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PRODUCTIVITY OF OIL WELLS IN ARBITRARILY SHAPED RESERVOIRS

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1. INTRODUCTION

A Boundary Element approach for predicting the productivity of oil wells arranged in complex configurations within irregularly shaped reservoirs were developed. The integral equations are written for boundary points as well as for the locations of the wells which are treated as point sources and sinks with specified pressures but unknown strengths. Using this approach, the solution to the resulting matrix gives the values of the nodal boundary pressure and their normal derivatives, as well as the unknown flow rates of all the wells.

2. PROBLEM FORMULATION

Consider a hypothetical two-dimensional homogeneous reservoir S having NSS sources and/or sinks located randomly within an arbitrarily shaped reservoir. The following assumptions were used in developing the theory: a) the reservoir is in steady-state flow with reservoir pressure above bubble points *i.e.* undersaturated condition; b) single phase fluid having small (and constant) compressibility and constant viscosity is flowing in the system; c) the reservoir has a uniform thickness and it has a finite boundary; and d) gravitational effects are negligible.

The differential equation describing the unknown functions *i.e.* pressure, at all points in the reservoir is obtained by the introduction of Darcy's law into the continuity equation. By imposing the conditions and assumptions stated above, the differential equation describing the pressure distribution in the reservoir is [1,2]:

$$\frac{\partial^2 p}{\partial X^2}+\frac{\partial^2 p}{\partial Y^2}+\frac{\mu}{k}\sum_{m=1}^{NSS} q_m\ \delta(X-X_m, Y-Y_m)=0 \qquad (1)$$

where p is pressure, μ is the dynamic viscosity of the fluid, k is the permeability, q_m is the flow rate of the m^{th} well per unit area (positive for injectors and negative for producers), δ is the Dirac delta function, X, Y are coordinates axes, and X_m , Y_m are coordinates of the m^{th} source and/or sink where m goes from 1 to NSS.

Equation (1) can be transformed into an integral equation by multiplying it with the free-space Green's function and integrating it twice by parts. The free-space Green's function is also called the fundamental solution [1,2,3] and is given as:

$$G=\frac{1}{2\pi}\ln\left(\frac{1}{r}\right) \qquad (2)$$

where r is the distance between a field point (X, Y) and a point of application of a unit charge (X_C, Y_C). After standard manipulation [1], equation (1) then becomes:

$$\alpha p\,(X_i, Y_i) = \frac{1}{2\pi} \sum_{j=1}^{N} \frac{\partial p}{\partial n_j} \int_{sj} \ln\left(\frac{1}{r_{i,j}}\right) ds \, -$$

$$\frac{1}{2\pi} \sum_{j=1}^{N} p_j \int_{sj} \frac{\partial}{\partial n}\left[\ln\left(\frac{1}{r_{i,j}}\right)\right] ds \; + \frac{1}{2\pi}\,\frac{\mu}{k} \sum_{m=1}^{NSS} q_m \ln\left(\frac{1}{r_{i,m}}\right) \qquad (3)$$

where the boundary of the reservoir is divided into N constant elements with constant properties as shown in Figure 1. α is the included angle at the i^{th} pivot point. It is assigned a value of ½ when the pivot point is on a smooth boundary (*i.e.* not on a corner), and a value of 1 when the pivot point is inside the problem domain. For simplicity, let

$$G_{i,j} = \frac{1}{2\pi} \int_{sj} \ln\left(\frac{1}{r_{i,j}}\right) ds \qquad (4)$$

$$H_{i,j} = \frac{1}{2\pi} \int_{sj} \frac{\partial}{\partial n}\left[\ln\left(\frac{1}{r_{i,j}}\right)\right] ds \qquad (5)$$

$$GSS_{i,m} = \frac{1}{2\pi} \ln\left(\frac{1}{r_{i,m}}\right) \qquad (6)$$

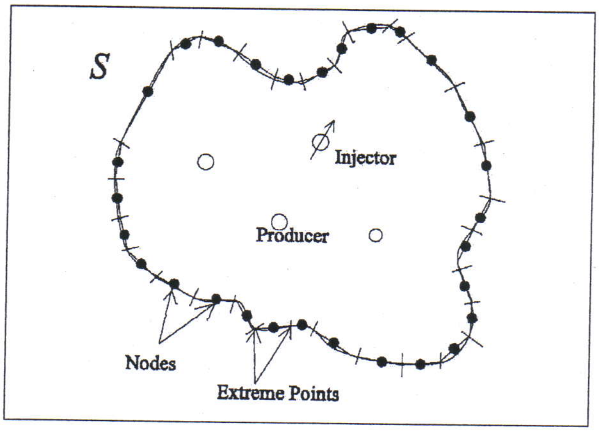

Figure 1: Reservoir having *NSS* sources and sinks where its boundary is divided into *N* segments or elements

