

UNIVERSITI SAINS MALAYSIA

Supplementary Semester Examination
Academic Session 2005/2006

June 2006

IUK 291E – Mathematic II
[Matematik II]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of FIVE (5) pages of printed material before you begin the examination.

Instructions:

1. Answer **ALL** questions. All questions can be answered either in Bahasa Malaysia OR English.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA (5) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Arahan:

1. Jawab **SEMUA** soalan. Semua soalan boleh dijawab dalam Bahasa Malaysia **ATAU** Bahasa Inggeris.

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Answer all questions.

1. (a) If $f(x,y) = \sin^{-1} xy$, verify that $f_{xy} = f_{yx}$ (10 marks)
- (b) Find all points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + y - 3z = 2$. (10 marks)
2. (a) Find the volume of the solid region common to the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $r = 2 \cos \theta$. (10 marks)
- (b) Evaluate the double integral $\int_0^{2\pi} \int_0^a \frac{1}{a+r} r \, dr \, d\theta$ (10 marks)
3. (a) Use variation of parameters to solve the differential equation $y'' - y' = 2 \cos^2 x$ that satisfies the initial conditions $y(0) = 0$, $y'(0) = 1$. (7 marks)
- (b) A dairy product produces whole milk and skim milk in quantities x and y liters, respectively. Suppose the price (in ringgit) of whole milk is $p(x) = 100 - x$ and that of skim milk is $q(y) = 100 - y$, and also assume that $C(x, y) = x^2 + xy + y^2$ is the joint-cost function of the commodities. Maximize the profit
- $$P(x, y) = p x + q y - C(x, y)$$
- (7 marks)
- (c) Use the Lagrange multipliers to find the minimum distance from the origin to the ellipse $5x^2 - 6xy + 5y^2 = 4$ (6 marks)

4. (a) Use the binomial series to obtain the power series expansion of $\frac{1}{\sqrt{1-x^2}}$
(10 marks)

- (b) Use partial fractions to find the Maclaurin series for the function

$$f(x) = \frac{2x-3}{x^2-3x+2}$$

(10 marks)

5. (a) Use the Ratio Test to test the series $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ for convergence.

(Hint: $\lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k = e$).

(6 marks)

- (b) What is a p-series? Specify the value p and tell whether the p-series

$$\sum_{k=1}^{\infty} \frac{1}{2k\sqrt{k}}$$

converges or diverges.

(6 marks)

- (c) A Cobb-Douglas production function is an output function of the form $Q(x,y) = cx^\alpha y^\beta$ with $\alpha + \beta = 1$. Show that such a function is maximized

with respect to the fixed cost $px + qy = k$ when $x = \frac{\alpha k}{p}$ and $y = \frac{\beta k}{q}$.

(8 marks)

Jawab semua soalan.

1. (a) Jika $f(x,y) = \sin^{-1} xy$, tentulahkan bahawa $f_{xy} = f_{yx}$ (10 markah)
- (b) Cari semua titik diatas sfera $x^2 + y^2 + z^2 = 1$ dimana satah tangen adalah selari dengan satah $2x + y - 3z = 2$. (10 markah)
2. (a) Cari isipadu pepejal bagi rantau yang sama pada sfera $x^2 + y^2 + z^2 = 4$ dan silinder $r = 2 \cos \theta$. (10 markah)
- (b) Nilaikan kamiran berganda $\int_0^{2\pi} \int_0^a \frac{1}{a+r} r dr d\theta$ (10 markah)
3. (a) Dengan menggunakan ubahan parameter, selesaikan persamaan pembezaan $y'' - y' = 2 \cos^2 x$ yang memenuhi syarat awal $y(0) = 0$, $y'(0) = 1$. (7 markah)
- (b) Satu hasil tenusu mengeluarkan susu pekat dan susu cair dalam kuantiti x dan y liter, masing-masing. Katakan harga (dalam ringgit) bagi susu pekat ialah $p(x) = 100 - x$ dan bagi susu cair ialah $q(y) = 100 - y$, dan anggapkan juga bahawa $C(x, y) = x^2 + xy + y^2$ ialah kos bersama bagi kedua-dua komoditi. Maksimumkan keuntungan
- $$P(x, y) = p x + q y - C(x, y)$$
- (7 markah)
- (c) Guna pendarab Lagrange untuk mencari jarak minimum dari asalan ke elips $5x^2 - 6xy + 5y^2 = 4$ (6 markah)

4. (a) Guna siri binomial untuk mendapatkan kembangan siri kuasa bagi $\frac{1}{\sqrt{1-x^2}}$ (10 markah)
- (b) Guna pecahan separa untuk mencari siri Maclaurin bagi fungsi $f(x) = \frac{2x-3}{x^2-3x+2}$ (10 markah)
5. (a) Guna Ujian Nisbah untuk menguji siri $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ bagi penumpuan. (Petunjuk: $\lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k = e$). (6 markah)
- (b) Terangkan siri $-p$? Dapatkan nilai p dan nyatakan samada siri- p bagi $\sum_{k=1}^{\infty} \frac{1}{2k\sqrt{k}}$ menumpu atau mencapah. (6 markah)
- (c) Suatu fungsi pengeluaran Cobb-Douglas ialah suatu fungsi output dalam bentuk $Q(x,y) = cx^\alpha y^\beta$ dengan $\alpha + \beta = 1$. Tunjukkan fungsi tersebut dimaksimumkan terhadap kos tetap $px + qy = k$ bila $x = \frac{\alpha k}{p}$ dan $y = \frac{\beta k}{q}$. (8 markah)