j = 1 / Y_{j-1} = 1)$ . It is evident that  $\pi_{00} + \pi_{01} = 1$  and similarly  $\pi_{10} + \pi_{11} = 1$ .

For covariate dependence, let us define the following notations:

$X'_i = [1, X_{i1}, \dots, X_{ip}]$  = vector of covariates for the  $i$ th person;

$\beta'_0 = [\beta_{00}, \beta_{01}, \dots, \beta_{0p}]$  = vector of parameters for the transition from 0,

$\beta'_1 = [\beta_{10}, \beta_{11}, \dots, \beta_{1p}]$  = vector of parameters for the transition from 1.

Then the transition probabilities can be defined in terms of function of the covariates as follows:

### Model for Increase in the growth of GDP

$$\pi_{01}(Y_j = 1 / Y_{j-1} = 0, X) = \frac{e^{\beta'_0 X}}{1 + e^{\beta'_0 X}}, \text{ and}$$

#### Model for Decrease in the growth of GDP

$$\pi_{11}(Y_j = 0 / Y_{j-1} = 1, X) = \frac{e^{\beta'_1 X}}{1 + e^{\beta'_1 X}}.$$

Table 5 shows the summary of the results for both increase and decrease in the growth of GDP for covariate dependent Markov models. As expected, growth of industry is positively associated with increase in GDP growth and negatively associated with decrease in GDP growth during the period 1980-2000. Similarly, increased growth in labor force appears to have negative association, while the opposite is true for the model on decrease in GDP growth. Use of energy appears to have significant association at 10 percent level.

#### Summary and Conclusion

This paper examines the factors influencing the change in economic growth of countries. The economic performance of countries may depend on factors related to capital investment, investment on human capital accumulation, expenditure on health, and many other factors that are associated directly or indirectly with the economic growth. This paper provides only a preliminary overview of the problems associated with the relationship with growth in GDP. The main purpose of this paper is to demonstrate different techniques that can be employed to explain such relationships. Due to data limitations, some of the important variables could not be used.

In this paper, three different methods have been used: (i) regression models, (ii) logistic regression models, and (iii) Markov models with covariate dependence. First two models take account of both cross-section data as well as data with a lag of five years. The third models uses the Markov model for explaining the transitions from low or moderate economic performance to high performance as well as transition from high economic performance to moderate or low performance. It is surprising that in all these models, it appears that growth in industry is the most dominating factor in explaining the growth in GDP. In some models, the role of growth in labor force seems to have statistically significant association. For the East Asian economies, the transition to high performance was preceded by the demographic transitions that led to high growth of labor force during the period of population momentum. The Markov models reveals the findings more explicitly due to use of repeated measures of economic performance. It provides two sets of equations for increase in the growth of GDP as well as decrease in the growth of GDP from the same model and thus the role of variables can be determined for both directions in the change of economic performance. With more detailed data, the advantage of the covariate dependent Markov model will be more precise and obvious.

Table 1: Regression Model for Estimates of Regression Models for Growth of GDP for Year 1980,1985,1990, 1995 and 2000.

Variable	Estimates (standard error)				
	1980	1985	1990	1995	2000
<b>Intercept</b>	0.80858 (0.70167)	0.83301 (0.51638)	1.23444*** (0.42579)	0.95373 (0.74983)	1.22931*** (0.38210)
<b>Growth of Industry</b>	0.47570*** (0.04325)	0.52173*** (0.03162)	0.53080*** (0.03443)	0.52964*** (0.03977)	0.55038*** (0.03010)
<b>Population Growth</b>	0.83314* (0.49722)	-0.12925 (0.39179)	0.00179*** (0.00042140)	-0.39395 (0.48711)	-0.05071 (0.24924)
<b>Labor Force Growth</b>	-0.41322 (0.49053)	0.38654 (0.38508)	-0.08754 (0.14535)	0.49647 (0.50052)	0.09998 (0.23716)
<b>Energy in Use (percapita)</b>	-0.00015486 (0.00010384)	0.00018287** (0.00009097)	-0.00002381 0.00007162	0.00013866 (0.00012298)	0.00003298 (0.00006546)

\*\*\* Significant at 1% level      \*\* Significant at 5% level      \* Significant at 10% level

Table 2 : Regression Model for Estimates of Regression Models for Growth of GDP during the Period 1980-1985,1985-1990,1990-1995 and 1995-2000.

