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MULTI-ANTIMONOPOLE SOLUTIONS OF THE SU(2) YANG-MILLS-HIGGS FIELD THEORY*

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Abstract

We report on the existence of multi-antimonopole solutions of the SU(2) Yang-Mills-Higgs field theory. These solutions are similiar in characteristics to the previously reported multimonopole solutions of the magnetic ansatz except for the change in sign of the multi-monopole charges. They are a different kind of BPS solutions, possess infinite energy, and mirror symmetries about the z-axis.

The SU(2) Yang-Mills-Higgs (YMH) field theory, with the Higgs field in the adjoint representation are known to possess both magnetic monopole and multimonopole solutions [1]. In general, configurations with a unit magnetic charge are spherically symmetric. The 't Hooft-Polyakov monopole solution is a numerical, spherically symmetric monopole solution of unit magnetic charge .

Multimonopole configurations with magnetic charges greater than unity possess at most axial symmetry [2]. Exact monopole and multimonopoles solutions exist in the BPS limit [1], [2]. Outside the BPS limit, when the Higgs field potential is non-vanishing only numerical solutions are known. Asymmetric multimonopole solutions are numerical even in the BPS limit [3]. Non-BPS monopoles-antimonopoles chain solutions and numerical BPS axially symmetric vortex rings solutions have also been reported [4].

We have reported on the existence of a different type of BPS static antimonopole-monopole-antimonopole solution. We have also shown that the extended magnetic ansatz possesses more multimonopole-antimonopole configurations. These exact solutions satisfy the first order Bogomol'nyi equations and possess infinite energy. In general, they possess axial and mirror symmetry and represent different combinations of monopoles, multimonopole, and antimonopoles [5]. Here we would like to show that the SU(2) YMH field theory also support multi-antimonopole solutions which are actually the anti-configurations of the monopoles solutions of Ref.[5].

The SU(2) YMH Lagrangian in 3+1 dimensions with vanishing Higgs mass and self interaction is

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}D^\mu\Phi^a D_\mu\Phi^a, \quad a,\ b,\ c = 1,2,3; \quad \mu,\nu,\alpha = 0,1,2,3; \tag{1}$$

$$D_\mu\Phi^a = \partial_\mu\Phi^a + \epsilon^{abc}A^b_\mu\Phi^c, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu, \tag{2}$$

where $D_\mu\Phi^a$ is the covariant derivative of the Higgs field, $F^a_{\mu\nu}$ is the gauge field strength and A^a_μ is the gauge potential. The gauge field coupling constant is scaled away and the metric used is $g_{\mu\nu} = (-+++)$. The equations of motion that follow from the Lagrangian (1) are

$$D^\mu F^a_{\mu\nu} = \partial^\mu F^a_{\mu\nu} + \epsilon^{abc}A^{b\mu}F^c_{\mu\nu} = \epsilon^{abc}\Phi^b D_\nu\Phi^c, \quad D^\mu D_\mu\Phi^a = 0. \tag{3}$$

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The tensor identified with the electromagnetic field as introduced by t'Hooft [1] is

$$\begin{aligned} F_{\mu\nu} &= \hat{\Phi}^a F^a_{\mu\nu} - \epsilon^{abc}\hat{\Phi}^a D_\mu \hat{\Phi}^b D_\nu \hat{\Phi}^c = \partial_\mu A_\nu - \partial_\nu A_\mu - \epsilon^{abc}\hat{\Phi}^a \partial_\mu \hat{\Phi}^b \partial_\nu \hat{\Phi}^c, & (4)\\ A_\mu &= \hat{\Phi}^a A^a_\mu; \quad \hat{\Phi}^a = \Phi^a/|\Phi|, \quad |\Phi| = \sqrt{\Phi^a\Phi^a}. & (5)\end{aligned}$$

