

UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua  
Sidang Akademik 2004/2005

Mac 2005

**JIK 317 – KIMIA KUANTUM DAN TEORI KUMPULAN**

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **EMPAT BELAS** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab LIMA soalan.

Setiap jawapan mesti dijawab di dalam buku jawapan yang disediakan.

Setiap soalan bernilai 20 markah dan markah subsoalan diperlihatkan di penghujung subsoalan itu.

1. (a) Jelaskan konsep kuantum. Perincikan kelemahan mekanik klasik dalam menjelaskan dapatan daripada ujikaji-ujikaji tertentu. Bagaimana konsep kuantum dapat mengatasinya?

(6 markah)

- (b) Jelaskan konsep kedualan zarah gelombang. Jelaskan bagaimana konsep ini dapat menjelaskan kesan fotoelektrik.

(6 markah)

- (c) Dengan mempertimbangkan persamaan gelombang

$$\frac{\partial^2 \psi(x, t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} (x, t)$$

Untuk tangsi panjang L yang diikatkan pada satu hujung yang digetarkan sehingga menghasilkan gelombang pegun, buktikan bahawa persamaan Schrödinger satu dimensi ialah

$$\frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2} (E - U) \psi = 0$$

(8 markah)

2. (a) Nyatakan dengan ringkas postulat-postulat mekanik kuantum.

(10 markah)

- (b) Jika  $\hat{A} = x$ ,  $\hat{B} = \frac{d}{dx}$  dan  $\hat{C} = 3$ , tunjukkan bahawa

(i)  $(\hat{A} + \hat{B}) + \hat{C} = \hat{A} + (\hat{B} + \hat{C})$

(ii)  $\hat{A}\hat{B} \neq \hat{B}\hat{A}$

(iii)  $(\hat{A}\hat{B})\hat{C} = \hat{A}(\hat{B}\hat{C})$

(4 markah)

(c) Jelaskan sebutan-sebutan berikut

- (i) ketumpatan kebarangkalian
- (ii) operator
- (iii) kehermitian sesuatu operator
- (iv) keortogonalan fungsi gelombang
- (v) keortonormalan fungsi gelombang
- (vi) nilai jangkaan

(6 markah)

3. (a) Untuk satu sistem yang terdiri daripada satu zarah dalam kotak 1 – dimensi, persamaan Schrödinger yang berkaitan ialah

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi(x) = 0 \quad \text{di mana } k^2 = \frac{2mE}{\hbar^2}$$

Dengan menggunakan syarat pormalan yang sesuai, tunjukkan bahawa penyelesaiannya ialah

$$\psi_n(x) = \frac{2}{L} \sin \frac{n\pi x}{L} \quad \text{di mana } L \text{ ialah kelebaran kotak.}$$

Tunjukkan juga bahawa tenaga zarah ini terkuantum mengikut persamaan di bawah.

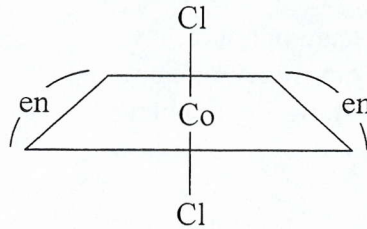
$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

(16 markah)

(b) Lakarkan kebarangkalian untuk mendapatkan zarah ini untuk beberapa nilai kuantum yang rendah.

(4 markah)

4. Takrifkan unsur simetri dan operasi simetri. Dengan menggunakan gambar rajah di bawah, kenalpasti unsur simetri yang wujud dalam struktur molekul tersebut :



