The Game Theoretical Argument of Informational Asymmetry Between A Borrower And A Lender*

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Abstract

The aim of this paper is to provide the possible game theoretical technical tool for a self-selection mechanism which is used to overcome informational asymmetry by sorting borrowers into groups according to the riskiness of their projects . The individual borrowers know more about their chances of success in their enterprise than a bank knows. The major problem in constructing such a sorting mechanism is to overcome the natural tendency of a particular risk-type group of borrowers to pretend that they belong to a different risk-type group, possibly in order to obtain more favorable treatment which would otherwise be reserved for members of the other group. The approach used to stop this disguising of one group of borrowers as members of another group is to make the loan contracts of the other group less favorable, so that each borrower finds it advantageous to stick to his own contract. The measure used in this paper's model to decrease the attraction of some contracts is a statement by a lender that he will grant only some of the loan application and that he will ration credit with some given probability of satisfying the loan application.

Keywords: Nash equilibrium, Smith and Stutzer model, utility functions.

1. The Model

The model is a standard two-period model, with periods indexed by t = 1, 2. There exist two groups of economic agents in a Smith and Stutzer model: lenders and borrowers. The utility function of lenders is described as a sum of their consumption in both periods. Letting c_t denote period t consumption, the lenders utility function is $U = c_1 + c_2$. Each lender is endowed with one unit of funds at t = 1, which can be either loaned out or consumed by a lender. This one unit of funds serves as a numerary in our model. The number of homogeneous lenders is N_1. There are $N_2 \leq N_1$ borrowers in the model. All borrowers are endowed with only one unit of their effort to expend at t = 1 and have no further funds of their own. Each borrower has access to an investment project. These projects are indivisible – their realization requires one unit of financial funds (which the borrower has to borrow from a lender) at t = 1 and one unit of effort. Additional inputs of funds or an effort would have no effect on a project output. At t = 2, each project is either a "success" or a "failure". A successful project returns y > 0 at t = 2, while an unsuccessful project produces zero.

In order to induce a source of information asymmetry needed for a functioning of this type of a model, we suppose that the borrowers are not homogeneous. The borrowers can be divided into two types, with type indexed by $i = H, L$. A type i borrowers has a probability P_i of operating a successful project. The values P_i satisfy $P_H < P_L$, so type H borrowers are "high (default) risk"

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borrowers. Each borrowers knows his own type, but not that of the others. This means that there exists information asymmetry between a borrower and lender. Again letting c_2 denote date 2 consumption, type i borrowers have utility functions given by

$$U = c_2 + \beta_i, \tag{1}$$

where β_i is a sure net return which is brought by an alternative employment opportunity for a borrower. It is just an opportunity cost of using the borrower's effort endowment. It is common knowledge to all borrowers and all lenders that the fraction of borrowers of type H is θ, such that $0<\theta<1$.

We assume that $N_1 > 1$, so there is a competition among lenders. Each lender offers a loan contract consisting of a pair (R_i, π_i), where R_i is the gross interest rate charged to a borrower of type i and π_i is the probability that a loan to a borrower of type i will be granted. The gross interest rate is paid by a borrower in the second period after a random return of a project becomes known. From a limited liability of a borrower it follows, that in case of failure, his payment to a lender is zero.

The probability of granting a loan is incorporated into a loan contract offer because it enables lenders to separate the borrowers using the self-selection mechanism. If the contract were to consist only of the interest rate there would be no way for a lender to distinguish between the two types of borrowers; all borrowers would apply for the same interest rate in equilibrium.

Each lender can provide at most one loan. Hence each lender can be viewed as making a choice of which type of borrower he would prefer to lend to; the lender can only choose between two types of borrower. Because the lenders are homogenous we can suppose that, if there exists a unique equilibrium contract (R_L, π_L), in an equilibrium, all lenders lending to type L borrowers will offer the same interest rate R_L and the same probability of granting a loan π_L. The same reasoning applies for the lenders borrowing to type H borrowers.

The game of this model has two stages:

Stage 1: Lenders choose a loan contract to offer, taking the offers of other lenders as given.

Stage 2: Borrowers observe the offers from stage 1 and then choose to apply for the loan contracts they view as most attractive. We assume that each borrower can apply for only one loan.