where X_i , Y_i are coordinates of any pivot point, $r_{i,j}$ is the distance between the pivot point and the j^{th} element where j runs from 1 to N, and $r_{i,m}$ is the distance between the pivot point and the m^{th} source and/or sink. Equation (3) now simplifies to:

$$\alpha p\,(X_i, Y_i) = \sum_{j=1}^{N} \frac{\partial p}{\partial n_j} G_{i,j} - \sum_{j=1}^{N} p_j H_{i,j} + \sum_{m=1}^{NSS} q_m\, GSS_{i,m} \qquad (7)$$

The boundary of the reservoir, S, can be of the type S_p or $S_{dp/dn}$ or a combination of the two types. Over the S_p type boundary, the pressure p is specified as constant throughout the element while dp/dn is unknown. Over the $S_{dp/dn}$ type boundary, the dp/dn is prescribed as constant and the pressure p is unknown. Similarly, the sources and/or sinks can also have known and unknown rates. For the known flow rate well, the well-bore pressure, p_w is unknown and for the unknown flow rate well, the well-bore pressure is prescribed.

The idea is to apply Equation (7) at all the boundary nodes ($\alpha = ½$), as well as at the entire source and/or sink locations ($\alpha = 1$). By doing so, a system of $N+NSS$ equations with $N+NSS$ unknown can be obtained and simplified to matrix form as follows:

$$[HGGSS] \ \vec{U} = \vec{A} \qquad (8)$$

where $[HGGSS]$ consists of the coefficients H, G and GSS. The vector $\vec{U}$ contains all the $N+NSS$ unknowns of p, dp/dn, p_w and q and $\vec{A}$ is a vector containing all the known values.

3. VALIDATION

The flow rates obtained from Muskat's analytical equations [4] are compared with the BEM solutions for a circular battery of wells located at a radius r = 50 feet in a circular reservoir as of radius R = 5,000 feet as shown in Figure 2. The wells in the battery are symmetric about the center. In order to have uniform pressures around the boundary of the reservoir, it was necessary to place the center of the battery at the reservoir center.

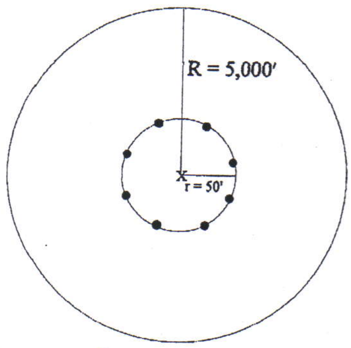

Figure 2: A circular battery of n wells at the center of a circular reservoir

The ratio of the total production of the battery, Q_n to the production of a single well, Q_1 is plotted against the number of wells and compared with the Muskat's results as shown in Figure 3. The perfect match of the plots in Figure 3 clearly shows that the BEM solutions agree with the Muskat's analytical solutions.

Even though regular well patterns and boundary geometries are presented in these example applications, this was done simply to allow comparison with published analytical solutions. The method is equally applicable to non-pattern well clusters arbitrarily located in reservoirs with irregular boundary shapes.

4. CONCLUSIONS

The concept of formulating differential equations at source and/or sink points as well as at boundary node points was investigated and found to give excellent results. The formulation has the advantage of calculating the unknown source and/or sink rates directly as part of the matrix solution.

Other potential uses include (i) the calculation of the production of individual wells within leases in a multiple lease reservoir and (ii) the identification of candidate wells in a field that may need work-over by comparing the predicted production rates with the actual field production rates.

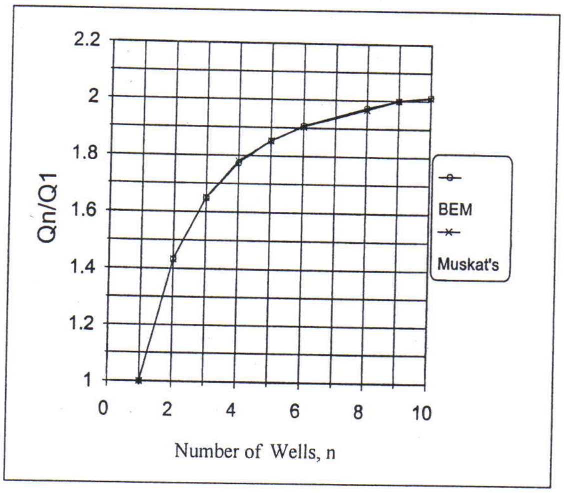

Figure 3: Comparison between Muskat's solution and the BEM solution

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