J-TYPE RANDOM 2,3 SATISFIABILITY REVERSE ANALYSIS TOPOLOGICAL-BASED METHOD IN DISCRETE HOPFIELD NEURAL NETWORK

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2025

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by

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Thesis submitted in fulfilment of the requirements for the degree of Master of Science

July 2025

ACKNOWLEDGEMENT

What a wonderful journey of these time and almost reaching to the final destination. First, I would like to express my deepest gratitude to my supervisor, Assoc. Prof. Dr. Mohd Shareduwan Mohd Kasihmuddin, for his invaluable and patient guidance, constant support, and insightful feedback throughout the process of this research. His expertise and encouragement have been instrumental in shaping this thesis and my growth as a researcher. I used to very struggle during my Master's life without goals, objectives, and so on. He always push me forward and encourage me to do better. He consistently motivated me to strive for excellence and always believe me that I can make more achievement. I am also deeply thankful to Assoc. Prof. Dr. Mohd. Asyraf Mansor for his insightful suggestions and assistance during the different stages. His guidance and opinions have significantly enriched the quality of this study.

I extend my sincere appreciation to the team members of the Artificial Intelligence Research Development Group (AIRDG) at School of Mathematical Sciences for providing me with the knowledge and resources necessary to undertake this research. Every activities that were hold in this group help me to increase the knowledge and improve the communication ability. Meanwhile, my heartfelt thanks go to my team members, Nur Ezlin Zamri, Atiqah Romli, Gao Yuan, Chang Yunjie, Liu Xiaoyan, Zheng Chengfeng, Chen Ju, and Guo Yueling. I always asking for help from them which encourage me to achieve more goals. Their consistent support and willingness to assist me were important in achieving my research goals. I am deeply thankful for their patience and kindness throughout this journey.

At last, I would like to extend my thanks to Universiti Sains Malaysia (USM) and the Institute of Postgraduate Studies (IPS) for providing me with the invaluable opportunity to pursue my studies. I am particularly thankful for the adequate resources and supportive environment that facilitated the successful completion of my research.

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LIST OF SYMBOLS

$\wedge$	AND
$\vee$	OR
$\neg$	NOT
l_i	Literal
C_i	Clause
$P_{GRAN3SAT}$	The G-type random 3 satisfiability logical structure
u	The total number of first order clause
v	The total number of second order clause
w	The total number of third order clause
$\mathbb{Z}^{+}$	The set of positive integers
$C_i^{(1)}$	First order clause
$C_i^{(2)}$	Second order clause
$C_i^{(3)}$	Third order clause
$N_{GRAN3SAT}$	The total number of literals
S_i	The i-th neuron state
W_{ijk}	The synaptic weight
ε_i	Threshold of neuron network
$E_{P_{GRAN3SAT}}$	The cost function of GRAN3SAT
$F_j^{(k=1,2,3)}$	The evaluation of neuron state
$L_{P_{GRAN3SAT}}$	The Lyapunov energy function of GRAN3SAT
$L_{P_{GRAN3SAT}}^{\min}$	The minimum energy of GRAN3SAT
dh_i^{new}	The updated local field

R	Relaxation rate
Tol	Tolerance value
O_{ij}	The expected frequency
r_{i+}	The marginal distribution for the variable in i-th row
r_{+j}	The marginal distribution for the variable in j-th column
df	The degrees of freedom
χ^2	The chi-square value
$S_i^{induced}$	The i-th induced logic
$P_{JRAN2,3SAT}$	The J-type random 3 satisfiability logical structure
$E_{P_{GRAN3SAT}}$	The cost function of JRAN2,3SAT
$h_i(t)$	The local field
$tanh(h_i)$	Hyperbolic tangent activation function
$H_{P_{\mathbf{JRAN2,3SAT}}}$	The Lyapunov energy function of JRAN2,3SAT
$h_{P_{JRAN2,3SAT}}^{\min}$	The minimum energy of JRAN2,3SAT
$\mathbb{R}^d$	A d-dimensional real vector space
X	The raw data
A	The covariance matrix
W_i	The principal component
λ_i	The eigenvalues
T	The transformed data
$d(x, y)$	The Euclidean distance
$D(C_1, C_2)$	The linkage criteria
f	The filter function
x_{ij}	The raw data
p_{ij}	The converted input

μ_k	The centroid value
$\mathcal{I}$	The overlapping interval
$Q^{best}_{JRAN2,3SAT}$	The best logic of JRAN2,3SAT
$Q^{best}_{HornSAT}$	The best logic of HornSAT
Q^{best}_{2SAT}	The best logic of 2SAT
Q^{train}	A set of logical formulas
TP	True positive
TN	True negative
FP	False positive
FN	False negative
$g(x)$	The sum of TP and TN
ACC	Accuracy
PRE	Precision
SEN	Sensitivity
SPC	Specificity
N_{trail}	The number of trails
Π	The global permutation operator
π	The local permutation operator
s_i^f	The final neuron state
$F1$	F1 score
$N_{\textbf{comb max}}$	Neuron combinations
$N_{\textbf{learn}}$	Number of leaning trail
$N_{\textbf{cl}}$	Current number of leaning trail
$N_{\textbf{test}}$	Number of testing trail
$f_{\max}$	Maximum fitness value

f_{c_i}	Current fitness value
E_c	Current energy value
N_g	Number of global minimum solution
N_l	Number of local minimum solution
N_{t_c}	Total trails during retrieval phase
$S_{jaccard}$	Jaccard index
HD	Hamming Distance
MCC	Matthews Correlation Coefficient
Z_m	The ratio of global minimum solution
N_{iloop}	Ideal loop value
N_{cloop}	Current loop value
S_i^{bm}	The benchmark neuron state
p	Overlapping percentage
A_i	Number of attributes
$+/-/=$	Win/Lose/Equal

LIST OF ABBREVIATIONS

2SAT	2 Satisfiability
2SATRA	2 Satisfiability Reverse Analysis Method
3SAT	3 Satisfiability
3SATRA	3 Satisfiability based Reverse Analysis Method
***r*2SAT**	Weighted Random k Satisfiability
A2SATRA	Log-linear Analysis 2 Satisfiability Reverse Analysis Method
AI	Artificial Intelligence
AIRDG	Artificial Intelligence Research Development Group
ANNs	Artificial Neural Networks
CAM	Content Addressable Memory
CNF	Conjunctive Normal Form
CNNs	Convolutional Neural Networks
DHNN	Discrete Hopfield Neural Network
DNF	Disjunctive Normal Form
E2SATRA	Energy 2 Satisfiability Reverse Analysis Method
ES	Exhaustive Search
GRAN3SAT	G-Type Random 3 Satisfiability
HD	Hamming Distance
HNN	Hopfield Neural Network
HornSAT	Horn Satisfiability
HTAF	Hyperbolic Tangent Activation Function
IPS	Institute of Postgraduate Studies