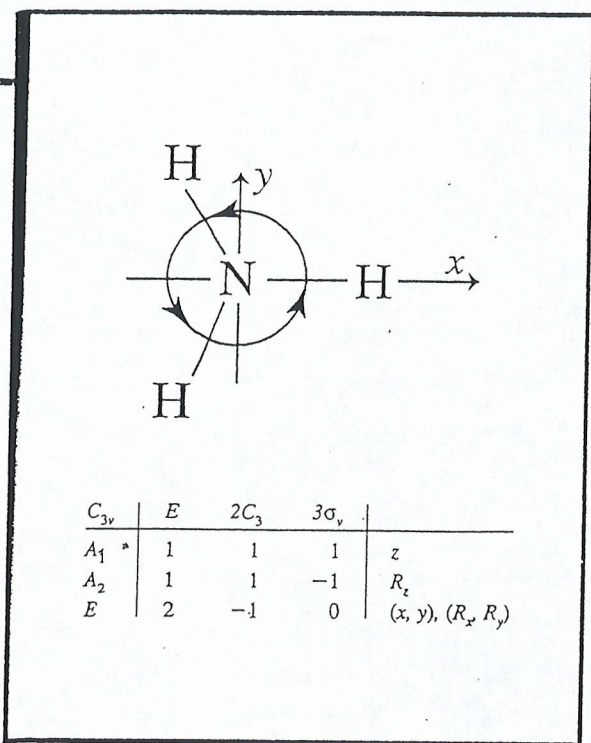
(20 markah)

5. Dengan merujuk kepada jadual karakter berkaitan, dapatkan satu perwakilan terturunkan untuk struktur  $\text{CH}_4$ . Seterusnya turunkan perwakilan terturunkan berkenaan. Tunjukkan semua langkah anda.

(20 markah)

6. (a) Dengan menunjukkan langkah, dapatkan orbital yang sesuai bagi pengikatan  $\pi$  (Pi) "luar satah" untuk  $\text{BCl}_3$ .
- (b) Dapatkan bilangan getaran perenggangan B-H hujung yang aktif dalam spektrum Raman untuk diborana  $\text{B}_2\text{H}_6$  yang mempunyai kumpulan fungsi  $\text{D}_{2h}$ .

(20 markah)



## Character Tables†

### 1. GROUPS OF LOW SYMMETRY

$C_1$	$E$
$A$	1

$C_s$	$E$	$\sigma_h$		
$A'$	1	1	$x, y, R_z$	$x^2, y^2, z^2, xy$
$A''$	1	-1	$z, R_x, R_y$	$yz, xz$

$C_i$	$E$	$i$		
$A_g$	1	1	$R_x, R_y, R_z$	$x^2, y^2, z^2, xy, xz, yz$
$A_u$	1	-1	$x, y, z$	

### 2. $C_n$ , $C_{nv}$ , AND $C_{nh}$ GROUPS

The  $C_n$  groups

$C_2$	$E$	$C_2$		
$A$	1	1	$z, R_z$	$x^2, y^2, z^2, xy$
$B$	1	-1	$x, y, R_x, R_y$	$yz, xz$

$C_3$	$E$	$C_3$	$C_3^2$		
$A$	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$E$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^* \\ 1 & \epsilon^* & \epsilon \end{Bmatrix}$			$(x, y), (R_x, R_y)$	$(x^2 - y^2, xy), (yz, xz)$

$\epsilon = e^{(2\pi i)/3}$

† $i = \sqrt{-1}$ ;  $\epsilon^* = \epsilon$  with  $-i$  substituted for  $i$ .

$C_4$	$E$	$C_4$	$C_2$	$C_4^3$		
$A$	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1		$(x^2 - y^2, xy)$
$E$	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$				$(x, y), (R_x, R_y)$	$(yz, xz)$

$C_5$	$E$	$C_5$	$C_5^2$	$C_5^3$	$C_5^4$		
$A$	1	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$E_1$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^2 & \epsilon^{2*} & \epsilon^* \\ 1 & \epsilon^* & \epsilon^{2*} & \epsilon^2 & \epsilon \end{Bmatrix}$					$(x, y), (R_x, R_y)$	$(yz, xz)$
$E_2$	$\begin{Bmatrix} 1 & \epsilon^2 & \epsilon^* & \epsilon & \epsilon^{2*} \\ 1 & \epsilon^{2*} & \epsilon & \epsilon^* & \epsilon^2 \end{Bmatrix}$						$(x^2 - y^2, xy)$

