UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2004/2005

March 2005

IUK 291E – Mathematics II
[Matematik II]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of SEVEN (7) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH (7) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Answer **FOUR (4)** questions. All questions can be answered either in Bahasa Malaysia or English.

*[Jawab **EMPAT (4)** soalan. Semua soalan boleh dijawab dalam Bahasa Malaysia atau Bahasa Inggeris].*

1. (a) Let f be the function defined by $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ for $(x,y) \neq (0,0)$.

(i) Find $\lim\limits_{(x,y)\to(2,1)} f(x,y)$

(5 marks)

(ii) Show that f has no limit at (0,0) by showing f(x,y) tends toward different numbers as $(x,y) \to (0,0)$ along each coordinate axis.

(5 marks)

(b) Let $z = e^{-t}(\sin\frac{x}{c} + \cos\frac{x}{c})$ where c is a constant. Show that z satisfies the equation $\dfrac{\partial z}{\partial t} = c^2 \dfrac{\partial^2 z}{\partial x^2}$

(8 marks)

(c) Find the sum of the series $\sum\limits_{k=0}^{\infty}\left[\left(\frac{-3}{8}\right)^k + \left(\frac{3}{4}\right)^{2k}\right]$

(7 marks)

(a) Biar f ditakrifkan sebagai fungsi $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ *untuk* $(x,y) \neq (0,0)$.

(i) Cari $\lim\limits_{(x,y)\to(2,1)} f(x,y)$

(5 markah)

(ii) Tunjukkan f tidak ada had pada (0,0) dengan menunjukkan f(x,y) menumpu kepada nombor berlainan apabila $(x,y) \to (0,0)$ *sepanjang setiap paksi koordinat.*

(5 markah)

...3/-

(b) *Biar* $z = e^{-t}(\sin\frac{x}{c} + \cos\frac{x}{c})$ *dimana c adalah pemalar. Tunjukkan bahawa z memenuhi persamaan* $\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2}$

(8 markah)

(c) *Cari jumlah bagi siri* $\sum_{k=0}^{\infty}\left[\left(\frac{-3}{8}\right)^k + \left(\frac{3}{4}\right)^{2k}\right]$

(7 markah)

2. (a) If $z = \frac{x}{x-y}$ and $w = \frac{y}{x-y}$, compute the total differentials dz and dw.

Why are both total differentials equal?

(8 marks)

(b) If $R = u + f(u^2v^2)$ and let $S = u^2v^2$, apply the chain rule to show that

$$u\frac{\partial R}{\partial u} - v\frac{\partial R}{\partial v} = u$$

(7 marks)

(c) Find the solution of the non-homogeneous differential equation $y'' + 2y' + 2y = \cos x$ that satisfies the initial conditions $y(0) = 0, y'(0) = -4$

(10 marks)

(a) *Jika* $z = \frac{x}{x-y}$ *dan* $w = \frac{y}{x-y}$, *dapatkan pembezaan keseluruhan dz dan dw.*
Kenapa kedua-dua pembezaan keseluruhan ini sama?

(8 markah)

(b) *Jika* $R = u + f(u^2v^2)$ *dan biarkan* $S = u^2v^2$, *guna petua rantai untuk menunjukkan* $u\frac{\partial R}{\partial u} - v\frac{\partial R}{\partial v} = u$

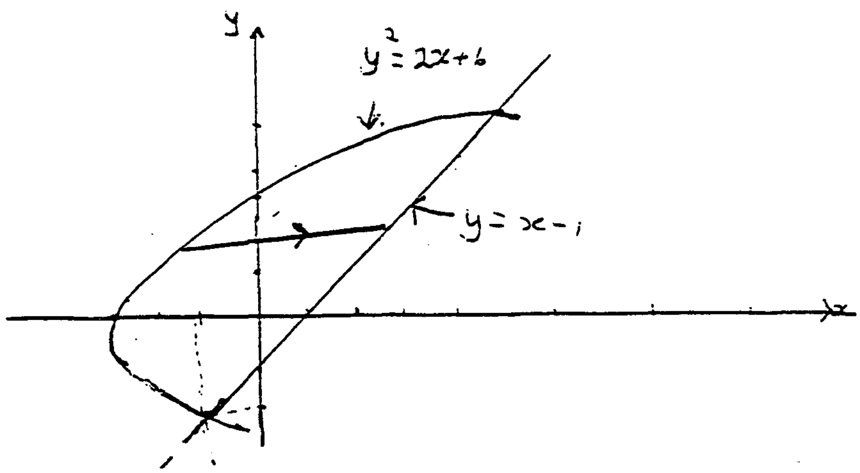
(7 markah)

(c) *Dapatkan penyelesaian bagi persamaan pembezaan tak seragam* $y'' + 2y' + 2y = \cos x$ *yang memenuhi syarat-syarat awal* $y(0) = 0$, $y'(0) = -4$

(10 markah)

3. (a) Evaluate $\iint_D xy\,dA$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$ as shown in the figure below.