Variable	Estimates (standard error)			
	1980-1985	1985-1990	1990-1995	1995-2000
<b>Intercept</b>	2.19365 (1.11311)	2.94733*** (0.91451)	1.82865 (1.53763)	2.41922*** (0.79922)
<b>Growth of Industry</b>	0.34073 (0.06949)	0.30810*** (0.05461)	0.13861 (0.12556)	0.18851*** (0.04239)
<b>Population Growth</b>	0.64148 (0.76319)	-0.93755 (0.67394)	0.00175 (0.00145)	-0.22683 (0.51920)
<b>Labor Force Growth</b>	-0.83000 (0.75429)	0.24039 (0.66343)	0.20549 (0.52906)	0.28121 (0.53349)
<b>Energy in Use (percapita)</b>	-0.00011388 (0.00016172)	-0.00006818 (0.00015756)	-0.00010532 0.00026058	0.00007276 (0.00013108)

\*\*\* Significant at 1% level      \*\* Significant at 5% level      \* Significant at 10% level

Table 3 : Estimates of Parameters of Logistics Regression Models for Growth of GDP in 1980,1985,1990,1995 and 2000.

Variable	Estimates (Standard error)				
	1980	1985	1990	1995	2000
<b>Intercept</b>	1.7713 ( 1.1235 )	5.5059*** ( 1.8889 )	7.8074*** ( 2.2701 )	1.9917* ( 1.1553 )	2.5159* ( 1.5248 )
<b>Growth of Industry</b>	-0.4916*** ( 0.1362 )	-0.8004*** ( 0.2710 )	-1.0917*** ( 0.2480 )	-0.4388*** ( 0.1193 )	-1.2099*** ( 0.3124 )
<b>Population Growth</b>	-1.2381 ( 1.0842 )	0.6147 ( 1.1009 )	-0.00262 ( 0.00390 )	1.5588** ( 0.6790 )	0.4689 ( 0.9897 )
<b>Labor Force Growth</b>	1.0753 ( 1.1016 )	-1.6654 ( 1.1328 )	-1.1950** ( 0.5883 )	-1.6747** ( 0.7044 )	-0.00620 ( 0.9366 )
<b>Energy in Use (percapita)</b>	-0.00006 ( 0.000153 )	-0.00022 ( 0.000234 )	-0.00053** ( 0.000259 )	-0.00009 ( 0.000221 )	0.000266 ( 0.000252 )

\*\*\* Significant at 1% level      \*\* Significant at 5% level      \* Significant at 10% level

Table 4 : Estimates for Lagged Logistic Regression Models for Binary Outcomes for the Periods 1980 – 2000.

Variable	1980-1985	1985-1990	1990-1995	1995-2000
<b>Intercept</b>				
p-value	0.0500**	0.0189**	0.1043	0.4233
<b>Growth of Industry</b>				
p-value	0.0019***	<0.0001***	0.1089	0.0003***
<b>Population Growth</b>				
p-value	0.1787	0.4765	0.3222	0.6466
<b>Labor Force Growth</b>				
p-value	0.3858	0.5179	0.1598	0.5326
<b>Energy in Use (percapita)</b>				
p-value	0.4479	0.1229	0.7487	0.5328

\*\*\* Significant at 1% level      \*\* Significant at 5% level      \* Significant at 10% level

Table 5 : Covariate Dependent Markov Model for Increasing and Decreasing in the Growth of GDP for the Period 1980-2000.

Variables	Estimates (Standard Error)	
	Increase	Decrease
<b>Intercept</b>	-2.0669*** ( 0.6567 )	2.9073*** ( 0.8925 )
<b>Growth of Industry</b>	-0.2007*** ( 0.0532 )	0.1208** ( 0.0538 )
<b>Population Growth</b>	-0.00568 ( 0.00493 )	0.00116 ( 0.00245 )
<b>Labor Force Growth</b>	0.6919*** ( 0.2261 )	-1.0248*** ( 0.2983 )
<b>Energy in Use (percapita)</b>	0.000268* ( 0.000159 )	0.000054 ( 0.000139 )

\*\*\* Significant at 1% level

\*\* Significant at 5% level

\* Significant at 10% level

## References

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