JRAN2,3SAT	J-Type Random 2,3 Satisfiability
JSATRA	Topology based J-Type Random 2,3 Satisfiability Reverse Analysis
LoL	League of Legends
MAE	Mean Absolute Error
MAJ2SAT	Major 2 Satisfiability
NFL	No Free Lunch Theorem
P2SATRA	Permutation 2 Satisfiability Reverse Analysis Method
PCA	Principal Component Analysis
PRO2SAT	Probabilistic 2 Satisfiability
R2,3SATRA	Random 2,3 Satisfiability Reverse Analysis
RA	Reverse Analysis
RAN2SAT	Random 2 Satisfiability
RAN3SAT	Random 3 Satisfiability
RMSE	Root Mean Square Error
RNN	Recurrent Neural Network
S2SATRA	Supervised 2 Satisfiability Reverse-based Analysis Method
SAT	Satisfiability
Std	Standard Deviation
TDA	Topological Data Analysis
TSP	Traveling Salesman Problem
UCI	UC Irvine Machine Learning Repository
USM	Universiti Sains Malaysia
WA	Wan Abdullah method
YRAN2SAT	Y-Type Random 2 Satisfiability

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SATISFIABILITI 2, 3 RAWAK KAEDAH BERTOPOLOGI ANALISIS BERBALIK JENIS-J DALAM RANGKAIAN NEURAL HOPFIELD DISKRIT

ABSTRAK

Kemajuan perwakilan logik satisfiabiliti dalam rangkaian diskrit neural Hopfield telah berkembang menjadi struktur yang lebih fleksibel, membolehkan penggunaan logik yang bersifat sistematik dan tidak sistematik. Namun, isu utama dalam perwakilan satisfiabiliti yang fleksibel ini adalah kewujudan klausa tertib pertama, yang merendahkan kualiti logik sebagai perwakilan neuron dan membawa kepada masalah overfitting. Oleh itu, tesis ini mencadangkan logik satisfiabiliti peringkat tinggi hibrid yang dinamakan satisfiabiliti 2,3 secara rawak jenis J. Peraturan logik yang dicadangkan ini menghasilkan struktur secara rawak berdasarkan klausa tertib kedua, tertib ketiga, atau gabungan kedua-duanya. Tingkah laku peraturan logik yang dicadangkan akan dinilai menggunakan pelbagai metrik prestasi, ralat latihan, ralat pengambilan semula, analisis tenaga, dan analisis kesamaan. Hasil simulasi menunjukkan bahawa model yang dicadangkan mencapai nisbah minimum global tertinggi dengan nilai purata sebanyak 0.4674. Model yang baru dicadangkan akan digabungkan dalam model perlombongan logik yang dikenali sebagai analisis berbalik satisfiabiliti 2,3 secara rawak kategori J berasaskan topologi. Kaedah pilihan attribut tak terselia dikenali sebagai analisis data bertopologi (ADB) digunakan untuk memilih atribut yang paling penting. Sementara itu, operasi permutasi meningkatkan keupayaan model perlombongan logik untuk mendapatkan logik teraruh terbaik yang dapat mengekstrak pola daripada data. Model perlombongan logik yang dicadangkan menunjukkan prestasi yang lebih unggul berbanding model-model perlombongan logik terkini dengan ketepatan purata sebanyak 0.8375 dalam pelbagai set data dunia sebenar yang dipilih.

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ABSTRACT

The development of satisfiability logical representation in Discrete Hopfield Neural Networks has evolved into a more flexible structure, allowing for both systematic and non-systematic logic. However, the main issue with the exiting flexible satisfiability representation is the appearance of first order clauses which will degrade the quality of the logical rule as a neuron representation and leads to overfitting issue. Therefore, this thesis proposes a hybrid higher order satisfiability logical rule named J-Type Random 2,3 Satisfiability. The proposed logical rule randomly generates structures based on either second order, third order, or a combination of both clauses. The behavior of the proposed logical rule will be evaluated using various performance metrics in terms of training error, retrieval error, energy analysis, and similarity analysis. Simulation results indicate that the proposed model achieves the highest global minimum ratio with an average value of 0.4674. The newly proposed model will be incorporated into the logic mining model known as topology based J-type random 2,3 satisfiability reverse analysis. The unsupervised attribute selection method called topological data analysis is employed in selecting most significant attributes. Meanwhile, permutation operation improves the ability of the logic mining model to retrieve the best induced logic that can extract patterns from the dataset. The proposed logic mining model demonstrates superior performance compared with existing state-of-the-art logic mining models with an average accuracy of 0.8375 in the selected various real-life datasets.

CHAPTER 1

INTRODUCTION

1.1 Overview

With the advancement of Artificial Intelligence (AI), AI-driven technologies have gained huge attention and increasingly influenced various aspects of modern life. AI enables the development of machines capable of performing tasks that traditionally require human intelligence, such as problem-solving, natural language processing, and knowledge acquisition. As research deepens, AI applications continue to expand across diverse fields. This chapter firstly introduces Artificial Neural Networks (ANNs), a representative concept within AI. Following this, the significance of the research is discussed, outlining the problem statement. This chapter will then present the research questions, research objectives, and methodologies that align with this thesis.

1.2 Introduction to Artificial Neural Network

ANNs are computational systems inspired by early theories on how the human brain processes sensory information. In biological systems, neurons form an complicated, interconnected network capable of sophisticated information processing. McCulloch and Pitts (1943) first introduced a mathematical model of a neuron, representing the neuron as a binary switch that activates based on input from connected neurons and associated synaptic weights. ANNs simulate this behavior on computers by organizing artificial neurons in layers, simulating these interactions to solve various tasks. Training ANNs typically involves supervised learning, a process where the network is provided with input-output pairs. The network adjusts the weights between neurons to reduce prediction errors, thereby allowing the network to generalize from example data. Over time, ANNs improves the prediction accuracy on the training and validation data through these weight adjustments, eventually reaching a point of convergence when performance stabilizes. Nowadays, ANNs play an essential role in various fields, such as healthcare, finance, industry, and autonomous systems, due to their effectiveness in applications like natural language processing, image recognition, computer vision, and classification (Zamri et al., 2024).