The Abelian electric field is $E_i = F_{0i}$, and the Abelian magnetic field is $B_i = -\frac{1}{2}\epsilon_{ijk}F_{jk}$. The topological magnetic current which is also the topological current density of the system is defined to be

$$k_\mu = \frac{1}{8\pi}\ \epsilon_{\mu\nu\rho\sigma}\ \epsilon_{abc}\ \partial^\nu \hat{\Phi}^a\ \partial^\rho \hat{\Phi}^b\ \partial^\sigma \hat{\Phi}^c, \qquad (6)$$

and the corresponding conserved topological magnetic charge is

$$M = \int d^3x\ k_0 = \frac{1}{8\pi}\oint d^2\sigma_i \left(\epsilon_{ijk}\epsilon^{abc}\hat{\Phi}^a\partial_j\hat{\Phi}^b\partial_k\hat{\Phi}^c\right) = \frac{1}{4\pi}\oint d^2\sigma_i\ B_i. \qquad (7)$$

The Bogomol'nyi equations, $B^a_i \pm D_i\Phi^a = 0$, when solved with the $\pm$ sign corresponds to monopoles and antimonopoles respectively for the usual BPS solutions. In our case the multimonopole [5] is solved with the "+" sign and the anti-multimonopole is solved with the "−" sign. The anti-multimonopole ansatz is obtained by modifying the ansatz of Ref.[5],

$$\begin{aligned} A^a_\mu &= -\frac{1}{r}\psi(r)\left(\hat{u}^a_\theta\hat{\phi}_\mu + \hat{u}^a_\phi\hat{\theta}_\mu\right) + \frac{1}{r}R(\theta)\left(\hat{u}^a_\phi\hat{r}_\mu + \hat{u}^a_r\hat{\phi}_\mu\right) + \frac{1}{r}G(\theta,\phi)\left(\hat{u}^a_r\hat{\theta}_\mu - \hat{u}^a_\theta\hat{r}_\mu\right),\\ \Phi^a &= \Phi_1\ \hat{u}^a_r + \Phi_2\ \hat{u}^a_\theta + \Phi_3\ \hat{u}^a_\phi; \qquad \hat{u}^a_r = \sin\theta\ \cos\phi\ \delta^a_1 - \sin\theta\ \sin\phi\ \delta^a_2 + \cos\theta\delta^a_3,\\ \hat{u}^a_\theta &= \cos\theta\ \cos\phi\ \delta^a_1 - \cos\theta\ \sin\phi\ \delta^a_2 - \sin\theta\ \delta^a_3, \quad \hat{u}^a_\phi = \sin\phi\ \delta^a_1 + \cos\phi\ \delta^a_2, \qquad (8)\end{aligned}$$

where $\Phi_1 = \frac{1}{r}\psi(r)$, $\Phi_2 = \frac{1}{r}R(\theta)$, $\Phi_3 = \frac{1}{r}G(\theta,\phi)$, $r = \sqrt{x^i x_i}$, $\theta = \cos^{-1}(x_3/r)$, and $\phi = \tan^{-1}(x_2/x_1)$. The ansatz will give $A_\mu = \hat{\Phi}^a A^a_\mu = 0$. Hence E_i is zero, B_i is independent of A^a_μ. To calculate for the Abelian magnetic field B_i, we write

$$\begin{aligned} \Phi^a &= \Phi_1\ \hat{u}^a_r + \Phi_2\ \hat{u}^a_\theta + \Phi_3\ \hat{u}^a_\phi\\ &= \tilde{\Phi}_1\ \delta^{a1} + \tilde{\Phi}_2\ \delta^{a2} + \tilde{\Phi}_3\ \delta^{a3}; \qquad (9)\end{aligned}$$

