
UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2002/2003

Februari 2003

IUK 291/4 - MATEMATIK II

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH (7) mukasurat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **LIMA (5)** soalan. Semua soalan boleh dijawab samada dalam Bahasa Inggeris ATAU Bahasa Malaysia ATAU kombinasi kedua-duanya.

1. (a) Let function f defined by

$$f(x, y) = x^2 + 2y^2$$

find the images of $(0,0)$, $(1,-2)$, and $(0,3)$. Also give the domain
(40 marks)

Katakan fungsi f ditakrifkan oleh

$$f(x, y) = x^2 + 2y^2$$

cari imej $(0, 0)$, $(1, -2)$ dan $(0, 3)$. Dapatkan juga domain bagi f

- (b) Find the limit, if it exists

$$(x, y, z) \xrightarrow{\text{lim}} (2,1,2) \frac{xz^2}{\sqrt{x^2 + y^2 + z^2}} \quad (20 \text{ marks})$$

Cari had sekiranya ada untuk

$$(x, y, z) \xrightarrow{\text{lim}} (2,1,2) \frac{xz^2}{\sqrt{x^2 + y^2 + z^2}}$$

- (c) Locate all relative extrema and saddle point of

$$f(x, y) = 3x^2 - 2xy + y^2 - 8y \quad (40 \text{ marks})$$

Tentukan semua lampauan relatif dan titik pelana bagi

$$f(x, y) = 3x^2 - 2xy + y^2 - 8y$$

2. (a) Let $f(x, y) = xy^2 - xz + y^2z$ and let $x = \cos t$, $y = \tan t$, and $z = e^t$

Find $\frac{df}{dt}$ (30 marks)

Jika $f(x, y) = xy^2 - xz + y^2z$ dan jika $x = \cos t$, $y = \tan t$, dan $z = e^t$

Cari $\frac{df}{dt}$

- (b) Find the directional derivative of the function at the given point in the direction of V .

$f(x, y) = x^2y^2$; $(2, -1)$; $V = \nabla f(2, -1)$ (40 marks)

Cari derivatif arah fungsi berikut pada titik yang diberikan dalam arah V .

$f(x, y) = x^2y^2$; $(2, -1)$; $V = \nabla f(2, -1)$

- (c) If the length l and width w of a rectangular box are each increased by 3% and the height h is decreased by 2%, then approximately how much does the volume change?

Note: The volume formula is $V = lwh$ (30 marks)

Jika panjang l dan lebar w suatu kotak segiempat tepat ditambah masing-masingnya 3% dan tinggi dikurangkan sebanyak 2% maka kira perubahan isipadu.

Nota: Formula isipadu ialah $V = lwh$

3. (a) Evaluate the double integral

$$\int_0^1 \int_0^x y\sqrt{x^2 - y^2} dy dx \quad (30 \text{ marks})$$

Nilaiikan pengkamiran gandadua ini

$$\int_0^1 \int_0^x y\sqrt{x^2 - y^2} dy dx$$

- (b) Evaluate the triple integral

$$\int_0^7 \int_0^{\sqrt{4-x^2}} \int_{-5+x^2+y^2}^{3-x^2-y^2} x dz dy dx \quad (30 \text{ marks})$$

Nilaiikan pengkamiran ganda tiga ini

$$\int_0^7 \int_0^{\sqrt{4-x^2}} \int_{-5+x^2+y^2}^{3-x^2-y^2} x dz dy dx$$

- (c) Evaluate $\iint_R (2x - y^2) dA$ over the triangular region R enclosed between the lines $y = -x + 1$, and $y = 3$

(40 marks)

Nilaiikan

$$\iint_R (2x - y^2) dA \text{ untuk kawasan tiga segi } R \text{ yang dibendung oleh } y = -x + 1, \text{ and } y = 3$$

4. (a) Find the interval of convergence and radius of convergence of

$$\sum_{k=0}^{\infty} k! x^k \quad (40 \text{ marks})$$

Cari jeda penumpuan dan jejari penumpuan bagi

$$\sum_{k=0}^{\infty} k! x^k$$

- (b) Express $\frac{1}{\sqrt{1+x}}$ as a binomial series (20 marks)

Nyatakan $\frac{1}{\sqrt{1+x}}$ sebagai siri binomial

- (c) Find the sum of the power series

$$\sum_{n=1}^{\infty} n x^{n-1} = 1 + 2x + 3x^2 + \dots \quad (40 \text{ marks})$$

Dapatkan jumlah siri kuasa berikut

$$\sum_{n=1}^{\infty} n x^{n-1} = 1 + 2x + 3x^2 + \dots$$

5. (a) Find the general solution of the equation

$$y'' + 4y' + 4y = 0$$

Then find the particular solution that satisfies the initial conditions
 $y(1) = 1$, $y'(1) = 2$

(40 marks)

Dapatkan selesaian am persamaan berikut

$$y'' + 4y' + 4y = 0$$

Kemudian cari selesaian khusus yang memenuhi keadaan awal
 $y(1) = 1$, $y'(1) = 2$

- (b) Find the Fourier series expansion for the function

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x \leq \pi \end{cases}$$

(40 marks)

Dapatkan kembangan siri Fourier bagi fungsi berikut

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x \leq \pi \end{cases}$$

(c) Given $f(x, y) = x^3 y^5 - 2x^2 y + x$, find

i. f_{xxy} ii. f_{yxy} iii. f_{yyy}

(20 marks)

Jika $f(x, y) = x^3 y^5 - 2x^2 y + x$, cari

i. f_{xxy} ii. f_{yxy} iii. f_{yyy}