According to the architecture, ANNs are typically classified into three main categories: feedforward neural networks, recurrent neural networks, and convolutional neural networks. Feedforward neural networks have a linear structure where data flows in a single direction from the input to the output layer, making them effective for tasks like basic pattern recognition. Recurrent neural networks contain feedback loops and allow them to retain information from previous inputs and thus process sequential data characteristic that makes them useful for time-dependent tasks such as language processing. Convolutional neural networks are designed for grid-like data, such as images, with an architecture optimized to capture spatial relationships, making them particularly effective for visual data processing tasks. In this thesis, Hopfield Neural Network (HNN) is central to this research. The network exemplifies a recurrent network with a unique feedback mechanism, where enables the model to function as an associative memory which can store and retrieve patterns based on input. Discrete Hopfield Neural Network (DHNN) builds upon the pattern storage and retrieval capabilities of recurrent networks, extending these functions to applications in intelligent systems. DHNN has seen extensive use in combinatorial optimization problems. The development of DHNN and their implementation with various technologies have demonstrated significant potential for enabling real-world applications.

Despite the remarkable progress in DHNN and their applications (Rusdi et al., 2023), significant challenges remain in developing systems that are both efficient and interpretable, particularly for solving various combinatorial optimization problems. Meanwhile, existing DHNN models face limitations in exploring diversified solution space that restrict their adaptability to real-world scenarios. These challenges highlight the need for innovative approaches to enhance the capabilities of DHNN and ultimately enable the network to handle more sophisticated problems while maintaining interpretability. By embedding novel logical structures into DHNN, this thesis aims to bridge the gap between logical reasoning and neural computation. This implementation not only advances the theoretical foundations of neural networks but also paves the way for practical applications in artificial intelligence. Ultimately, this thesis results in a broader philosophical standpoint, whereby:

Enhancing neural networks with flexible and effective logical structures.

In simpler terms, neural networks can be optimized as an intelligent computational model when a flexible mechanism is applied. Hence, a novel intelligent system can be developed by introducing mechanisms into the DHNN that regulate the relationships among neurons. This can be accomplished through various approaches, such as designing flexible logical structures, formulating optimized neuron update functions, and developing effective logic mining models. In the following section, the problem statement will explore each of these perspectives in detail, providing a comprehensive discussion of the challenges and opportunities associated with this research.

1.3 Problem Statement

ANNs are often regarded as black-box models due to the lack of interpretability and the unknown mechanisms behind neuron updates. This has motivated researchers to explore ways to explain the connections within these networks. Satisfiability as a representative logical rule can be implemented into neural networks to make the networks more interpretable and to uncover the inner relationships among neurons. Based on these central features, the following four problem statements can be presented.

Problem Statement 1: Inflexible logical rule to represent variables.

Explanation of Problem Statement 1: Logical rules are powerful tools for representing intelligent systems in an interpretable and computationally efficient manner. Satisfiability (SAT) logic as demonstrated by Abdullah (1993) which has been used to model the behavior of neural networks. Incorporating logical rules into DHNN offers potential for developing more robust and effective models. For instance, a variety of logical rules enable the representation of diverse knowledge structures. Systematic and non-systematic logical rules have been explored separately: systematic logical rule uses a fixed number of literals per clause, whereas non-systematic logical rule allows for a flexible number of literals across different clauses. The representative models for systematic and non-systematic are: 2 Satisfiability (2SAT) logic proposed by Kasihmuddin et al. (2019) and Random 2 Satisfiability (RAN2SAT) logic proposed by Sathasivam et al. (2020). In particular, random implies the non-systematic. Limited neuron combinations reduce the ability of the network to explore wide range of search space. Systematic logical rules tend to yield consistent behavior, whereas non-systematic logical rules offer a broader solution space but with less predictability (Zamri et al., 2022). Therefore, combining both types as a hybrid logical rule presents a promising direction. Furthermore, higher order clauses have shown better potential in finding global minima, while first order clauses often result in suboptimal solutions and more local minimum solutions (Guo et al., 2022). However, existing models like G-Type Random 3 Satisfiability (GRAN3SAT) (Gao et al., 2022) and Random 3 Satisfiability (RAN3SAT) (Karim et al., 2021) still include first order clauses, while models with higher order clauses such as 3 Satisfiability (3SAT) (Mansor et al., 2017) may increase computational cost and risk information loss. Hence, there is a need for a flexible logical rule that combines systematic and non-systematic features, avoids first order clauses, and minimizes training iterations. This work aims to optimize such logical structures to help DHNN achieve optimal synaptic weights with reduced error and greater global minima convergence.

Problem Statement 2: DHNN suffers from limited interpretability and a high tendency to converge to local minima due to the absence of robust logical rules.

Explanation of Problem Statement 2: DHNN is a fully connected binary neural network with symmetric synaptic weights, operating based on the Hebbian learning rule (Hopfield and Tank, 1985), where weights are adjusted according to neuron activation correlations. DHNN iteratively updates neuron states until a stable state or global minimum solution is reached, representing a solution to a given combinatorial optimization problem (Tank and Hopfield, 1986). However, DHNN is prone to becoming trapped in local minima, leading to suboptimal solutions due to the iterative update process. To address this issue, incorporating logical rules into DHNN offers a promising enhancement. Satisfiability logical rule can provide truth-value assignments to variables in logical formulas, which directly correspond to neuron connectivity in the network. This approach helps improve the ability of the network to find optimal solutions by converging toward more effective global minima. Specifically, higher order clauses characterize high probability to obtain satisfied interpretation which have been shown to reduce iterations and lead to more optimal synaptic weights compared to first order clauses, which often result in ineffective solutions and local minima (Mansor et al., 2017). The first attempts at implementing logic programming into neural networks were made by Abdullah (1992), and this area continues to develop. However, current studies solely focus on systematic or non-systematic logical rule which limits the search space of the optimal solutions. By implementing hybrid higher order logical rules with DHNN, the model can potentially handle optimization problems more effectively and discover optimal global solutions in fewer iterations. This approach not only improves convergence but also broadens the practical applicability of DHNN in areas such as healthcare, business, and social science, particularly in complex classification and knowledge extraction tasks.

Problem Statement 3: Ineffective logical rules hinder the logic mining model from accurately retrieving patterns in real-life datasets.

Explanation of Problem Statement 3: Implementing logical rules into DHNN has shown strong potential for extracting meaningful patterns from real-world datasets. This approach allows for effective representation of the knowledge in question without misinterpretation of the patterns. The process involves identifying a logical rule that accurately captures the underlying datasets, a technique commonly referred to as a logic mining model. However, systematic logical rules with fixed order clauses often struggle to generalize across complex datasets (Kasihmuddin et al., 2019). While non-systematic logical rules demonstrate considerable efficacy with simulated data, the flexibility of the model still constrains the capacity of the logic to generalize more datasets (Guo et al., 2022). Existing logic mining models primarily rely on systematic rules such as 2SAT (Kho et al., 2020) or 3SAT (Zamri et al., 2020b). None of the existing works explore non-systematic logical rules in real-life datasets, resulting in a failure to identify optimal attribute combinations that represent dataset patterns. This limitation requires a more flexible logical structure to find more possibilities of optimal patterns. The simulated experimental results demonstrate that hybrid higher order logical rule may generate better performance in real-life classification tasks. This implementation can lead to a broader range of potential induced logical rules for effectively representing the overall behavior of datasets. Hybrid higher order logical rules are needed because the search space for induced logic can be expanded by using flexible logical rule, enabling a broader exploration of attribute combinations that lead to optimal dataset representations. Flexible logical structure helps uncover relationships between different neurons. Through reverse analysis approach, the most effective logic combination can be identified.

