FLUTTER MODELING AND SIMULATION OF WING SECTION USING BONDGRAPH TECHNIQUE

by

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Nomenclature

A = Area of bar

b = Wing section chord

[C] = Damping Matrix

C(k) = Theodorsen function

C_α = Pitch damping

C_h = Heave damping

d.o.f = Degree of freedom

E = Young Modulus

F = Force

h = Vertical translation or heaving

i = $\sqrt{-1}$

I = Inertia Component

I_α = Moment of Inertia

k = Reduced frequency

[K] = Stiffness Matrix

k_α = Pitch stiffness

k_h = Heave stiffness

L = Lift force centered at mean aerodynamic chord

l = Length

m = Mass

[M] = Mass matrix

ρ = Density of Air

P = Pressure

P_m = Modal Momentum

q = Dynamic pressure

R = Resistor

S' = Non-dimensionalized Laplace variable

S_E = Effort source component

S_F = Flow Source component

t = Time

TF = Transformer Component

Vcr = Critical flutter speed

ω = Frequency in rad/sec

ω_α = Pitch frequency in rad/s

ω_h = heave frequency in rad/s

x_α = Distance between elastic axis and center of gravity in semi-chord

ε = Strain

σ = Stress

μ = Mass ratio

η = Degree of freedom

ABSTRAK

Permodelan dan Simulasi Flutter untuk Kerajang Sayap Menggunakan Kaedah Bondgraph

Tesis ini berkaitan dengan pemodelan flutter untuk kerajang sayap dengan menggunakan kaedah Bondgraph. Flutter adalah ketidakstabilan dinamik suatu struktur yang terdedah kepada aliran udara. Apabila terdedah kepada aliran udara, suatu struktur boleh dan akan bergetar dengan amplitud yang meningkat jika terdapat gangguan daya luar. Getaran tersebut akan berterusan sehingga struktur tersebut rosak atau musnah. Yang menariknya, getaran dengan amplitud yang meningkat hanya akan berlaku apabila suatu struktur terdedah kepada aliran udara melebihi halaju tertentu yang diberi nama halaju kritikal flutter. Pada halaju ini, suatu struktur dikatakan mengalami ketidakstabilan dinamik. Tetapi pada halaju kurang dari halaju kritikal, getaran yang berlaku akan teredam dengan sendirinya. Objektif penyelidikan ini ialah untuk meramalkan sambutan kerajang sayap yang terdedah kepada aliran udara. Sambutan akan diperolehi dengan menggunakan kaedah Bondgraph. Bondgraph digunakan untuk memodelkan sistem flutter untuk dua darjah kebebasan menerusi gambarajah. Gambarajah ini akan dipasangkan dengan penuras daya aerodinamik yang berubah-ubah. Fungsi penuras ini ialah membekalkan daya aerodinamik yang betul bagi kedudukan kerajang sayap semasa simulasi. Simulasi dilakukan berdasarkan pada halaju kurang dari halaju kritikal flutter, halaju kritikal dan halaju melebihi halaju kritiikal. Sebelum semua itu boleh dilakukan, halaju flutter akan diramalkan terlebih dahulu. Selepas itu daya aerodinamik dianggarkan ke domain Laplace untuk digunakan bagi membina penuras aerodinamik. Seterusnya, penuras ini dipasangkan pada Bondgraph dan disimulasikan. Keputusan sambutan yang diperolehi melalui kaedah Bondgraphs menunjukan persamaan dengan teori flutter. Oleh itu kaedah Bondgraph boleh diguna pakai untuk memperolehi sambutan flutter bagi kerajang sayap. Untuk pembangunan seterusnya, teknik ini boleh digunakan untuk memodelkan flutter berbilang darjah kebebasan.

ABSTRACT

Flutter Modeling and Simulation of Wing Section Using Bondgraph Technique

This thesis is about the investigation of the flutter modeling of wing section or airfoil using Bondgraph technique. Flutter is the dynamic instability of a structure exposed to airflow. When exposed to airflow, a structure can and will vibrate with its amplitude increasing when there exist any disturbance that creates an oscillation. The vibration will continue until the structure is damaged or destroyed. Interestingly, the vibration with increasing amplitude is only possible beyond a certain speed called critical flutter speed. Beyond this critical speed, the stability of the structure is said to be dynamically unstable. However, below the critical speed, any vibration that occurred will be damped out eventually. The research goal is to predict the vibration response of an airfoil exposed to airflow. The response is obtained using the Bondgraph modeling technique. Bondgraph is used to model the two degrees of freedom flutter system graphically. This Bondgraph representation is coupled with the unsteady aerodynamic filter. The function of this filter is to supply the correct aerodynamic forces with respect to the position of the airfoil in simulation. The simulation is done based on the speed below the critical flutter speed, at the critical flutter speed and above the critical speed. Before any of that can be done, the flutter speed is anticipated first. Then the aerodynamic forces are transformed into the Laplace domain. The transformed forces are used to build the unsteady aerodynamic filter. This filter is then coupled with the Bondgraph representation before being simulated. The result obtained from Bondgraph technique and validation show good correlation with the theory of flutter. Thus Bondgraph can be used to predict the flutter response of wing section. For further improvement, this technique can be applied to model multi degree of freedom flutter.