(10 marks)



(b) Let $f(x) = \dfrac{5+x}{2-x-x^2}$

(i) By using partial fractions, express f(x) as sum of two terms.

(5 marks)

(ii) Express both of the terms in (i) as the sum of a geometric series.

(5 marks)

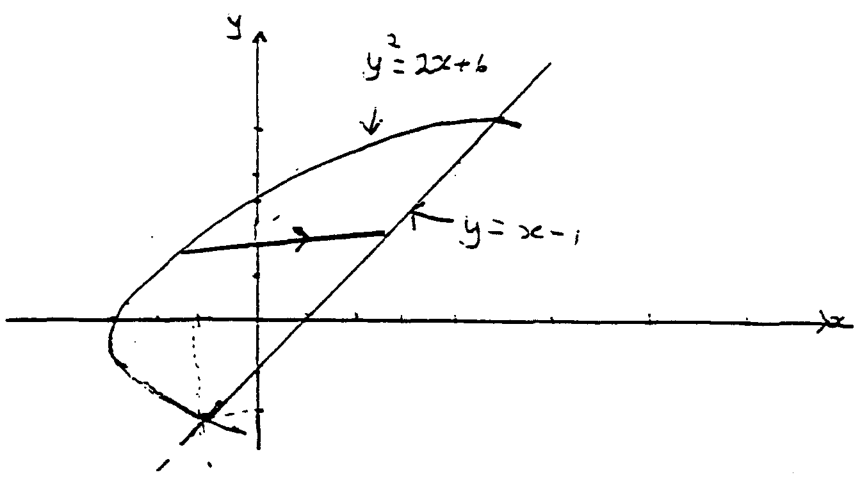
(iii) Use the result in (ii) to show that the Maclaurin series for f(x) is

$$\sum_{k=0}^{\alpha}\left[2x^k + (-1)^k \frac{1}{2}\left(\frac{x}{2}\right)^k\right]$$

(5 marks)

(a) Nilaikan $\iint_D xy\,dA$ *dimana D ialah kawasan yang dibatasi oleh garis y = x-1 dan lengkungan parabola* $y^2 = 2x+6$ *seperti yang ditunjukkan oleh gambarajah di bawah;*

(10 markah)



(b) *Biar* $f(x) = \dfrac{5+x}{2-x-x^2}$

(i) *Dengan menggunakan pecahan separa, nyatakan f(x) sebagai jumlah dua sebutan.*

(5 markah)

(ii) *Nyatakan dua sebutan yang diperolehi daripada (i) sebagai jumlah siri geometrik*

(5 markah)

(iii) *Guna keputusan daripada (ii) untuk menunjukkan siri Maclaurin bagi f(x) ialah* $\sum_{k=0}^{\alpha}\left[2x^k + (-1)^k \frac{1}{2}\left(\frac{x}{2}\right)^k\right]$

(5 markah)

4. (a) Obtain the Fourier series expansion for the function

$$f(x) = \begin{cases} 0 & \text{for } -4 \le x < 0 \\ 2 & \text{for } 0 \le x < 4 \end{cases}$$

for which the period is 8.

(8 marks)

(b) (i) Find all the critical points on the graph of $f(x,y) = 8x^3 - 24xy + y^3$

(5 marks)

(ii) Use the second partial test to classify each point

(5 marks)

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(c) A cylindrical can is to hold 4 Πcm^3 of orange juice. The cost per square centimeter of constructing the metal top and bottom is twice the cost per square centimeter of constructing the cardboard side. What are the dimensions of the least expensive can. Assume x and y is the radius and height of the cylinder respectively.

(7 marks)

(a) Dapatkan kembangan siri Fourier bagi fungsi

$$f(x) = 0 \quad bagi \quad -4 \leq x < 0$$
$$= 2 \quad bagi \quad 0 \leq x < 4$$

dimana kalaannya ialah 8.

(8 markah)

(b) (i) Cari semua titik kritikal di atas graf $f(x,y) = 8x^3 - 24xy + y^3$.

(5 markah)

(ii) Guna ujian separa kedua untuk mengkelaskan setiap titik.

(5 markah)

(c) Satu tin berbentuk silinder boleh diisi dengan $4\Pi cm^3$ *jus oren. Harga kos satu persegi sentimeter logam bagi membina atas dan bawah tin ialah dua kali harga bagi membina sisi tin. Apakah dimensi tin yang boleh memberikan harga kos yang paling rendah? Anggap x dan y sebagai jejari dan tinggi bagi tin silinder masing-masing.*

(7 markah)

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