$\epsilon = e^{(2\pi i)/5}$

$C_6$	$E$	$C_6$	$C_3$	$C_2$	$C_3^2$	$C_6^5$		
$A$	1	1	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1	1	-1		
$E_1$	$\begin{Bmatrix} 1 & \epsilon & -\epsilon^* & -1 & -\epsilon & \epsilon^* \\ 1 & \epsilon^* & -\epsilon & -1 & -\epsilon^* & \epsilon \end{Bmatrix}$						$(x, y), (R_x, R_y)$	$(yz, xz)$
$E_2$	$\begin{Bmatrix} 1 & -\epsilon^* & -\epsilon & 1 & -\epsilon^* & -\epsilon \\ 1 & -\epsilon & -\epsilon^* & 1 & -\epsilon & -\epsilon^* \end{Bmatrix}$							$(x^2 - y^2, xy)$

$\epsilon = e^{(\pi i)/3}$

$C_7$	$E$	$C_7$	$C_7^2$	$C_7^3$	$C_7^4$	$C_7^5$	$C_7^6$		
$A$	1	1	1	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$E_1$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^2 & \epsilon^3 & \epsilon^{3*} & \epsilon^{2*} & \epsilon^* \\ 1 & \epsilon^* & \epsilon^{2*} & \epsilon^{3*} & \epsilon^3 & \epsilon^2 & \epsilon \end{Bmatrix}$							$(x, y), (R_x, R_y)$	$(yz, xz)$
$E_2$	$\begin{Bmatrix} 1 & \epsilon^2 & \epsilon^{3*} & \epsilon^* & \epsilon & \epsilon^3 & \epsilon^{2*} \\ 1 & \epsilon^{2*} & \epsilon^3 & \epsilon & \epsilon^* & \epsilon^{3*} & \epsilon^2 \end{Bmatrix}$								$(x^2 - y^2, xy)$
$E_3$	$\begin{Bmatrix} 1 & \epsilon^3 & \epsilon^* & \epsilon^2 & \epsilon^{2*} & \epsilon & \epsilon^{3*} \\ 1 & \epsilon^{3*} & \epsilon & \epsilon^{2*} & \epsilon^2 & \epsilon^* & \epsilon^3 \end{Bmatrix}$								

$\epsilon = e^{(2\pi i)/7}$

$C_8$	$E$	$C_8$	$C_4$	$C_2$	$C_4^3$	$C_8^3$	$C_8^5$	$C_8^7$		
$A$	1	1	1	1	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1	1	-1	-1	-1		
$E_1$	$\begin{Bmatrix} 1 & \epsilon & i & -1 & -i & -\epsilon^* & -\epsilon & \epsilon^* \\ 1 & \epsilon^* & -i & -1 & i & -\epsilon & -\epsilon^* & \epsilon \end{Bmatrix}$								$(x, y), (R_x, R_y)$	$(yz, xz)$
$E_2$	$\begin{Bmatrix} 1 & i & -1 & 1 & -1 & -i & i & -i \\ 1 & -i & -1 & 1 & -1 & i & -i & i \end{Bmatrix}$									$(x^2 - y^2, xy)$
$E_3$	$\begin{Bmatrix} 1 & -\epsilon & i & -1 & -i & \epsilon^* & \epsilon & -\epsilon^* \\ 1 & -\epsilon^* & -i & -1 & i & \epsilon & \epsilon^* & -\epsilon^* \end{Bmatrix}$									

$\epsilon = e^{(\pi i)/4}$

The  $C_{2n}$  groups

$C_{2n}$	$E$	$C_2$	$\sigma_v(xz)$	$\sigma'_v(yz)$		
$A_1$	1	1	1	1	$z$	$x^2, y^2, z^2$
$A_2$	1	1	-1	-1	$R_z$	$xy$
$B_1$	1	-1	1	-1	$x, R_y$	$xz$
$B_2$	1	-1	-1	1	$y, R_x$	$yz$

$C_{3v}$	$E$	$2C_3$	$3\sigma_v$		
$A_1$	1	1	1	$z$	$x^2 + y^2, z^2$
$A_2$	1	1	-1	$R_z$	
$E$	2	-1	0	$(x, y), (R_x, R_y)$	$(x^2 - y^2, xy), (xz, yz)$

$C_{4v}$	$E$	$2C_4$	$C_2$	$2\sigma_v$	$2\sigma_d$		
$A_1$	1	1	1	1	1	$z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	$R_z$	
$B_1$	1	-1	1	1	-1		$x^2 - y^2$
$B_2$	1	-1	1	-1	1		$xy$
$E$	2	0	-2	0	0	$(x, y), (R_x, R_y)$	$(xz, yz)$