Chapter 1

Introduction

1.1 Aeroelastic Flutter

Aeroelasticity has been a problem in aircraft design since the early stage of flying. The problem was largely unknown in the early days because the aircraft at that time were flying at low speed and had rigid structures (Teichman, 1941) However, the problem started to become serious when aircraft speed increases and the wing structures become less rigid. The word “aeroelasticity” is defined as the mutual interaction of aerodynamic forces, inertial forces and elastic forces on a structure. One of the reasons why aircraft structure is not rigidly build is because rigid structures are usually heavier compared to less rigid structure. Heavy aircraft cost more to operate when compared to lighter aircraft. As a result, many aircrafts are prone to experience many aeroelastic phenomena such as buffeting, divergence and flutter. Among those phenomena, flutter is considered the most dangerous of all (Kussner, 1936).

In engineering terms, flutter means a vibration that amplifies. Early studies showed that flutter has nothing to do with the vibration set up by the inertia forces of the aircraft engine (Teichmann, 1941). Flutter is a phenomenon where a structure experiences an aerodynamically induced vibration and can be destructive. Bisplinghoff (1996) defined flutter as the dynamic instability of a structure at a speed called the flutter speed. Many structures such as suspension bridges and aircraft wings that are exposed to airflow have a potential of experiencing flutter. The collapsed of the Tay Bridge in Dundee where a train with 65 people on board plunged into the river beneath was resulted from flutter. Another similar accident happened when Tacoma Narrow Suspension Bridge collapsed due to the same reason. Recently, a United States Air force F-117 Nighthawk Stealth aircraft lost its right wing before crashing towards spectators at an air show as shown in Figure 1.2. Later investigation showed it was also caused by flutter (Farhat, 2001).

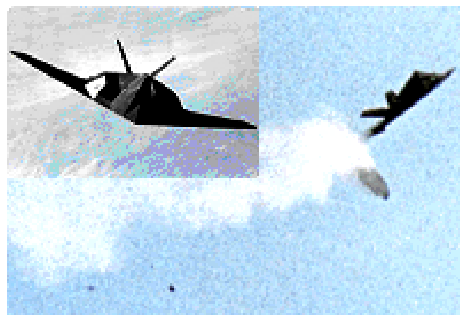

Figure 1.1 F-117 Nighthawk Flutter Accident (Hindman, 2003)

Aeroelastic flutter phenomenon is only possible when a structure is free to rotate about at least two axes or has two degrees of freedom oscillation. The reason is single degree of freedom oscillation will be damped out by the aerodynamic forces (Theodorsen, 1934). On aircraft, flutter usually occurs on the aerodynamic surfaces such as wing, vertical and horizontal tail and or canard wing. Aircraft aerodynamics surfaces are constructed so that they can carry the loads that are produced in flight and they are also exposed to absorb the energy from the airflow. The aerodynamic forces that can induce flutter are related to the dynamic pressure, or the airspeed, of the airplane. If flutter-inducing forces are present they will increase the amplitude as the airspeed increases. Aerodynamic surfaces structure such as the wing can be viewed as a beam connected to a spring extending from the fuselage as shown graphically in Figure 1.2. If one taps the beam with a hammer, it will vibrate at a frequency, which relates to the stiffness of the spring. A spring with high stiffness will vibrate at a higher frequency than a less stiff spring. This vibrating frequency is known as the natural frequency of the system. In theory, flutter will usually occur at or near the natural frequency of a structure (NASA, 1997).