Problem Statement 4: Insufficient attribute selection method and limited attribute combinations constrain the performance of the logic mining model.

<u>Explanation of Problem Statement 4</u>: Logic mining model can be separated into three phases: pre-processing phase, training phase, and retrieval phase. Each phase presents challenges that can affect overall performance. First of all, the selection of relevant attributes is a crucial aspect of building effective neural network models, as this process significantly impacts performance and the ability to generalize to unseen data. However, most existing logic mining models such as 2 Satisfiability Reverse Analysis Method (2SATRA) (Kho et al., 2020) and Energy 2 Satisfiability Reverse Analysis Method (E2SATRA) (Jamaludin et al., 2021) rely on random attribute selection. This approach risks excluding important attributes and ultimately limits the ability of the model

to uncover underlying patterns. While some models like Supervised 2 Satisfiability Reverse-based Analysis Method (S2SATRA) (Kasihmuddin et al., 2022) and Log-linear Analysis 2 Satisfiability Reverse Analysis Method (A2SATRA) (Jamaludin et al., 2022) use correlation and log-linear methods for attribute selection, these models focus on linear or non-linear relationships between attributes and outcomes. These techniques often rely on assumptions about data distribution. To overcome this, an unsupervised attribute selection method is needed to uncover hidden relationships without relying on predefined assumptions. Additionally, while previous models have solely focused on optimizing true positives during the training phase (Jamaludin et al., 2023) which may lose the consideration of negative cases that are accurately identified. Moreover, the failure to account for interrelationships among attributes can hinder the discovery of optimal attribute combinations, limiting the search space for potential solutions. Thus, an improved logic mining model should incorporate efficient unsupervised attribute selection, consider both true positives and negatives, and explore diverse attribute combinations to enhance performance and generalizability across real-life datasets.

1.4 Research Question

In order to establish a link between the research problems and research objectives, this section will propose four important research questions. These questions are generated from the problem statement in section 3 of this chapter, and this thesis will focus on solving these questions. The research questions are listed as follows:

RQ1. What kind of logical rule can be formulated by combining systematic and non-systematic logical features to contribute the higher order clause without considering the first order clause in logic phase to generate a random set of literals?

RQ2. How does the hybrid higher order logical structure govern the Discrete Hopfield Neural Network to effectively achieve the purpose of storing and retrieving patterns?

RQ3. How can the implementation of a hybrid higher order logical rule in a logic mining model be used to retrieve the best induced logic for effectively extracting patterns from a dataset?

RQ4. How does the geometrical topology be used to connect different attributes that leads to the connection of the second and third order clauses in logic mining model?

1.5 Research Objectives

This thesis focuses on modeling a DHNN using a hybrid of systematic and non-systematic logical rules. The proposed model will be implemented into the DHNN and the performance will be analyzed under various conditions. The logical rule developed will then be implemented into a logic mining model for application in real-world scenarios. An unsupervised attribute selection method combined with permutation operations will be implemented to enhance the performance of the proposed logic mining model. The model will be examined on a variety of datasets and be evaluated using various metrics. To address the research challenges outlined, four research objectives will be set. The research objectives were written based on the Specific, Measurable, Achievable, Relevant, and Time-bound (SMART) framework.

RO1. To formulate a novel systematic and non-systematic logical rule, referred to as J-type Random 2,3 Satisfiability, which incorporates second and third order literals per clause. The proposed J-type Random 2,3 Satisfiability logical rule will store information externally and will evaluate as True only if all the clauses within the rule are satisfiable.

RO2. To implement the proposed J-type random 2,3 satisfiability into the Discrete Hopfield Neural Network by minimizing the corresponding energy function, where the logical rule governs the DHNN toward global minimum solutions within a large search space.

RO3. To propose J-type Random 2,3 Satisfiability logical structure within the logic mining model and apply to real-life scenarios, named Random 2,3 Satisfiability Reverse Analysis. The effectiveness and advantages of incorporating a hybrid higher order logical rule will be demonstrated by comparing the proposed model with existing models.

RO4. To develop a new logic mining model named the Topology based Reverse Analysis Method. Topological Data Analysis attribute selection method and permutation operation will be utilized to enhance the performance. The proposed logic mining model will extract patterns in the form of induced logic from the datasets. This induced logic will possess the capability to effectively classify and predict outcomes.

1.6 Methodology

The primary methodology for enhancing DHNN with a flexible logical structure involves formulating a hybrid higher order logical rule, known as J-Type Random 2,3 Satisfiability (JRAN2,3SAT) which is implemented into DHNN to guide the convergence of the network to a stable state representing the global minimum solution. The validated optimal logical structure will then be applied to real-life datasets. This process involves thorough data preprocessing and permutation optimization to retrieve the optimal induced logic that accurately represents the patterns within the data. Specifically, the following four methodologies will provide the details.

<u>Methodology 1</u>

In recent years, various types of SAT logical rules have been proposed, each designed to model different aspects of ANNs. This makes exploring diverse SAT representations essential as the logical rule can effectively capture the underlying relationships in neural networks. In this thesis, a new type of logical rule will be introduced, drawing inspiration from the work of Karim et al. (2021), which proposed RANkSAT, and the work of Gao et al. (2022), which introduced GRAN3SAT. Both of these models incorporate higher order clause logical rules, with a key feature that is non-systematic. However, the proposed model combines both systematic and non-systematic logical rules to offer more flexibility and robustness. Specifically, the novel logical rule implements two systematic logical rules 2SAT and 3SAT with a non-systematic rule that blends both 2SAT and 3SAT. These rules will appear in random combinations within the logical formula, providing a dynamic and adaptive structure. The logic in this proposed model will first be defined based on the combination of these logical rules. To improve the interpretability, the logic will be converted into Boolean algebra which is specifically in Conjunctive Normal Form (CNF). In this form, literals are connected by disjunctive operators (OR, denoted as $\vee$), and clauses are linked by conjunctive operators (AND, denoted as $\wedge$). This formulation allows for a clear representation of the logical structure, which is essential for further processing within the neural network model. For instance, given the description of the rules above, one can derive various examples of logic formulas that exemplify the combination of the different logical rules. The proposed JRAN2,3SAT differs in several important ways from the previously discussed models, RANkSAT and GRAN3SAT. In RANkSAT, the combinations of clauses are restricted to specific sets, such as $k = 1, 3$, $k = 2, 3$, and $k = 1, 2, 3$. Additionally, this model imposes limits on the ratio of positive literals to negative literals, such as a 1:1 or 2:1 ratio, as well as a predefined distribution of clause orders (e.g., a 1:1 or 1:2 ratio between first and third order clauses). These constraints reduce the flexibility of RANkSAT in modeling more complicated patterns. On the other hand, GRAN3SAT allows for more flexibility by not imposing these limitations, offering greater freedom in applying logical rules. JRAN2,3SAT proposed in this thesis combines the systematic and non-systematic features found in both RANkSAT and GRAN3SAT. The model allows for combinations of clause orders, specifically $k = 2$, $k = 3$, and $k = 2, 3$, removing first order clauses entirely and focusing on higher order clauses. This operation enhances the ability of the model to capture more logical relationships within the data. Furthermore, JRAN2,3SAT does not impose any predefined ratios for positive to negative literals or for the number of clauses of each order. This lack of restrictions increases the flexibility of the model and enables better adaptation to different datasets without overfitting or introducing bias.