$C_{5v}$	$E$	$2C_5$	$2C_5^2$	$5\sigma_v$		
$A_1$	1	1	1	1	$z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	$R_z$	
$E_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y), (R_x, R_y)$	$(xz, yz)$
$E_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$

$C_{6v}$	$E$	$2C_6$	$2C_3$	$C_2$	$3\sigma_v$	$3\sigma_d$		
$A_1$	1	1	1	1	1	1	$z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	-1	$R_z$	
$B_1$	1	-1	1	-1	1	-1		
$B_2$	1	-1	1	-1	-1	1		
$E_1$	2	1	-1	-2	0	0	$(x, y), (R_x, R_y)$	$(xz, yz)$
$E_2$	2	-1	-1	2	0	0		$(x^2 - y^2, xy)$

The  $C_{nh}$  groups

$C_{2h}$	$E$	$C_2$	$i$	$\sigma_h$		
$A_g$	1	1	1	1	$R_z$	$x^2, y^2, z^2, xy$
$B_g$	1	-1	1	-1	$R_x, R_y$	$xz, yz$
$A_u$	1	1	-1	-1	$z$	
$B_u$	1	-1	-1	1	$x, y$	

$C_{3h}$	$E$	$C_3$	$C_3^2$	$\sigma_h$	$S_3$	$S_3^5$		
$A'$	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$E'$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right.$	$(x, y)$	$(x^2 - y^2, xy)$
$A''$	1	1	1	-1	-1	-1	$z$	
$E''$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} -\epsilon \\ -\epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} -\epsilon^* \\ -\epsilon \end{array} \right.$	$(R_x, R_y)$	$(xz, yz)$

$\epsilon = e^{(2\pi i)/3}$

$C_{4h}$	$E$	$C_4$	$C_2$	$C_4^3$	$i$	$S_4^3$	$\sigma_h$	$S_4$		
$A_g$	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$B_g$	1	-1	1	-1	1	-1	1	-1		$x^2 - y^2, xy$
$E_g$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right.$	$(R_x, R_y)$	$(xz, yz)$
$A_u$	1	1	1	1	-1	-1	-1	-1	$z$	
$B_u$	1	-1	1	-1	-1	1	-1	1		
$E_u$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right.$	$(x, y)$	



$D_6$	$E$	$2C_6$	$2C_3$	$C_2$	$3C_2'$	$3C_2''$		
$A_1$	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	-1	$z, R_z$	
$B_1$	1	-1	1	-1	1	-1		
$B_2$	1	-1	1	-1	-1	1		
$E_1$	2	1	-1	-2	0	0	$(x, y), (R_x, R_y)$	$(xz, yz)$
$E_2$	2	-1	-1	2	0	0		$(x^2 - y^2, xy)$

The  $D_{nd}$  groups

$D_{2d}$	$E$	$2S_4$	$C_2$	$2C_2'$	$2\sigma_d$		
$A_1$	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	$R_z$	
$B_1$	1	-1	1	1	-1		$x^2 - y^2$
$B_2$	1	-1	1	-1	1	$z$	$xy$
$E$	2	0	-2	0	0	$(x, y), (R_x, R_y)$	$(xz, yz)$

$D_{3d}$	$E$	$2C_3$	$3C_2$	$i$	$2S_6$	$3\sigma_d$		
$A_{1g}$	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	-1	1	1	-1	$R_z$	
$E_g$	2	-1	0	2	-1	0	$(R_x, R_y)$	$(x^2 - y^2, xy), (xz, yz)$
$A_{1u}$	1	1	1	-1	-1	-1		
$A_{2u}$	1	1	-1	-1	-1	1	$z$	
$E_u$	2	-1	0	-2	1	0	$(x, y)$	

$D_{4d}$	$E$	$2S_8$	$2C_4$	$2S_8^3$	$C_2$	$4C_2'$	$4\sigma_d$		
$A_1$	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	1	1	-1	-1	$R_z$	
$B_1$	1	-1	1	-1	1	1	-1		
$B_2$	1	-1	1	-1	1	-1	1	$z$	
$E_1$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0	$(x, y)$	
$E_2$	2	0	-2	0	2	0	0		$(x^2 - y^2, xy)$
$E_3$	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0	$(R_x, R_y)$	$(xz, yz)$