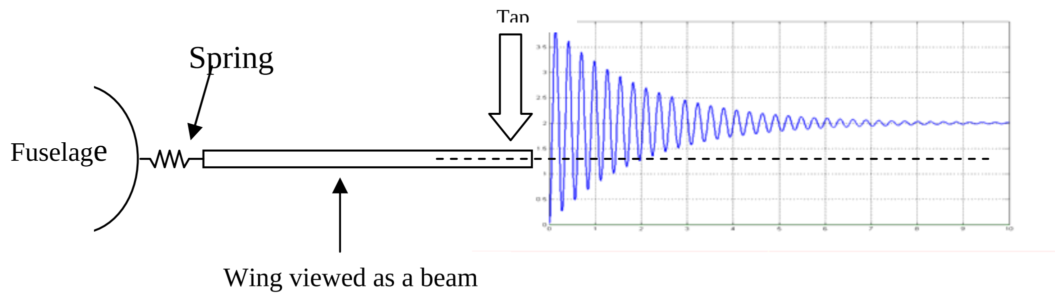

Figure 1.2 Wing viewed as a Blade with response when tapped

Flutter characteristics can be examined by tapping the surface at steadily faster airspeeds, then watching how fast the vibrations damp out. The vibrations will take longer to decay as the airspeed approaches a possible resonant condition. In this way potential flutter can be approached safely without actually experiencing sustained flutter (NASA, 1997). Another important fact about flutter is that it can only occur above the flutter critical speed. Below the critical speed, the vibration of aircraft wing that is subjected to external forces such as gust or sudden maneuver will damp out, as the vibration will give its energy to the airflow. Above the critical speed, the vibration tends to absorb the energy from the air stream and continue vibrating with increasing amplitude (Bisplinghoff *et al.*, 1996). The external disturbances will trigger a structure to oscillate and the oscillation amplitude will increase due to the absorption of energy. No oscillation means flutter will not occur because there is no oscillation and the flow cannot induce its to larger amplitude.

The vibration with increasing amplitude can lead to failure of wing structures through extreme deformation. Furthermore, the mild flutter or flutter with constant amplitude can cause the structure to experience structural fatigue and fail eventually. Due to this fact and the number of aircraft accident caused by flutter, the aviation authorities have decided that all aircraft must undergo aeroelastic flutter analysis or the prediction of flutter speed for safety reasons. This also motivates many researchers to study flutter in order to get a better understanding on the phenomena. Aeroelastic flutter analysis is not precise and it requires flight verification so that flutter will not occur within the operational flight speed.

Although the critical flutter speed is very essential in aeroelastic flutter analysis, many have sought to predict the response of flutter nowadays. Currently the method of obtaining the response of flutter is by using the state space, which involves mathematical modeling. Although this method is capable of predicting flutter response, the process of obtaining a mathematical model and the solution for the flutter system seems difficult and slow. Another possible approach in obtaining the response of flutter will be used in this research. The difference approach is called the Bondgraph technique. The unique feature of Bondgraph is that, it does not involve any Mathematical model from the structural point of view.

1.2 Dynamic Modeling

Mathematical modeling is essential in solving any problems regarding the dynamic system including flutter. A system such as flutter is called dynamics because its present output depends on the past input. If the system is not in equilibrium, the output changes with time. The basic concept of solving a dynamic system starts with the understanding of the physical phenomena. When the physical phenomenon or real system is fully understood, then a physical model can be built to represent the phenomenon. A physical model usually consists of a diagram representing a system model. The system model on the other hand is a simplified physical phenomenon, which reflects not all but some of the real system characteristics. Only important features to the study area are considered while others are left out (Karnopp *et al.*, 1990). The system model should also be competent to support the solution of a specific problem (Breedveld, 2003). The physical model obtained is then used to derive the Mathematical model. Usually, mathematical model for dynamics system is described in terms of differential equations. A solution for the mathematical model can then be obtained analytically or numerically before being interpreted.

In using Bondgraph, mathematical modeling is not required. The dynamics modeling of physical system in Bondgraph is represented as graphical modeling. Graphical modeling is based on the inclusion of physical laws in the process of modeling a physical system such as in Figure 1.3 for a car suspension system.

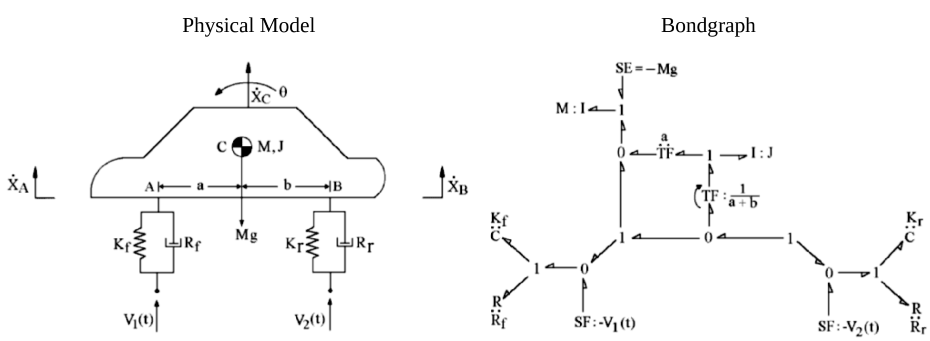

Figure 1.3 Example of Bondgraph for Car Suspension System (Breedveld, 2003)