As a solution to this game, a standard Nash equilibrium definition applies: a Nash equilibrium is a set of contract offers (R_i^*, π_i^*), for $i = L,H$, such that given these offers, no lender has an incentive to offer a different loan contract. Generally there are three kinds of outcome of this game:

1. No equilibrium.
2. Pooling equilibrium, in which both types of borrowers choose the same contract. This means that borrowers are pooled together and information about the type of each borrower is not revealed.
3. Separating equilibrium, in which each type of borrower chooses his type specific contract. Borrowers are in this way separated into two groups and information about the type of each borrower is revealed

2. The Equilibrium in the Absence of a Government Intervention

In the absence of government intervention, the expected utility of a type i borrower receiving a type j loan contract will be:

$$U_{ij} = \pi_j P_i (y - R_j) + (1 - \pi_j)\beta_i, \qquad i,j = L,H. \tag{2}$$

The first term in (2) $\pi_j P_i(y - R_j)$ represents the expected utility from operating a project funded by a lender. The second term $(1-\pi_j)\beta_i$ is an expected utility from a utilization of outside opportunities occurring when the borrower does not obtain a loan.

In a separating equilibrium, each borrower of type i will receive either no contract or the type i contract, i.e. the contract of his type. In equilibrium, U_{ii} will be maximized subject to the self-selection constraints:

$$U_{ii} \geq U_{ij}, \qquad i,j = L, H, \ i \neq j, \tag{3.i}$$

and a zero-profit condition for lenders serving either type of borrower:

$$P_i R_i - 1 = 0. \qquad i = L, H. \tag{4}$$

Self-selection constraints mean that the type i borrower does not obtain a higher utility by obtaining a type j borrower contract. So as long as the self-selection constraints are satisfied, each borrower in separating equilibrium reveals his type by choosing the contract designed for his type.

A zero-profit condition is brought about by a competition between lenders. Its meaning is evident: The expected revenue obtained in the second period, which implies the lender's expected second-period consumption $c_2 = P_i R_j$, is equal to the lender's opportunity cost of his loan. This opportunity cost is given by a lender's first-period consumption $c_1 = 1$, which would be possible if the lender did not loan his one unit of fund's endowment.

The only binding constraint in (3.i) is the constraint for maximization problem of a low-risk borrower

$$U_{HH} = U_{HL}, \tag{3.ii}$$

which says that, in order to separate low-risk borrowers from high-risk borrowers, the contract designated for a low-risk borrower has to be such that the high-risk borrower cannot improve his utility by a deviation from a contract designated for him to the contract designated for a low-risk borrower.

In order to solve the equilibrium value of π_L we substitute the appropriate expected utilities from (2) into (3.ii) using the equilibrium value of $\pi_H{}^* = 1$:

$$1P_H(y - R_H{}^*) + (1-1)\beta_H = \pi_L{}^* P_H(y - R_L{}^*) + (1 - \pi_L{}^*)\beta_H. \tag{5.i}$$

We substitute in an equation (5.i) for $R_i{}^*$ from (4) and we obtain

$$P_H(y - 1/P_H) = \pi_L{}^* P_H(y - 1/P_L) + (1 - \pi_L{}^*)\beta_H. \tag{5.ii}$$

Finally we express from (5.ii) the equilibrium value of a probability of granting a loan contract to a low-risk borrower

$$\pi_L{}^* = \frac{y - \left(\dfrac{1}{P_H} + \dfrac{\beta_H}{P_H}\right)}{y - \left(\dfrac{1}{P_L} + \dfrac{\beta_H}{P_H}\right)} < 1. \tag{5.iii}$$

Because $P_H < P_L$ it follows that $\pi_L{}^* < 1$. This means that a positive fraction $(1 - \pi_L{}^*)$ of low-risk borrowers willing to pay the market risk-adjusted interest rate $R_L{}^*$ will not receive loans, while a fraction $\pi_L{}^*$ of otherwise identical borrowers will receive loans. Thus, lenders sort borrowers into risk classes ex post, by making it tougher to obtain low-interest loans, by granting only a fraction of loan applications that desire the low interest rate. If this credit rationing were not in place, high-risk borrowers would apply for the lower interest loan designated for low-risk borrowers and there would be no separating equilibrium.

Recalling that θ is the population fraction of high-risk borrowers, the pooling contract (R_θ, π_θ) earns non-negative profit to a lender, if the size of interest rate R_θ and an expected probability of success in (4) are such that

$$[\theta P_H + (1-\theta)P_L]R_\theta \geq 1, \tag{6}$$

where the term $[\theta P_H + (1-\theta)P_L]$ is an expected probability of success of a borrower of an unknown type.

If the proportion of low-risk borrowers is high enough, the problem of a cross-subsidization of high risk-borrowers by low-risk borrowers diminishes.

3. Government Interventions

3.1. Non-Targeted Loan Guarantees

We suppose that the government offers to guarantee a fraction α of the amount of each private loan made to borrowers. The utility function of a borrower is still (2), because the borrower does not care if his loan is guaranteed or not. He is only interested in the probability of obtaining a loan and in the required interest rate on it.