$$\begin{aligned} \tilde{\Phi}_1 &= \sin\theta\cos n\phi\ \Phi_1 + \cos\theta\cos n\phi\ \Phi_2 - \sin n\phi\ \Phi_3 = |\Phi|\cos\alpha\sin\beta\\ \tilde{\Phi}_2 &= \sin\theta\sin n\phi\ \Phi_1 + \cos\theta\sin n\phi\ \Phi_2 + \cos n\phi\ \Phi_3 = |\Phi|\cos\alpha\cos\beta\\ \tilde{\Phi}_3 &= \cos\theta\ \Phi_1 - \sin\theta\ \Phi_2 = |\Phi|\sin\alpha. \qquad (10)\end{aligned}$$

and the Higgs unit vector is simplified to

$$\hat{\Phi}^a = \cos\alpha\sin\beta\ \delta^{a1} + \cos\alpha\cos\beta\ \delta^{a2} + \sin\alpha\ \delta^{a3}. \qquad (11)$$

The Abelian magnetic field and the magnetic charge at large distances reduce to

$$\begin{aligned} B_i &= \frac{1}{r^2\sin\theta}\left\{\frac{\partial\sin\alpha}{\partial\theta}\frac{\partial\beta}{\partial\phi} - \frac{\partial\sin\alpha}{\partial\phi}\frac{\partial\beta}{\partial\theta}\right\}\hat{r}_i + \frac{1}{r\sin\theta}\left\{\frac{\partial\sin\alpha}{\partial\phi}\frac{\partial\beta}{\partial r} - \frac{\partial\sin\alpha}{\partial r}\frac{\partial\beta}{\partial\phi}\right\}\hat{\theta}_i\\ &+ \frac{1}{r}\left\{\frac{\partial\sin\alpha}{\partial r}\frac{\partial\beta}{\partial\theta} - \frac{\partial\sin\alpha}{\partial\theta}\frac{\partial\beta}{\partial r}\right\}\hat{\phi}_i,\\ \sin\alpha &= \frac{\psi\cos\theta - R\sin\theta}{\sqrt{\psi^2+R^2+G^2}},\ \beta = \gamma + \phi,\ \gamma = \tan^{-1}\left(\frac{\psi\sin\theta + R\cos\theta}{G}\right), \quad \text{and} \qquad (12)\\ M &= \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left(\frac{\partial\sin\alpha}{\partial\theta}\frac{\partial\beta}{\partial\phi} - \frac{\partial\sin\alpha}{\partial\phi}\frac{\partial\beta}{\partial\theta}\right)d\theta d\phi, \quad \text{respectively.} \qquad (13)\end{aligned}$$

The gauge fixing condition that we used here is the radiation or Coulomb gauge, $\partial^i A_i^a = 0$, $A_0^a = 0$. The ansatz (8) reduced the Bogomol'nyi equations with the negative sign to just four first order differential equations,

$$\begin{aligned} r\frac{\partial\psi}{\partial r} + \psi - \psi^2 &= -p, \quad \frac{\partial R}{\partial\theta} + R\cot\theta - R^2 = p - b^2\csc^2\theta, \\ \frac{\partial G}{\partial\theta} + G\cot\theta &= 0, \quad \frac{\partial G}{\partial\phi}\csc\theta + G^2 = -b^2\csc^2\theta; \qquad p, b = \text{constants}. \end{aligned} \tag{14}$$

The solutions obtained for the four classes of anti-multimonopole, labeled as anti-A1, anti-B1, anti-B2, and anti-A2 [5] are

$$\begin{aligned} G(\theta,\phi) &= -b\csc\theta\tan b\phi, \quad \psi(r) = \frac{(m+1) - mr^{2m+1}}{1 + r^{2m+1}}, \quad p = m(m+1), \\ b = (m-1),\ R &= \tan\theta - (m-1)\cot\theta; \quad b = (m),\ R = -m\cot\theta; \\ b = (m+1),\ R &= (m+1)\cot\theta; \quad b = (m+2),\ R = \tan\theta + (m+2)\cot\theta, \end{aligned} \tag{15}$$