Furthermore physical modeling helps to keep consistency and prevents the modeler from making certain mistakes, which would be physically meaningless (Stramigioli, 2002). The analysis of the problem such as aeroelastic flutter phenomenon can be carried out using graphical modeling or Bondgraph. The idea is to consider the aeroelastic phenomenon as a dynamic system consisting of subsystems and components that are interconnected and also interact with each other through energy flow mechanism (Pagwiwoko *et al*, 2002). Figure 1.4 shows the concept of the dynamic modeling using conventional and Bondgraph technique.

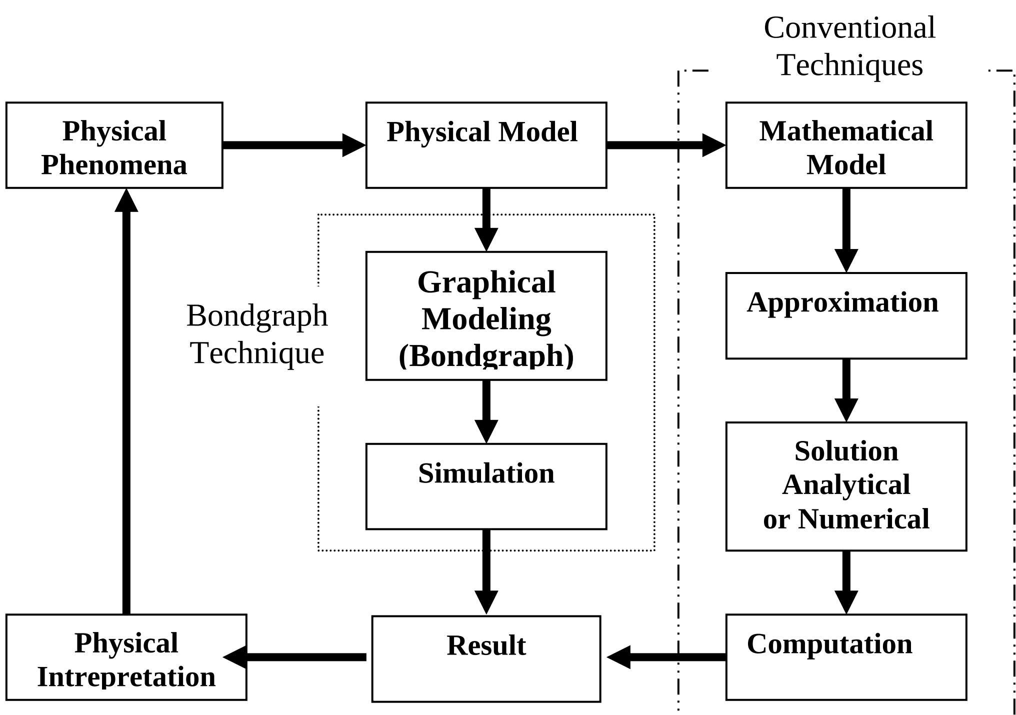

Figure 1.4 Modeling Techniques

Another advantage in using the graphical modeling technique is that it can be connected to block diagram. This feature plays a crucial role, as block diagrams are capable of performing differential and algebraic mathematical operation for simulation in most of the software in the market today (Stramigioli, 2002). In this research, the advantage is used to the full extent when the flutter graphical model is attached to the aerodynamic filter. The transfer function built block diagram is used to supply the forces acting on the Aero-elastic flutter system. In this research, Bondgraph for aeroelastic flutter system is obtained using finite mode representation. In Finite Mode, the graphical model or Bondgraph is quite the same and depends generally on the degree of freedom of a system (Karnopp *et al*,. 1990).

1.3 Research Aim

The focus of this research is to apply the Bondgraph technique on the bending-torsion flutter phenomena of the wing section. Bondgraph Technique reduces the time required to obtain a model of a system and it's solution. Implementation of such a technique will result in significant savings in the time required to predict flutter response in the aeroelastic analysis.