The zero profit condition for lenders in this case is no longer given by (4), but by

$$P_i R_i + \alpha(1-P_i)R_i - 1 = 0, \; i = L, H. \tag{7}$$

The first term in (7), $(P_i R_i)$, is an expected revenue to a lender from a successful project. The second term, $\alpha(1-P_i)R_i$, is an expected return to a lender from a guaranteed portion of an unsuccessful project. The opportunity cost of lending one unit of funds is 1, which is a third term in (7).

The social consequences of this program are as follows: by increasing π_L, the expected number of funded projects will increase, thus increasing an expected output and consumption. A reasonable measure of efficiency must consider the consumer welfare derived from increased efficiency is by evaluating changes in the expected output of funded projects minus the cost of inputs employed in production. These costs of inputs per additional investment project operated are one unit of capital investment plus the opportunity cost of effort β. Total welfare defined in this way can be written as V^* (V^G) for case without guarantees (with guarantees):

$$V^* = (1-\theta)\pi_L{}^*[P_L y - (1+\beta_L)] + \theta\pi_H{}^*[P_H y - (1+\beta_H)]$$

$$(V^G = (1-\theta)\pi_L^G[P_L y - (1+\beta_L)] + \theta\pi_H^G[P_H y - (1+\beta_H)]),$$

where $\pi_H{}^* = \pi_H^G = 1$.

Thus the expected change in efficiency arising from the loan guarantee program as compared to a situation without government intervention is a change in total welfare:

$$V^G - V^* = (1-\theta)(\pi_L^G - \pi_L{}^*)[P_L y - (1+\beta_L)]. \tag{8}$$

The expression (8) says that the change in the efficiency is given as an expected net benefit from one low-risk project, given that a project is financed and undertaken, $[P_L y - (1+\beta_L)]$, multiplied by an increase in a probability of obtaining a finance for a low-risk project under a loan guarantee regime $(\pi_L^G - \pi_L^*)$, multiplied by a fraction of low-risk borrowers in a population $(1-\theta)$.

The efficiency measure used in this model can be rationalized by assuming that consumers' expected utilities are linear in consumption (i.e. project output y). We have also implicitly assumed that the government's losses on its loan programs are financed by a nondistortionary lump sum taxation, which means that the government's losses are just a transfer payment, which is neutral from the efficiency evaluation point of view. In the absence of ideal lump sum taxes our efficiency measure has to be adjusted for the inefficiency of a real world government taxes used to finance government losses on a loan guarantees program.

3.2. Direct Targeted Loans

Suppose the government offers to finance at an interest rate R_g a fraction α of loans denied by private lenders. It means that π_j a fraction of type j borrowers' projects is financed by loans from commercial lenders, $\alpha(1-\pi_j)$ percent is financed by a government finance and $(1-\alpha)(1-\pi_j)$ percent of type j borrowers' projects is not financed at all, and consequently not undertaken. This policy is similar to actual "targeted" direct loan programs, which attempt to verify that loans are granted only to those borrowers who cannot obtain financing from commercial lenders.

The zero profit condition is again given by (4), so equilibrium interest rates are the same as in the model without government intervention

$$R_i^D = R_i^* = 1/P_i \quad (i = L,H).$$

The expected utility of a type i borrower given type j contract will not be (2) like in the model without a government intervention, but it will be rather:

$$U_{ij} = \pi_j P_i(y-R_j)+(1-\pi_j)P_i\alpha(y-R_g)+(1-\pi_j)(1-\alpha)\beta_i, \quad i,j = L,H. \tag{9}$$

The first term in (9) is the same as a first term in (2) and represents the expected utility of a borrower derived from operating a project funded through a commercial lender. The second term is the expected utility from a government-funded project. The third term is the expected utility of outside opportunities occurring when the project is not undertaken.

We assume that the marginal expected utility associated with an increase in the probability of obtaining a direct government loan $\left(\frac{\partial U_{ii}}{\partial \alpha}\right)$ is positive

$$\frac{\partial U_{ii}}{\partial \alpha} = (1-\pi_i)P_i(y-R_g)-\beta_i(1-\pi_i) > 0, \tag{9.i}$$

so that direct loans will be taken when offered. A sufficient condition for this is obtained by expressing y from (9.i):

$$y > R_g + \frac{\beta_i}{P_i}. \tag{9.ii}$$

The high-risk borrowers are again not rationed and $\pi_H^D = 1$. In order to obtain the equilibrium value of π_L^D we substitute appropriate utilities from (9) into a single binding self-selection constraint $U_{HH} = U_{HL}$ using condition $\pi_H^D = 1$:

$$P_H(y-R_H) = \pi_L P_H(y-R_L)+(1-\pi_L)P_H\alpha(y-R_g)+(1-\pi_L)(1-\alpha)\beta_H. \tag{9.iii}$$

After using a zero profit condition (4) to substitute for R_i in (9.iii) and after some algebraic manipulations we obtain

$$\pi_L^D = \frac{y-\left[\frac{1}{P_H}+\frac{\beta_H}{P_H}+\alpha(y-R_g-\frac{\beta_H}{P_H}\right]}{y-\left[\frac{1}{P_L}+\frac{\beta_H}{P_H}+\alpha(y-R_g-\frac{\beta_H}{P_H}\right]} \tag{10}$$

which reduces to (5.iii) when $\alpha = 0$.