$D_{5d}$	$E$	$2C_5$	$2C_5^2$	$5C_2$	$i$	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$		
$A_{1g}$	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	-1	1	1	1	-1	$R_z$	
$E_{1g}$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(R_x, R_y)$	$(xz, yz)$
$E_{2g}$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$
$A_{1u}$	1	1	1	1	-1	-1	-1	-1		
$A_{2u}$	1	1	1	-1	-1	-1	-1	1	$z$	
$E_{1u}$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	$(x, y)$	
$E_{2u}$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0		

$D_{6d}$	$E$	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12}^5$	$C_2$	$6C_2'$	$6\sigma_d$		
$A_1$	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2$	1	1	1	1	1	1	1	-1	-1	$R_z$	
$B_1$	1	-1	1	-1	1	-1	1	1	-1		
$B_2$	1	-1	1	-1	1	-1	1	-1	1	$z$	
$E_1$	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	0	0	$(x, y)$	
$E_2$	2	1	-1	-2	-1	1	2	0	0		$(x^2 - y^2, xy)$
$E_3$	2	0	-2	0	2	0	-2	0	0		
$E_4$	2	-1	-1	2	-1	-1	2	0	0		
$E_5$	2	$-\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	0	$(R_x, R_y)$	$(xz, yz)$

The  $D_{nh}$  groups

$D_{2h}$	$E$	$C_2(z)$	$C_2(y)$	$C_2(x)$	$i$	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
$A_g$	1	1	1	1	1	1	1	1		$x^2, y^2, z^2$
$B_{1g}$	1	1	-1	-1	1	1	-1	-1	$R_z$	$xy$
$B_{2g}$	1	-1	1	-1	1	-1	1	-1	$R_y$	$xz$
$B_{3g}$	1	-1	-1	1	1	-1	-1	1	$R_x$	$yz$
$A_u$	1	1	1	1	-1	-1	-1	-1		
$B_{1u}$	1	1	-1	-1	-1	-1	1	1	$z$	
$B_{2u}$	1	-1	1	-1	-1	1	-1	1	$y$	
$B_{3u}$	1	-1	-1	1	-1	1	1	-1	$x$	

$D_{3h}$	$E$	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$		
$A_1'$	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2'$	1	1	-1	1	1	-1	$R_z$	
$E'$	2	-1	0	2	-1	0	$(x, y)$	$(x^2 - y^2, xy)$
$A_1''$	1	1	1	-1	-1	-1		
$A_2''$	1	1	-1	-1	-1	1	$z$	
$E''$	2	-1	0	-2	1	0	$(R_x, R_y)$	$(xz, yz)$

$D_{4h}$	$E$	$2C_4$	$C_2$	$2C_2'$	$2C_2''$	$i$	$2S_4$	$\sigma_h$	$2\sigma_v$	$2\sigma_d$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	-1	-1	1	1	1	-1	-1	$R_z$	
$B_{1g}$	1	-1	1	1	-1	1	-1	1	1	-1		$x^2 - y^2$
$B_{2g}$	1	-1	1	-1	1	1	-1	1	-1	1		$xy$
$E_g$	2	0	-2	0	0	2	0	-2	0	0	$(R_x, R_y)$	$(xz, yz)$
$A_{1u}$	1	1	1	1	1	-1	-1	-1	-1	-1		
$A_{2u}$	1	1	1	-1	-1	-1	-1	-1	1	1	$z$	
$B_{1u}$	1	-1	1	1	-1	-1	1	-1	-1	1		
$B_{2u}$	1	-1	1	-1	1	-1	1	-1	1	-1		
$E_u$	2	0	-2	0	0	-2	0	2	0	0	$(x, y)$	

$D_{5h}$	$E$	$2C_5$	$2C_2^2$	$5C_2$	$\sigma_h$	$2S_5$	$2S_5^3$	$5\sigma_v$		
$A_1'$	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
$A_2'$	1	1	1	-1	1	1	1	-1	$R_z$	
$E_1'$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y)$	
$E_2'$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, xy)$
$A_1''$	1	1	1	1	-1	-1	-1	-1		
$A_2''$	1	1	1	-1	-1	-1	-1	1	$z$	
$E_1''$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	$(R_x, R_y)$	$(xz, yz)$
$E_2''$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0		