1.4 Research Objectives

The objective of this research is to demonstrate that Bondgraph can be applied to obtain the aeroelastic flutter response of wing section. The main task to achieve the objective is to obtain the graphical model for aeroelastic flutter using Finite Mode Bondgraph first. The filter representing the unsteady aerodynamic forces is then built and coupled with the Bondgraph model. The response is obtained through simulation and validated. Taking the critical flutter speed as reference, the Bondgraph is simulated to prove the obtained response correlates with flutter theory.

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1.5 Thesis Organization

This thesis is organized into 6 chapters. Chapter 2 discussed the literature review of the various methods and techniques use to conduct the aeroelastic flutter analysis. Graphical modeling is also introduced in this chapter. The methods are discussed in general as to give the theoretical insight of each method and how it contributes to the flutter modeling and analysis.

Chapter 3 described the methodology and steps taken to fulfill the objectives of this research. Bondgraph theory is also explained thoroughly in this chapter. This chapter covered the basic of Bondgraph to the Finite Mode Bondgraph, which is used in this research to obtain the aeroelastic flutter response. The Bondgraph construction method is also explained in detail using single degree of freedom vibration system as an example. The modeling and simulation of aeroelastic flutter is discussed in Chapter 4. The modeling is based on the application of finite Mode Bondgraph. The pre simulation and simulation tasks is also explained.

Chapter 5 discussed the result and its validity obtained from this research. The result from each method is presented and discussed. The validity is done by comparing the result obtained from this research with the result from other work done by aeroelastician. Finally the last chapter laid out the summary and conclusion of this thesis. Furthermore, this chapter also touched on the improvement and future work that can be carried out from this research.

Chapter 2

Literature Review

2.1 Introduction

Since the crash of the first aircraft due to the oscillation of wing structure, many researchers have contributed their efforts to better understand the phenomena and try to prevent the accident from occurring again. Their efforts managed to reduce the number of accident caused by the wing oscillation but still flutter keeps occurring. The latest case involved F-117 Stealth Aircraft which is the most sophisticated plane in the U.S Air force inventory (Farhat, 2001). The accident proves that flutter is still a major threat to the aircraft; its pilot and passengers even 100 years after the first invention of aircraft.

2.2 Early Flutter Development

The first flutter study were made by Lanchester, Bairstow and Fage who tried to solve the mystery of oscillating wing on World War I Handley page bomber in 1916. Two years later, Birnbaum managed to tackle the problem of the oscillating wing by introducing the concept of reduced frequency (Kussner, 1935). The aerodynamic forces acting on the oscillating wing are expressed using this non-dimensional parameter (Kussner, 1935). A major breakthrough in flutter calculation came when the non-stationary airfoil theory was developed based on the work of Kutta and Joukowsky. But the researchers at that time still found it difficult to predict the critical speed, because the representation of the unsteady aerodynamic forces was complex and required tedious work to obtain the solution.

2.3 Unsteady Aerodynamic Forces

The problem in the unsteady aerodynamics was solved when Theodore Theodorsen determined the unsteady aerodynamic forces of the oscillating wing. The velocity potentials are developed for the flow around the oscillating airfoil by resolving the solution into certain definite integrals. The theory is based on the Kutta condition and potential flow. The most significant thing is that, Theodorsen managed to present the solution in the simple form. In addition, this solution is expressed by means of non-dimensional parameter of the reduced frequency, k. (Theodorsen, 1934). The unsteady aerodynamic forces of the section model are calculated based on the linearized thin-airfoil theory (Nam, 2001). Theodorsen modeled the wing section as flat plate and assumed the flat airfoil is oscillating about the shear center or elastic axis as in Figure 2.1.

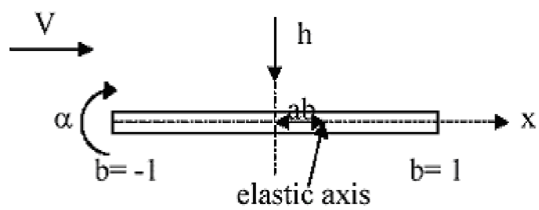

Figure 2.1 Flat Airfoil (Theodorsen, 1934)

The unsteady flow is composed of two flow components, non-circulatory and circulatory flow. The Non-circulatory flow component can be expressed through the sources and sinks. The circulatory flow on the other hand is related to the flat vorticity extending from the trailing edge to the infinity. A bound vortex distribution is employed over the airfoil and a vortex distribution over the airfoil wake Λ to satisfy the Kutta Condition for the Circulatory flow. Kutta condition requires that no infinite velocities exist at the trailing edge of the airfoil. To consider the wake, an assumption is made where a bound vortex ΛΓ is located $1/X_0$ and a shed Vortex, ΔΓ at X_0 such as in figure 2.2.