Scope Methodology 1

Despite the advantages, JRAN2,3SAT also has several certain development can be achieved. One key component is that the clauses must be connected in CNF to enable the computation of synaptic weights using Wan Abdullah method (WA) (Abdullah, 1992). As a result, any logical representations that cannot be converted into Boolean algebra may not be considered in the analysis. Additionally, excluding first order clauses may reduce diversity and complexity, which could contribute to exploring a wider range of possible solutions. Although this exclusion helps prevent the model from converging to suboptimal synaptic weights, the capacity of the logical rule to represent certain complicated patterns may be reduced which could be important for capturing the full complexity of the dataset.

Methodology 2

The next step after proposing the new logical rule is to implement within the framework of DHNN. This process begins by defining the inconsistency of the logical formulas and deriving the associated cost function. Once the logical clauses are checked for satisfaction, the next step is to determine the synaptic weights using WA method (Abdullah, 1993). This approach involves comparing the derived cost function with the Lyapunov energy function to compute the synaptic weights. The cost function is known, but the coefficients in the energy function are initially unknown. By setting the cost function and the energy function equal, the optimal synaptic weights can be calculated. This method introduces an efficient computation process of synaptic weights, which are crucial for the training process of the network. One key aspect of the proposed logical structure is the ability to achieve optimal synaptic weights with fewer iterations, which significantly improves the convergence speed of DHNN. Once the synaptic weights are calculated, they are stored as Content Addressable Memory (CAM), which serves as the memory structure for DHNN. CAM stores the relationships among neurons which are also represented by synaptic weights, allowing the information to be retrieved during the retrieval phase. The network then enters the retrieval phase, where the local field of each neuron is computed and output scaling is achieved through Hyperbolic Tangent Activation Function (HTAF) (Kasihmuddin et al., 2019). The local field computation for each neuron is determined based on the synaptic weights. Once the local fields are calculated, the minimum energy for the network is computed using a specific formula. The neuron states are then updated through HTAF, producing a new output. This process continues iteratively until the desired results are achieved, known as the relaxation phase. Among these iterations, the one yielding the highest number of global minimum solutions is selected as the final output. The final energy of the network is determined by calculating the absolute value of the difference between the local field and the minimum energy. If this difference is less than a predefined tolerance, the solution is considered to have reached the global minimum. Otherwise, the network may become trapped in a local minimum, limiting the ability of the network to converge to the optimal solution.

Scope Methodology 2

While this methodology provides a systematic and effective approach for implementing DHNN, there are several developments can be achieved when ensuring the reproducibility of the results in this thesis. First, the new model specifically considers only 2SAT and 3SAT logical rules per clause, deliberately excluding first order clauses to minimize their potential impact on the training phase of DHNN. This feature ensures that the training process remains focused on higher order relationships within the data. Additionally, the distribution of negative literals in the formula is a crucial factor, as described in the work of Zamri et al. (2022) on $r2$SAT. The ratio of positive to negative literals in the clauses can significantly influence the performance and convergence of DHNN, making careful control and analysis of the negative and positive literal distribution within the proposed logical rule. By considering these developments and optimizing the training and retrieval phases in the future, DHNN can achieve robust performance while accurately capturing the logic embedded in the data.

Methodology 3

With the successful establishment and implementation of the novel JRAN2,3SAT in DHNN, the next key step is to explore the application of the proposed model to real-life datasets. While initial experiments have relied on simulated data to evaluate the performance of the proposed model, applying the model to real-world data will truly demonstrate the capability of classification and prediction. To begin with, a real-life dataset is selected, with a predetermined number of attributes chosen for inclusion in the model. Worth noting that not all available attributes are included in the model, as this may lead to overfitting and an increase in computational complexity. To address this, only the most relevant and informative attributes are selected. These selected attributes are then converted into bipolar form by using the k-means clustering (Kasihmuddin et al., 2022). Once the data is appropriately preprocessed, the dataset is divided into two parts: training data and testing data. This split ensures that the model can learn from one set of data and be evaluated on another, maintaining the consistency of the retrieval phase. Before entering the training phase, a core concept in evaluating model performance must be addressed: the confusion matrix. By calculating the confusion matrix at two stages of the model, the effectiveness of the model can be assessed in both the training and retrieval phases (Caelen, 2017). The confusion matrix consists of four main elements: True Positive (TP), True Negative (TN), False Positive (FP), and False Negative (FN). Specifically, by optimizing the synaptic weights in the network based on the confusion matrix, the best logical rules can be identified for the training phase. The criteria for evaluating the best logic during training will focus on maximizing the sum of TP and TN, as this reflects the ability of the network to correctly classify both positive and negative instances. Once the optimal synaptic weights and the best logic have been determined, they are stored in CAM, ensuring that the network has access to this information during the retrieval phase. The retrieval phase focuses on generating the best-induced logic that accurately represents the underlying structure of the dataset (Zamri et al., 2022). During this phase, the neuron states are updated iteratively, and the accuracy of the model is monitored to determine the best-induced logic. The induced logic is considered optimal when the highest accuracy is achieved through the logic.

Scope Methodology 3

Simulated data were utilized to justify JRAN2,3SAT in DHNN model, applying the model to real-life datasets introduces additional challenges and complexities. One of the most critical challenges is data preprocessing. Real-world datasets often require extensive cleaning, normalization, and transformation to ensure that they are suitable for input into the logic mining model. This preprocessing step is essential for converting raw data into a structured format that can be effectively utilized by DHNN. Moreover, there are several areas within the model that can be optimized to further enhance the ability of the model to classify or predict outcomes. These optimizations may involve refining the attribute selection process, adjusting the parameters for improved convergence, or enhancing the training algorithms to reduce overfitting (Zamri et al., 2024). By addressing these challenges, the proposed logic mining model can achieve even better performance when applied to a variety of real-life datasets, showcasing the potential for real-world applications in classification and prediction tasks.

