UNIVERSITI SAINS MALAYSIA

Supplementary Semester Examination
Academic Session 2004/2005

May 2005

IUK 291E – Mathematic II
[Matematik II]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of <u>SIX</u> (6) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>ENAM</u> (6) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions:
1.   Answer **FOUR (4)** questions. All questions can be answered either in Bahasa Malaysia or English.

Arahan:
1.   Jawab EMPAT (4) soalan. Semua soalan boleh dijawab dalam Bahasa Malaysia atau Bahasa Inggeris.

Answer <u>ALL</u> questions.

Jawab <u>SEMUA</u> soalan.

1. (a) By using the Ratio Test, show that the series $\sum_{k=1}^{\infty} \dfrac{2^k}{k!}$ converge.

(10 marks)

(b) Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \dfrac{2^k x^k}{k!}$

(10marks)

(c) Let $z = e^{-t}(\sin\dfrac{x}{c} + \cos\dfrac{x}{c})$ where c is a constant. Show that z satisfies the

equation $\dfrac{\partial z}{\partial t} = c^2 \dfrac{\partial^2 z}{\partial x^2}$

(5 marks)

(a) *Dengan menggunakan Ujian Nisbah, tunjukkan bahawa* $\sum_{k=1}^{\infty} \dfrac{2^k}{k!}$ *menumpu.*

(10 markah)

(b) *Cari jeda penumpuan bagi siri kuasa* $\sum_{k=1}^{\infty} \dfrac{2^k x^k}{k!}$

(10 markah)

(c) *Biar* $z = e^{-t}(\sin\dfrac{x}{c} + \cos\dfrac{x}{c})$ *dimana c adalah pemalar. Tunjukkan bahawa z memenuhi persamaan* $\dfrac{\partial z}{\partial t} = c^2 \dfrac{\partial^2 z}{\partial x^2}$

(5 markah)

2. (a) Find the Maclaurin series for f(x) = cos x. By letting u = 2x, find the first four terms of the Maclaurin series for $\cos^2 x$.

(Use the trigonometric identity $\cos^2 x = \dfrac{1}{2} + \dfrac{1}{2}\cos 2x$)

(10 marks)

(b) Let f(x, y, z) = xy +yz +xz, and let P_0 denote the point (1,2,-1).

Find (i) the gradient of f at P_0

(ii) the directional derivative of f in the direction from P_0 toward the point Q (-1, 1 -1).

(8 marks)

(c) Find the solution of the non-homogeneous differential equation $y'' + 2y' + 2y = \cos x$ that satisfies the initial conditions y(0)=0, y'(0)=-4

(7 marks)

(a) *Dapatkan siri Maclaurin bagi f(x) = cos x. Biarkan u = 2x, dapatkan empat sebutan pertama siri Maclaurin bagi $\cos^2 x$.*

(Guna identity trigonometri $\cos^2 x = \dfrac{1}{2} + \dfrac{1}{2}\cos 2x$)

(10 markah)

(b) *Biar f(x, y, z) = xy +yz +xz, dan P_0 mewakili titik (1,2,-1).*

Cari (i) kecerunan f dititik P_0

(ii) terbitan berarah f dari arah P_0 ke arah titik Q (-1, 1, -1).

(8 markah)

(c) *Dapatkan penyelesaian bagi persamaan pembezaan tak seragam $y'' + 2y' + 2y = \cos x$ yang memenuhi syarat-syarat awal y(0) = 0, y'(0) = -4*

(7 markah)

3. (a) Evaluate $\int_0^{\frac{\pi}{3}} \int_0^{y^2} \frac{1}{y} \sin \frac{x}{y} dx dy$

(5 marks)

(b) Evaluate $\int_0^2 \int_0^x \int_0^{x+y} xyz \, dz \, dy \, dx$

(5 marks)

(c) If y is differentiable function of x such that sin (x + y) + cos (x − y) = y, find $\frac{dy}{dx}$

(5 marks)

(d) A right circular cylinder is changing in such a way that its radius r is increasing at the rate of 3 in./min and its height h is decreasing at the rate of 5in./min. At what rate is the volume of the cylinder changing when the radius is 10 in. and the height 8 in? (Volume of right circular cylinder is $V = \pi r^2 h$)

(10marks)

(a) Nilaikan $\int_0^{\frac{\pi}{3}} \int_0^{y^2} \frac{1}{y} \sin \frac{x}{y} dx dy$

(5 markah)

(b) Nilaikan $\int_0^2 \int_0^x \int_0^{x+y} xyz \, dz \, dy \, dx$

(5 markah)

(c) Jika y suatu fungsi terbeza bagi x dimana sin (x + y) + cos (x − y) = y, cari $\frac{dy}{dx}$

(5 markah)

(d) Jejari r bagi suatu silinder bulat-tepat monokok dengan kadar 3in./min. dan tinggi h menyusut dengan kadar 5in./min. Cari kadar perubahan isipadu silinder apabila jejari ialah 10in. dan tingginya ialah 8inci. (Isipadu silinder bulat-tepat ialah $V = \pi r^2 h$)

(10 markah)

...5/-

4.   (a)   Obtain the Fourier series expansion for the function

$$f(x) = 0 \quad \text{for} \quad -4 \leq x < 0$$

$$= 2 \quad \text{for} \quad 0 \leq x < 4$$

for which the period is 8.

(8marks)

(b)   (i)   Find all the critical points on the graph of
$$f(x,y) = (x-1)(y-1)(x+y-1)$$

(5 marks)

(ii)   Use the second partial test to classify each point

(5marks)

(c)   A cylindrical can is to hold $4 \, \Pi \, cm^3$ of orange juice. The cost per square centimeter of constructing the metal top and bottom is twice the cost per square centimeter of constructing the cardboard side. What are the dimensions of the least expensive can. Assume x and y is the radius and height of the cylinder respectively.

(7marks)

(a)   *Dapatkan kembangan siri Fourier bagi fungsi*

$$f(x) = 0 \quad bagi \quad -4 \leq x < 0$$

$$= 2 \quad bagi \quad 0 \leq x < 4$$

dimana kalaannya ialah 8.

(8markah)

(b)   (i)   *Cari semua titik kritikal diatas graf*
$$f(x,y) = (x-1)(y-1)(x+y-1)$$

(5 markah)

(ii)   *Guna ujian separa kedua untuk mengkelaskan setiap titik.*
(5markah)

...6/-

(c)   Satu tin berbentuk silinder boleh diisi dengan $4\Pi\ cm^3$ jus oren. Harga kos satu persegi sentimeter logam bagi membina atas dan bawah tin ialah dua kali harga bagi membina sisi tin. Apakah dimensi tin yang boleh memberikan harga kos yang paling rendah? Anggap x dan y sebagai jejari dan tinggi tin silinder masing-masing.

(7markah)

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