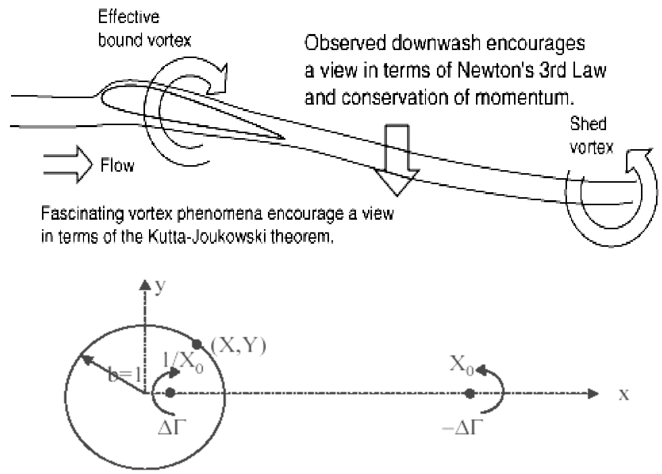

Figure 2.2 Circulatory Flow Vortex (Theodorsen, 1934)

For each flow component, the velocity potential is calculated using Bernoulli theorem (Theodorsen, 1934). For non-circulatory flow, the forces are obtained by performing integration from leading edge to the trailing edge whereas for the Circulatory flow, the integration is perform from leading edge to the infinity. Since its development, this representation of the unsteady aerodynamic forces for two-dimensional wing has been used widely in the study of the oscillating wing section and it is effective for modeling aerodynamic forces for wing section (Thompson *et al.*, 2002). The representation is accurate for low speed, linear incompressible flow (O'Neil, 1998). For compressible flow, a compressibility correction is required to be able to represent the forces correctly (Garrick, 1946). With the development, the prediction of the flutter speed is made to be possible.

2.4 Flutter Speed Prediction

The flutter speed prediction is a process of determining the flutter stability boundary for a structure that is expose in airflow. It can be performed using K-Method, P-K Method or G-Method. The work presented by Theodorsen and Garrick (1934) has opened the opportunity for the solution of the flutter problem. The three methods mentioned are capable of predicting the flutter speed for a wing section. K-method was used by Smilg and Wesserman and is also known as the Air Material Command method (Fung, 1969). In this method the prediction of the flutter speed is made possible by introducing dimensionless coefficients and the artificial structural damping coefficient into the equation of motion. The simplified equation is then solved by obtaining it's eigenvalues.

K-method only requires a straightforward complex eigenvalues analysis to be done for all values of reduced frequency k. This method assumes the artificial damping first (Scanlan *et al.*, 1968). Flutter speed is located at the point where the value of the damping becomes positive. The determinant is obtained by expanding the equation of motion for flutter system and simplify the equation by assuming $\lambda = \frac{1+ig}{\omega^2}$. Because of the straightforward eigenvalue analysis, this method has the advantage of computational efficiency. The eigenvalues for the characteristic equation of motion in equilibrium represent a point on the flutter boundary if the corresponding value of g equals to the assumed value of g. The general solution for the characteristic equation is given by the 2^{nd} order polynomial. By solving the polynomial, the roots will yield result in the form of complex numbers. The two complex roots will represent the two modes, which are heaving and pitching modes. From there, the values of frequencies, ω and damping, g can be computed. These series of value of the frequency and the structural damping for torsion and heaving mode are obtained for all values of the reduced frequency. The frequency and damping are then plotted against the air speed. The curves plotted are known as V-g and V-ω curve. Both P-k and G method will also yield these curves. The significant of V-g curve is that the critical flutter speed is reached when the value of the damping is zero or at g = 0 in V-g curves as shown in Figure 2.3.