$D_{6h}$	$E$	$2C_6$	$2C_3$	$C_2$	$3C_2'$	$3C_2''$	$i$	$2S_3$	$\hat{2}S_6$	$\sigma_h$	$3\sigma_d$	$3\sigma_v$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	1	-1	-1	1	1	1	1	-1	-1		
$B_{1g}$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
$B_{2g}$	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1		
$E_{1g}$	2	1	-1	-2	0	0	2	1	-1	-2	0	0	$(R_x, R_y)$	$(xz, yz)$ $(x^2 - y^2, xy)$
$E_{2g}$	2	-1	-1	2	0	0	2	-1	-1	2	0	0		
$A_{1u}$	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$z$	
$A_{2u}$	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1		
$B_{1u}$	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
$B_{2u}$	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
$E_{1u}$	2	1	-1	-2	0	0	-2	-1	1	2	0	0	$(x, y)$	
$E_{2u}$	2	-1	-1	2	0	0	-2	1	1	-2	0	0		

$D_{3h}$	$E$	$2C_3$	$2C_3^2$	$2C_2$	$C_2$	$4C_2'$	$4C_2''$	$i$	$2S_6$	$2S_6^5$	$2S_4$	$\sigma_h$	$4\sigma_d$	$4\sigma_v$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1		
$B_{1g}$	1	-1	-1	1	1	1	-1	1	-1	-1	1	1	1	-1		
$B_{2g}$	1	-1	-1	1	1	-1	1	1	-1	-1	1	1	-1	1		
$E_{1g}$	2	$\sqrt{2}$	$-\sqrt{2}$	0	-2	0	0	2	$\sqrt{2}$	$-\sqrt{2}$	0	-2	0	0	$(R_x, R_y)$	$(xz, yz)$ $(x^2 - y^2, xy)$
$E_{2g}$	2	0	0	-2	2	0	0	2	0	0	-2	2	0	0		
$E_{3g}$	2	$-\sqrt{2}$	$\sqrt{2}$	0	-2	0	0	2	$-\sqrt{2}$	$\sqrt{2}$	0	-2	0	0		
$A_{1u}$	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	$z$	
$A_{2u}$	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1		
$B_{1u}$	1	-1	-1	1	1	1	-1	-1	1	1	-1	-1	-1	1		
$B_{2u}$	1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1		
$E_{1u}$	2	$\sqrt{2}$	$-\sqrt{2}$	0	-2	0	0	-2	$-\sqrt{2}$	$\sqrt{2}$	0	2	0	0	$(x, y)$	
$E_{2u}$	2	0	0	-2	2	0	0	-2	0	0	2	-2	0	0		
$E_{3u}$	2	$-\sqrt{2}$	$\sqrt{2}$	0	-2	0	0	-2	$\sqrt{2}$	$-\sqrt{2}$	0	2	0	0		

$C_{nv}$	$E$	$2C_n^\phi$	...	$\infty\sigma_v$		
$A_1 \equiv \Sigma^+$	1	1	...	1	$z$	$x^2 + y^2, z^2$
$A_2 \equiv \Sigma^-$	1	1	...	-1	$R_z$	
$E_1 \equiv \Pi$	2	$2 \cos \phi$	...	0	$(x, y), (R_x, R_y)$	$(xz, yz)$
$E_2 \equiv \Delta$	2	$2 \cos 2\phi$	...	0		$(x^2 - y^2, xy)$
$E_3 \equiv \Phi$	2	$2 \cos 3\phi$	...	0		
...	...	...	...	...		