Methodology 4

During the pre-processing phase of the logic mining model, selecting appropriate attributes is crucial for determining the quality of neurons that are incorporated into the network. Supervised attribute selection methods, while effective in certain contexts, typically rely on specific data distributions which can limit their applicability across a broader range of datasets. In this thesis, an unsupervised attribute selection technique called Topological Data Analysis (TDA) is employed to overcome these limitations. TDA focuses on the intrinsic structure of the data without making any distributional assumptions, enabling the approach to identify patterns and relationships that are especially valuable for complicated data. Specifically, the process involves reducing dimensionality, applying clustering techniques, and constructing topological graphs (Mannucci et al., 1998). The way of reducing dimensionality is Principle Component Analysis (PCA). PCA reduces data dimensionality by identifying and retaining the most important directions which also refer to principal components that capture the maximum variance in the data, discarding less significant dimensions. This is done through linear transformations to project data into a lower-dimensional space while preserving its essential structure. In the topological graphs, a required number of points are used to represent the attribute classes. One attribute representing each class is randomly selected due to identical topological features. These selected attributes are treated as neurons and sent to the network for processing. These methods emphasize attributes contributing to the shape and connectivity of the data. Additionally, a permutation operation is used to explore different attribute combinations, expanding the search space and improving the chances of identifying the best induced logic for a given dataset (Jamaludin et al., 2023). By exchanging attribute positions and recording their locations, various attribute combinations can be explored, and the specific combination that generates better performance will be identified. These combined techniques in the proposed logic mining model are designed to enhance performance, enabling the model to generalize better when applied to real-life datasets.

Scope Methodology 4

Although the proposed logic mining model demonstrates strong performance, there are further opportunities for optimization. For instance, employing different unsupervised attribute selection methods could generate insightful results. One such method is unsupervised attribute selection using attribute similarity, which is based on the maximum information compression index to identify and remove redundant features (Mitra et al., 2002). Meanwhile, the diversity of the induced logic retrieved by the model can be enhanced not only through permutation operations but also by storing multiple sets of the best logic during the training phase, based on predefined criteria for selecting the best logic. Additionally, incorporating metaheuristic algorithms in the retrieval phase could further diversify the best induced logic. By leveraging such algorithms, the model can explore a broader solution space and potentially uncover multiple high-quality logic configurations that better capture the underlying patterns in the data. These optimizations have the potential to significantly improve the predictive and classification accuracy of the model across diverse applications.

Figure 1.1 provides the flowchart of the overall research framework for this thesis. From the philosophical standpoint presented in this chapter, the thesis is designed to achieve four primary research objectives step by step. Each objective highlights key components that guide the formulation of the research methodology.

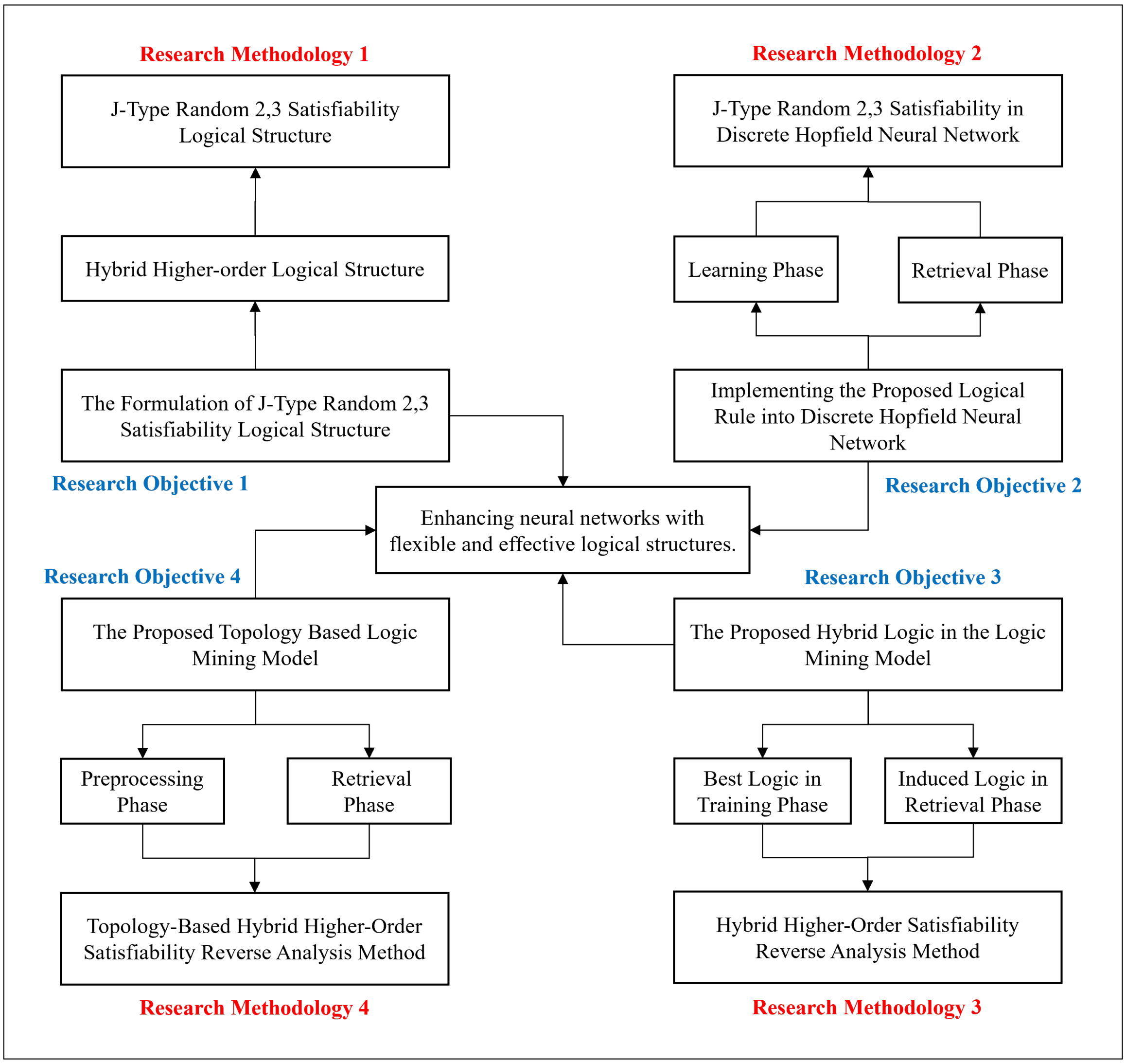

Figure 1.1: Diagram of overall research in this thesis.