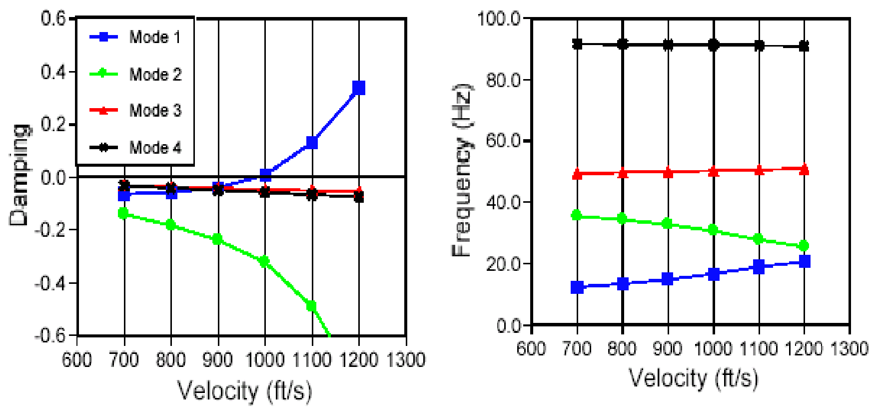

Figure 2.3 V-g and V- ω Curves (ZONA, 2001)

Divergence oscillation will occur when the corresponding value of damping first become positive. Mild or destructive flutter can also be known. From the V-ω curves, the tendency of flutter to occur is shown when both the frequencies start to close in on each other. The K technique is widely used and only abandon if the physical interpretation of the result is questionable (Dowell *et al.*, 1978).

In 1965 Irwin and Guyett presented another method to anticipate flutter speed. This method is called the P-K method. The method is an approximate method to find the decay rate solution (ZONA, 2001). To reduce and simplify the P-K method equation, the structural modal damping effect is excluded identical with the K-Method. But it can easily be included for some cases. The non-dimensional Laplace parameter is expressed as s' = g + ik where γ is the decay rate coefficient which is different from the previous method. Mathematically, this method is inconsistent because the non-dimensional parameter s' is expressed in terms of damped sinusoidal motion. Another reason for inconsistency is the aerodynamic forces are based on the undamped simple Harmonic motion. Rodden (1969) later modified the method when he added an aerodynamic damping matrix into the governing equation (ZONA, 2001).

The added aerodynamic damping matrix is represented by g in the aerodynamic forces term. The equation is solved for complex roots s' at several given values of the velocity and density. The lining up process is done by matching the reduced frequency k to the imaginary part of s' for every structural mode (ZONA, 2001). Such a process requires repeated interpolation of the aerodynamic forces from the range of reduced frequency k. When all the values of damping and frequency are acquired, the V-ω and V-g curves are plotted. The flutter boundary lies at the point where the value of damping equal to zero on the speed axis.

G-Method is a method where the first order damping is derived from Laplace domain unsteady aerodynamic forces. The flutter boundary is provided when the value of damping is equal to zero. The solution for this method begins by substituting p =g + ik into the governing equation. This will resulted a second order linear system equation in term of damping (ZONA, 2001). The solution only exists when the imaginary value for damping is equal to zero. This condition can be acquired by rewriting the 2^{nd} order equation into the form of state space. Then, a technique of reduced frequency sweeps is introduced. This technique seeks the condition where the damping is zero by solving the eigenvalues. The sweeping starts from zero reduced frequency of the unsteady aerodynamic forces with an increment value defined by the user and stop at its maximum value. The frequency and damping is then obtained. Then the V-ω and V-g curves can be plotted. The flutter condition occurs where the value of g equal to zero on the x-axis.

Although the three methods discussed above used different approach to obtain the plotted values of V-g and V-ω curves, they share the same goal. The goal is to locate the point where the damping value equal to zero. K-Method uses artificial damping to indicate the required damping for the harmonic motion. The damping values do not represent any physical meaning except when the damping value lies at the flutter boundary (Scanlan, *et al.,* 1968). In terms of computational time, G-method is the last option to choose from. This is followed by P-K method and K-Method provides the quickest solution (Nam, 2001). In addition, the solution technique for K-method is efficient and robust when compared with other techniques (ZONA, 2001). The disadvantage in P-K method is that it produces a discontinuity for bending mode in the damping curve. It is resulted from the aerodynamic lag root because the lining up process skips the bending mode during computation. Furthermore, the aerodynamic damping is not valid at high value of reduced frequency. At high aerodynamic forces, this method is known to produce unrealistic roots (ZONA, 2001). The discontinuity in damping curves does not occur in G-Method because the eigenvalue tracking is done by applying the Predictor-Corrector Scheme. If the eigenvalue changes sharply and creates discontinuity, the scheme will be activated to compute the damping value by reducing the size of increment of the reduce frequency by a factor. In contrast, Both P-K and G-Method provides smooth curves for torsion mode. K-Method disadvantage lies in the form of difficulty tracing the eigenvalue from the reduced frequency list when the curves loop around themselves for certain system and produce abnormal curve (Looye, 1998). Ironically, all methods discussed above do agree on one aspect, which is the flutter boundary. The reason is when the value of damping is equal to zero; the flutter equation in all the methods is reduced to the same form. This is why the critical flutter speed from the three methods is always in good agreement (ZONA, 2001).