$D_{\infty h}$	$E$	$2C_\infty^\phi$	...	$\infty\sigma_v$	$i$	$2S_\infty^\phi$	...	$\infty C_2$		
$\Sigma_g^+$	1	1	...	1	1	1	...	1	$R_z$	$x^2 + y^2, z^2$
$\Sigma_g^-$	1	1	...	-1	1	1	...	-1	$(R_x, R_y)$	
$\Pi_g$	2	$2 \cos \phi$	...	0	2	$-2 \cos \phi$	...	0		$(xz, yz)$
$\Delta_g$	2	$2 \cos 2\phi$	...	0	2	$2 \cos 2\phi$	...	0		$(x^2 - y^2, xy)$
...	...	...	...	...	...	...	...	...		
$\Sigma_u^+$	1	1	...	1	-1	-1	...	-1	$z$	
$\Sigma_u^-$	1	1	...	-1	-1	-1	...	1		
$\Pi_u$	2	$2 \cos \phi$	...	0	-2	$2 \cos \phi$	...	0	$(x, y)$	
$\Delta_u$	2	$2 \cos 2\phi$	...	0	-2	$-2 \cos 2\phi$	...	0		
...	...	...	...	...	...	...	...	...		

5.  $S_{2n}$  GROUPS

$S_4$	$E$	$S_4$	$C_2$	$S_4^3$		
$A$	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1	$z$	$x^2 - y^2, xy$
$E$	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$				$(x, y), (R_x, R_y)$	$(xz, yz)$

$S_6$	$E$	$C_3$	$C_3^2$	$i$	$S_6^5$	$S_6$		
$A_g$	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$E_g$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^* & 1 & \epsilon & \epsilon^* \\ 1 & \epsilon^* & \epsilon & 1 & \epsilon^* & \epsilon \end{Bmatrix}$						$(R_x, R_y)$	$(x^2 - y^2, xy), (xz, yz)$
$A_u$	1	1	1	-1	-1	-1	$z$	
$E_u$	$\begin{Bmatrix} 1 & \epsilon & \epsilon^* & -1 & -\epsilon & -\epsilon^* \\ 1 & \epsilon^* & \epsilon & -1 & -\epsilon^* & -\epsilon \end{Bmatrix}$						$(x, y)$	

$\epsilon = e^{(2\pi i)/3}$

$S_8$	$E$	$S_8$	$C_4$	$S_8^3$	$C_2$	$S_8^5$	$C_4^3$	$S_8^7$		
$A$	1	1	1	1	1	1	1	1	$R_z$	$x^2 + y^2, z^2$
$B$	1	-1	1	-1	1	-1	1	-1	$z$	
$E_1$	$\begin{Bmatrix} 1 & \epsilon & i & -\epsilon^* & -1 & -\epsilon & -i & \epsilon^* \\ 1 & \epsilon^* & -i & -\epsilon & -1 & -\epsilon^* & i & \epsilon \end{Bmatrix}$								$(x, y), (R_x, R_y)$	
$E_2$	$\begin{Bmatrix} 1 & i & -1 & -i & 1 & i & -1 & -i \\ 1 & -i & -1 & i & 1 & -i & -1 & i \end{Bmatrix}$									$(x^2 - y^2, xy)$
$E_3$	$\begin{Bmatrix} 1 & -\epsilon^* & -i & \epsilon & -1 & \epsilon^* & i & -\epsilon \\ 1 & -\epsilon & i & \epsilon^* & -1 & \epsilon & -i & -\epsilon^* \end{Bmatrix}$									$(xz, yz)$

$\epsilon = e^{(2\pi i)/8}$

6. TETRAHEDRAL, OCTAHEDRAL, AND ICOSAHEDRAL GROUPS

$T$	$E$	$4C_3$	$4C_3^2$	$3C_2$		
$A$	1	1	1	1		$x^2 + y^2 + z^2$
$E$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$		$\left. \begin{array}{l} (2z^2 - x^2 - y^2, \\ x^2 - y^2) \end{array} \right\}$
$T$	3	0	0	-1	$(R_x, R_y, R_z), (x, y, z)$	$(xy, xz, yz)$

$\epsilon = e^{(2\pi i)/3}$

$T_d$	$E$	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
$A_1$	1	1	1	1	1		$x^2 + y^2 + z^2$
$A_2$	1	1	1	-1	-1		
$E$	2	-1	2	0	0		$\left. \begin{array}{l} (2z^2 - x^2 - y^2, \\ x^2 - y^2) \end{array} \right\}$
$T_1$	3	0	-1	1	-1	$(R_x, R_y, R_z)$	
$T_2$	3	0	-1	-1	1	$(x, y, z)$	$(xy, xz, yz)$