respectively. The topological parameter m is restricted to a half-integer for G to be a single value function and it fixes the magnetic charges of the solutions. The monopoles and antimonopoles are associated with the number of zeros of Φ^a enclosed by the sphere at infinity. The Higgs field vanishes as $1/r$ at large r. However, the magnetic charges depends only on the unit vector of the Higgs field, $\hat{\Phi}^a$. The anti-multimonopole is located at the origin of the coordinate axes where the Higgs field is singular. The magnetic fields of the anti-A2, $m = 0$ and anti-B2, $m = 1$ configurations are shown in Fig.(1) and Fig.(2) respectively. A point plot of the net magnetic charges M at large r versus the magnetic charges M_0 at $r = 0$ for all the solutions discussed are as shown in Fig.(3).

These new anti-multimonopoles solutions are similar in characteristics to the multimonoples solutions of Ref.[5] with all the magnetic charge of the poles and hence the direction of the magnetic field, B_i, reversed. The detail study of the magnetic charges and magnetic fields of the A1, B1, B2, and A2 configurations had been done in Ref.[5]. Hence to every possible multimonopole configurations of the ansatz of Ref.[5] there always exist an anti-version of these configurations.

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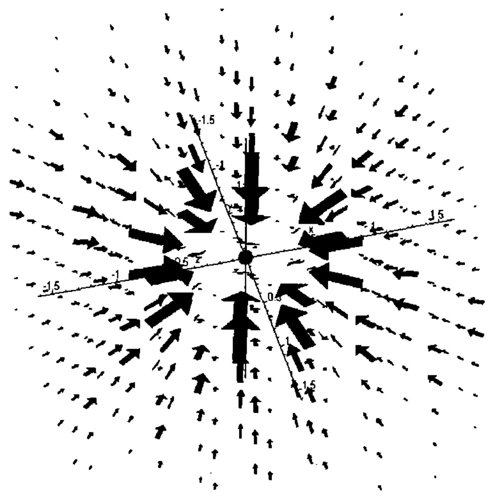

Figure 1: The magnetic field of the anti-A2, m=0 solution showing the 3-antimonopole at $r = 0$.

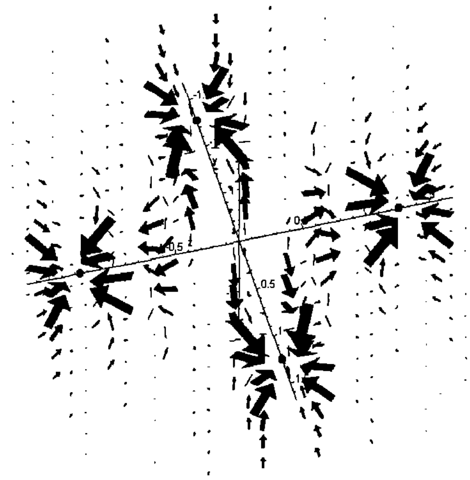

Figure 2: The magnetic field of the anti-B2, m=1 solution with four finitely separated anti-monopoles.

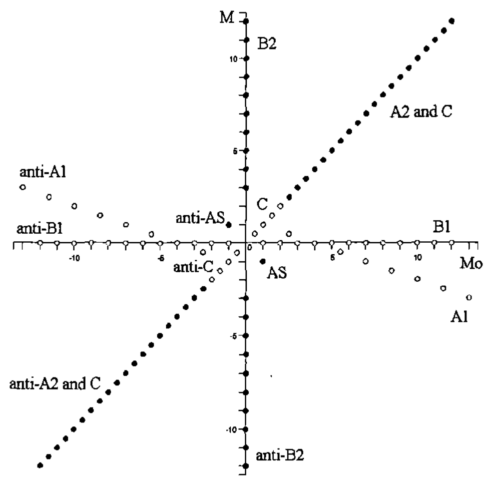

Figure 3: A point plot of the net magnetic charges M at large r versus the magnetic charges M_0 at $r = 0$ for all the existing monopole solutions of the magnetic ansatz.