1.7 Organization of Thesis

The organization of this thesis is as follows. Chapter 1 provides an introduction, starting with an overview of ANN, followed by a discussion on the significance of the research. To establish the motivation for this 1 thesis, the problem statement is addressed to highlight existing challenges in current research. Each problem statement is linked to a corresponding research question, research objective, and an overview of the methodology, which are presented in the next three sections.

Chapter 2 provides a comprehensive literature review organized into three primary domains relevant to this study: DHNN, the implementation of satisfiability logical rules within DHNN, and existing logic mining models. The first domain discusses previous studies in DHNN, exploring their applications, limitations, and potential. The second domain reviews various implementations of satisfiability logical rules in DHNN, examining existing approaches for their structure, limitations, and applicability. The final domain examines notable logic mining models, evaluating their methodologies and the challenges they address in logical inference and classification tasks.

Chapter 3 elaborates on the foundational work inspiring the proposed model, starting with the formulation of GRAN3SAT model (Gao et al., 2022), which is a key influence on the novel logical rule proposed in this thesis. The implementation of GRAN3SAT into DHNN will be discussed. The last part of the chapter describes the methodologies of current logic mining models, discussing their approach to data processing, attribute selection, and result evaluation.

Chapter 4 introduces the core contributions of this thesis which is the proposed JRAN2,3SAT logical rule, a hybrid higher order logical rule designed to cover systematic and non-systematic logical structures. This chapter details the design and the structure of the model and operational logic when applied in DHNN and introduces the associated logic mining model that leverages this hybrid rule to improve classification and knowledge extraction tasks.

Chapter 5 analyzes the performance of the proposed logical rule, beginning with the experimental setup. It then outlines the parameter settings and provides a brief introduction to the baseline models used for comparison. The results and discussion focus on three primary stages: the training phase, retrieval phase, and similarity analysis. The following section presents the Friedman test to determine whether significant differences exist among the baseline models across various metrics. Finally, the impact of different ratios of second and third order clauses is examined.

Chapter 6 focuses on evaluating the proposed performance of the model across several metrics, providing a thorough review of the datasets selected from repository datasets, performance metrics definitions, and parameter settings employed. This chapter also introduces baseline logic mining models for comparative evaluation and briefly describes their structure and key differences. The chapter then provides a systematic performance comparison which presents and analyzes results between the proposed and baseline models.

Chapter 7 concludes the thesis with two sections: conclusion and future work. Each section is structured around the four primary research objectives, discussing the summary of the study, potential optimizations, and future research directions. The conclusion highlights the contributions of this thesis which are mapping with objectives. The future work section identifies areas where the model can be further expanded. Therefore, this section emphasizes opportunities for improvement and broader applicability. Additionally, this section explores the potential applications in more diverse and complicated datasets.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter will conduct a literature review on the domains central to the proposed model. The first section will explore the development of DHNN in addressing various optimization tasks, introducing the foundational work. The next section will review Satisfiability which serves as a neuron representation and is implemented within DHNN. The specific operation is utilizing these logical rules to govern the process of the neural network. The following section will examine advancements in logic mining models, focusing on the application of well-structured logical rules to real-life datasets through reverse analysis approaches. Specifically, supervised and unsupervised logic mining models will be discussed separately, focusing on their attribute selection methods and primary objective functions within the network. Throughout this chapter, the research gaps in each domain will be identified and discussed.

2.2 Discrete Hopfield Neural Network

The HNN was introduced by Hopfield and Tank (1985) which is a type of Recurrent Neural Network (RNN). RNN is characterized by the output from one step is passed back into the network as input for the next step. This looping structure allows the network to remember information over time. DHNN follows this neuron update mechanism and shows specific features. Firstly, DHNN is a fully connected network with binary neurons. These neurons have bidirectional connections with synaptic weights which signifies the ability of DHNN to learn and store information. The network is trained using a Hebbian learning rule (Hopfield and Tank, 1985). DHNN works by iteratively updating the states of the neurons until a stable state is reached where the neuron outputs stop changing, which is also called the global minimum solution. The steady state will satisfy the constraints of the network. The synaptic weights of DHNN are symmetrical with no self-connections, and an asynchronous manner of neuron update that ensures the convergence of DHNN. Convergence property guarantees that the iterative process does not loop infinitely. A crucial role of DHNN is minimizing a configurational energy function. Energy property as a measure of the network stability and convergence plays a crucial role in DHNN which has been shown by Abdullah (1994). Rational measurement in terms of energy can enhance the optimization efficiency of DHNN. Worth mentioning that Folli et al. (2017) showed that the trade-offs between efficiency and effectiveness of associative memory in DHNN depend on the number of patterns stored in the network and the method used to adjust the connection weights. DHNN can thus be used for solving combinatorial optimization problems (Tank and Hopfield, 1986), and adequately reasonable solutions to complicated problems can be found in a linear time or even less. DHNN is one of the earlist models to provide a potential solution for Traveling Salesman Problem (TSP). Unfortunately, the current DHNN lacks a symbolic framework to describe the neural connectivity which limits the interpretability of the mechanisms. As stated in Bao and Zhao (2024), this study concludes that binary associative memory for DHNN exhibits a limited but predictable storage capacity with the retrieval performance heavily dependent on the network configuration and the learning rule applied. This study highlights the trade-off between capacity and stability. Without a clear symbolic rule to represent neuron interactions, understanding the decision-making process within the network becomes challenging. At the same time, the main weakness of DHNN is that the network may converge to a suboptimal solution that is not the best possible solution to the problem. As a result, the network will take many iterations for the network to converge to a stable state. Therefore, inserting new logical rules into DHNN which act as a symbolic instruction to govern the network, thereby enhancing the representation and storage capacity of information. A logical rule can be used to create a structure to train the DHNN and find an assignment of truth values to the variables in the logical formula such that the formula evaluates to true (Aloul et al., 2006). This assignment of truth values will then correspond to the connectivity of the neurons in DHNN.