Proposition 1: *The government lending crowds out commercial lending and for $R_g < R_L^D$ this crowding out is on a greater than one-to one basis.*

From Proposition 1 it follows that $\pi_L^D < \pi_L^* < 1$. Also, the social welfare effects of direct loans are more complex than those of loan guarantees. For while the additional funding of projects by the government will increase net output, the reduction in π_L implies a reduction of the number of projects financed through private commercial lenders. The change in an efficiency as compared to a situation without an intervention is thus given by replacing π_L^G in (8) by term $\pi_L^D + (1-\pi_L^D)\alpha$:

$$(1-\theta)[\pi_L^D + (1-\pi_L^D)\alpha - \pi_L^*][P_L y - (1+\beta_L)]. \quad (11)$$

Proposition 2: *The change in efficiency (11) has the same sign as the government's expected profit on government loans $(P_L R_g - 1)$, which in turn has the opposite side of the expected change in borrowers' utility caused by an introduction of direct targeted government loans $(U_{LL}^D - U_{LL}^*)$:*

$$sign((12)) = sign(P_L R_g - 1) = -sign(U_{LL}^D - U_{LL}^*). \quad (12)$$

It follows from Proposition 2 that efficiency is increased only when the government obtains profits and the utility of low-risk borrowers decreases in comparison with a situation without intervention. In that case, low-risk borrowers as a group will expect ex ante to be worse off, both because of the reduced probability of receiving private lenders loans (π_L) and because of an interest rate $R_g > R_L$.

In case when government programs are aimed at aiding the group of low-risk borrowers (who are rejected by private lenders) to increase their utility, according to equation (12), the government inevitably incurs losses. This also means, according to (12), a decrease in a social economic efficiency.

4. Conclusions

The principal result is that the welfare effects of credit support are not qualitatively indifferent to the determination of eligibility for government support or to the method of support chosen by a government. These results could look counterintuitive at first. One could expect that a targeted program should achieve better results and be more cost effective (not counting cost, which has to be incurred to distinguish between a targeted group and the rest of a population of borrowers) than non-specialized global programs open to all borrowers. Also one could intuitively argue that the support should be targeted to the most efficient group of low-risk borrowers, whose credit applications were rejected by lenders.

The main reason for the seemingly counterintuitive result of a presented model consists of the existence of informational asymmetry and a consequent need for a lender to create a mechanism which would identify the risk class of a borrower. The mechanism used by a lender to achieve a self-selection of borrowers into two risk groups is a reduction of a probability of granting a low-risk loan. This means the introduction of credit rationing for low-risk borrowers.

If the government offers subsidized credit (either direct credit or guaranteed loans) only to a proportion of the low-risk borrowers who were rejected by private lenders, this government intervention makes a low-risk contract more attractive to high-risk borrowers. Therefore, in order to restore incentive compatibility (to enable a separation between low and high-risk borrowers) some other aspect of the low-risk contract must become less desirable. That means that the overall probability of obtaining a loan has to fall. In this way increased subsidies to the rationed borrowers raise the extent of rationing. The loans from commercial lenders are crowded out on a greater than one-to-one basis. This is an equilibrium response and it is due to the existence of the incentive-compatibility constraint.

Targeted support faces an inevitable trade off either to increase the utility of some borrowers and to decrease the chance of other borrowers to obtain a loan and, in addition, to decrease the overall social efficiency or to increase the social efficiency by decreasing the expected utility of low-risk borrowers.

References

Innes, Robert, *Investment and Government Intervention in Credit Markets When There is Asymmetric Information*, Journal of Public Economics, 46 (3), pp. 347-381, 1991.

Smith, Bruce, and Stutzer, Michal, *Credit Rationing and Government Loan Programs: A Welfare Analysis*, Working paper 395, Federal Reserve Bank of Minneapolis, 1988.

Stiglitz, Joseph, E., and Weiss, Andrew, *Credit Rationing in Markets with Imperfect Information*, American Economic Review, 71, pp. 393-411, 1981.

Stiglitz, Joseph, E., and Weiss, Andrew, *Credit Rationing and Collateral. In Recent Development in Corporate Finance*, Cambridge University Press, New York, 1986.

Rothschild, Michael, and Stiglitz, Joseph, E., *Equilibrium in Competitive Insurance markets: An essay on the Economics of Imperfect Information*, Quarterly Journal of Economics, 90 (4), pp. 629-649, 1976.