$T_h$	$E$	$4C_3$	$4C_3^2$	$3C_2$	$i$	$4S_6$	$4S_6^5$	$3\sigma_h$		
$A_g$	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
$A_u$	1	1	1	1	-1	-1	-1	-1		
$E_g$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$		$\left. \begin{array}{l} (2z^2 - x^2 - y^2, \\ x^2 - y^2) \end{array} \right\}$
$E_u$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon \\ \epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} \epsilon^* \\ \epsilon \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} -\epsilon \\ -\epsilon^* \end{array} \right.$	$\left\{ \begin{array}{l} -\epsilon^* \\ -\epsilon \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$		
$T_g$	3	0	0	-1	3	0	0	-1	$(R_x, R_y, R_z)$	$(xy, xz, yz)$
$T_u$	3	0	0	-1	-3	0	0	1	$(x, y, z)$	

$\epsilon = e^{(2\pi i)/3}$

$O$	$E$	$6C_4$	$3C_2 (= C_4^2)$	$8C_3$	$6C_2$		
$A_1$	1	1	1	1	1		$x^2 + y^2 + z^2$
$A_2$	1	-1	1	1	-1		
$E$	2	0	2	-1	0		$\left. \begin{array}{l} (2z^2 - x^2 - y^2, \\ x^2 - y^2) \end{array} \right\}$
$T_1$	3	1	-1	0	-1	$(R_x, R_y, R_z), (x, y, z)$	
$T_2$	3	-1	-1	0	1		$(xy, xz, yz)$

$O_h$	$E$	$8C_3$	$6C_2$	$6C_4$	$3C_2(=C_4^2)$	$i$	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
$A_{2g}$	1	1	-1	-1	1	1	-1	1	1	-1		
$E_g$	2	-1	0	0	2	2	0	-1	2	0		$(2z^2 - x^2 - y^2,$ $x^2 - y^2)$
$T_{1g}$	3	0	-1	1	-1	3	1	0	-1	-1	$(R_x, R_y, R_z)$	
$T_{2g}$	3	0	1	-1	-1	3	-1	0	-1	1		$(xy, xz, yz)$
$A_{1u}$	1	1	1	1	1	-1	-1	-1	-1	-1		
$A_{2u}$	1	1	-1	-1	1	-1	1	-1	-1	1		
$E_u$	2	-1	0	0	2	-2	0	1	-2	0		
$T_{1u}$	3	0	-1	1	-1	-3	-1	0	1	1	$(x, y, z)$	
$T_{2u}$	3	0	1	-1	-1	-3	1	0	1	-1		

$I$	$E$	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$		
$A$	1	1	1	1	1		$x^2 + y^2 + z^2$
$T_1$	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	$(x, y, z), (R_x, R_y, R_z)$	
$T_2$	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1		
$G$	4	-1	-1	1	0		
$H$	5	0	0	-1	1		$(xy, xz, yz, x^2 - y^2, 2z^2 - x^2 - y^2)$

$I_h$	$E$	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	$i$	$12S_{10}$	$12S_{10}^3$	$20S_6$	$15\sigma$		
$A_g$	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
$T_{1g}$	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	$(R_x, R_y, R_z)$	
$T_{2g}$	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1		
$G_g$	4	-1	-1	1	0	4	-1	-1	1	0		
$H_g$	5	0	0	-1	1	5	0	0	-1	1		$(2z^2 - x^2 - y^2,$ $x^2 - y^2,$ $xy, xz, yz)$
$A_u$	1	1	1	1	1	-1	-1	-1	-1	-1		
$T_{1u}$	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	-3	$-\frac{1}{2}(1 - \sqrt{5})$	$-\frac{1}{2}(1 + \sqrt{5})$	0	1	$(x, y, z)$	
$T_{2u}$	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	-3	$-\frac{1}{2}(1 + \sqrt{5})$	$-\frac{1}{2}(1 - \sqrt{5})$	0	1		
$G_u$	4	-1	-1	1	0	-4	1	1	-1	0		
$H_u$	5	0	0	-1	1	-5	0	0	1	-1		

- 00000000 -