2.3 Satisfiability Logical Rule in Discrete Hopfield Neural Network

Satisfiability (SAT) is a well-known representation in the field of logic and computer science. The logical rule entails determining whether there exists an interpretation that can render a given Boolean formula valid. A Boolean formula is a mathematical expression that comprises variables, operators, and parentheses. Abdullah (1992) made the first attempt at logic programming on a neural network. This kind of work was continued by Sathasivam (2010). In this context, the concept of Horn Satisfiability (HornSAT) in DHNN with scheduled relaxation during the retrieval phase was introduced. In order to ensure the convergence property of DHNN, a relaxation method was implemented in this study. Kasihmuddin et al. (2019) proposed a systematic logical rule that contains a restricted number (which is two) of literals per clause (2SAT) in DHNN. In the continued research of systematic logical rule, Mansor et al. (2017) explored a higher order systematic logical rule to deal with three literals per clause, which is 3 Satisfiability (3SAT). Worth mentioning that this work demonstrates the unnecessity of exploring higher order of kSAT for $k > 3$. As a result, this theorem can be effectively employed to reduce any higher order logic to lower order logic. One of the new variants of logical rule in DHNN is the non-systematic logical rule. To explore more diverse solutions in the search space. Sathasivam et al. (2020) proposed RAN2SAT in DHNN, which can solve vital case complexity of optimization problems. RAN2SAT contains first and second order logical rules that can increase the diversity of the synaptic weight. The investigation of RANkSAT with DHNN for higher orders of k has been proposed by Karim et al. (2021), which is named RAN3SAT. The proposed RAN3SAT was reported to outperform RAN2SAT in terms of global minimum ratio. Except for the previous works of non-systematic logical rules, the study by Alway et al. (2022) introduced a variant of random k-Satisfiability called Major 2 Satisfiability (MAJ2SAT), formulas in which the majority of clauses are expressed in second order, with only a small number remaining as first order clauses. The findings of the study suggest that as the use of second order logic increases, MAJ2SAT is capable of providing a greater range of neuron variations, thus making the logical rule a suitable model for improving the self-learning process. Inspired by these works, Gao et al. (2022) proposed a hybrid model which is GRAN3SAT. In this model, the neuron structure of the conventional DHNN has a larger storage capacity and is well-suited for exploring high-dimensional applications.

Another creative structure has been showcased by Zamri et al. (2022). Researchers frequently overlook the role of negative literals within a logical structure. This is due to the fact that negative literals are often indirectly associated with faults or errors in achieving a satisfiability formula (Chen et al., 2023). This study investigated the logical rule that inserts ratio to indicate the distribution of negative literals that exists in each clause and proposed a novel logical rule named Weighted Random k Satisfiability (r2SAT). r2SAT has a dynamic distribution of negative literals that will facilitate the production of global minimum solutions with diverse final neuron states. According to the result, embedding r2SAT in DHNN can obtain more diversified final neuron states. Building on this study, Chen et al. (2023) proposed a systematic logical rule called Probabilistic 2 Satisfiability (PRO2SAT) logical rule, which defines the proportion of negative literals within logical clauses. Their findings show that varying the proportion of negative literals can lead to significantly different results. This underscores the importance of considering the balance of negative literals in logic-based neural network frameworks. Recently, Guo et al. (2022) proposed a logical rule that can represent both systematic and non-systematic which called Y-Type Random 2 Satisfiability (YRAN2SAT). By integrating both the features of systematic and non-systematic logical rules, there is greater flexibility in terms of solution space. This work shows that YRAN2SAT has the ability to represent new logical rules, which have a high potential for solution diversity. Unfortunately, the current work by Sathasivam et al. (2020) posed a learning issue when there is an existence of first order clause. In another development, the work by Gao et al. (2022) showed great potential in higher order clause when the logic is represented only in first order clauses, the learning capability of DHNN will collapse as the number of first order clauses increases. Worth mentioning, that based on the No Free Lunch Theorem (NFL) (Wolpert and Macready, 1997), there is no single logical rule that can perfectly represent all datasets in various scenarios. Therefore, there is a need to propose a more flexible logic that has a higher order clause to avoid the neuron from being trapped into local minimum solution and obtain diversified final solutions. By excluding first order clauses will enhance the ability of retrieving more global minimum solutions.

2.4 Logic Mining

Data mining is a powerful tool for the discovery of patterns, correlations, and trends within large datasets, which is often employed in a variety of fields, including finance, healthcare, and marketing. The approach enables the extraction of meaningful information from raw data, thereby facilitating decision-making processes and predictive analysis. Over time, a multitude of data mining techniques have been developed. For example, classification algorithms categorize data into predefined classes (Loh, 2011), whereas clustering techniques group similar data points based on specific characteristics (Wang et al., 2006). Association rule mining uncovers relationships between variables within datasets, while regression analysis predicts continuous outcomes based on input variables (Razi and Athappilly, 2005). Unlike conventional data mining, which primarily seeks to identify statistical correlations or clusters, logic mining focuses on revealing the logical relationships among variables. This approach is particularly valuable in fields that demand an in-depth understanding of the logical structure of data, such as knowledge representation (Zamri et al., 2024), rule-based systems, and expert systems. Despite extensive research in data mining, traditional methods often fall short of offering comprehensible explanations for their results. This limitation highlights the importance of incorporating symbolic rules into logic mining techniques.

In order to facilitate the application of satisfiability in DHNN models in real-world scenarios, logic mining model was developed. The initial contributions to this field were made by Sathasivam and Abdullah (2011), who introduced a Reverse Analysis (RA) to derive logical rules from datasets by randomly selecting attributes to serve as neurons. This approach aimed to generate an optimal logic representing dataset patterns. However, only limited success has been achieved in identifying the optimal logic for dataset representation, due to the potentially infinite number of induced logics during the retrieval phase. Building upon this foundation, Kho et al. (2020) introduced the 2 Satisfiability Reverse Analysis Method (2SATRA) by combining 2SAT and Reverse Analysis approaches. This model was effectively employed in classifying outcomes in League of Legends (LoL) games, illustrating the potential for a 2SAT logical rule to represent clauses. Through the analysis of induced logic which highlights attribute logical relationships, the study provides insights into optimal game-winning strategies. Following this study, a similar model was explored by Zamri et al. (2020a), who proposed 3 Satisfiability based Reverse Analysis Method (3SATRA). 3SATRA employs systematic logical rule which is formulated based on the empirical patterns observed in the training data. The optimal induced logic represents the overall behavior of the datasets, which will be used to train the network. The selection of this logic is primarily determined by accuracy, with higher accuracy indicating superior logic. Based on 2SATRA, Jamaludin et al. (2021) proposed an Energy 2 Satisfiability Reverse Analysis Method (E2SATRA), which considers only the final neuron states that fall into global minimum solutions. The energy operator guarantees the quality of the final best induced logic. Nevertheless, the conventional sequence of attributes employed in these studies constrains the search space for identifying optimal combinations of attributes. To address this limitation, Jamaludin et al. (2023) proposed Permutation 2 Satisfiability Reverse Analysis Method (P2SATRA) which introduced a permutation operator during the training phase. By examining various attribute combinations, thereby expanding the search space for optimal neuron combinations and enhancing the potential for achieving superior performance in real-world datasets.

While these developments marked significant progress in logic mining approaches, issues concerning the interpretability of induced logic persist. Attribute selection as a crucial step in machine learning has received considerable attention. Various techniques have been employed for the purpose of attribute selection. In the context of logic mining, supervised learning approaches were first introduced, Kasihmuddin et