2.5 Rational Aerodynamic Approximation

Rational aerodynamic approximation is required in order to be able to cast the dynamic aeroelastic equation into the Laplace domain or state space form. The approximation is developed base on the original aerodynamic data. The goal is to approximate the frequency domain of unsteady aerodynamic forces in terms of the rational function of the Laplace variable. When the rational function is applied the number of states increases due to the number of augmented aerodynamic states required to represent the unsteady aerodynamic forces neatly. Currently there are three rational aerodynamic approximation methods which are the Minimum State method, the Roger’s method and the Modified Pade’ method. The methods to be discussed are the Minimum State method and Roger’s method. Modified

Pade's method is not going to be discussed because it is similar with the Roger's method. Generally, all these methods approximate the forces with some errors.

Karpel (1981) developed an approach to approximate the aerodynamic forces into the Laplace variable in 1981. The Minimum State method approximates the unsteady aerodynamic forces by using a specific equation consist of matrix coefficients with transfer function. The matrix coefficients are real numbers and determine using least squares fit. The number of the augmented states is equal to the order of the aerodynamic root matrix. The aerodynamic data in the harmonic oscillations must be known before the approximation can be applied (ZONA, 2001). When compared to Roger's method, this method is known to produce less augmented states (Nam, 2001). The process of approximation starts with the replacement of reduced frequency by s. The unknowns in the equation above are solved by means of iteration. The iteration starts with an initial guess of the first unknown term. In term of computing times, this technique takes more time than the later technique. The guessing starts with one term in each row and each column. For any given first unknown value, the other matrix coefficients can be calculated using column-by-column least square solutions. The value of the second unknown term calculated is used to update the matrix coefficients by performing, this time, row-by-row least square fit. The least square fitting sequence is repeated until the specified number of iteration is acquired (ZONA, 2001).

Another approach is the Roger's Rational Approximation Function or also known as the Non-Critical Pole function. It was developed by Roger in 1977. He approximated the unsteady aerodynamic forces using a certain equation that consist of transfer functions with lagging terms and matrix coefficients. The equations are given in chapter 4.4. The non-dimensionlized Laplace variable is represented by s'. Where it is equal to the reduced frequency. The aerodynamic poles or aerodynamic lagging terms are pre-selected in the range of the reduced frequency of interest. This method also uses Least Square fitting technique to fit the unsteady aerodynamic data neatly (Nam, 2001). The complex form of the aerodynamic forces in terms of reduced frequency is separated into real and imaginary parts. Roger's method also caused the number of the augmented states to increase. The states increment and the level of accuracy depend on the number of lagging terms selected (Chen *et al.,* 2000). Usually the number of lagging terms used for good fitting is 4. The Modified Pade` method is closely similar to Roger's Method. It also uses the Least Square Fitting procedure.

Roger's method is simple and neatly transform the unsteady aerodynamic forces from the frequency domain to time domain. The penalty is in the form of the increase in the augmented states which is required in order to describe the states of fluids and to fit the data nicely (Pagwiwoko *et al.,* 2002). Furthermore, the lagging terms are selected arbitrarily and this can cause a small different in curve fitting result if using different lag term values. In Minimum State, the approximation does not produce an exact fit with the unsteady aerodynamic forces at certain reduced frequencies value. This lead to the introduction of constraints to be used in order to fit the data exactly. Up to three constraints can be used with the reduction technique, which resulted in the reduction of computing time (ZONA, 2001). These constraints are not required in the approximation function but often used to obtain good results (Karpel *et al*., 1995). Apart from that, this method sometimes requires nonlinear optimization to obtain the solution for the two unknown matrices. This explains why Roger's method is preferable. Furthermore, least squares fits tend to produce smaller percentage of errors at data points of large numerical value (ZONA, 2001).

2.6 Flutter Response

The determination of flutter boundary can provide valuable information about the stability condition within the airspeed range. As the aircraft becomes more complex and design requirement increases, flutter prediction is not enough. Now, flutter analysis result should include the time history deflection or response. Flutter response is capable of revealing many flutter characteristics such as non-linear effects, limit cycle oscillation, catastrophic or benign flutter as well as the condition which undamped oscillation might appear at velocities below critical speed (Marzocca *et al.*, 2001). In addition, flutter response also enables flutter suppression using control surfaces to be carried out as the flutter behavior can be predicted (Newson, 2002). Flutter response can be obtained via state space or the functional series technique. Functional series or also known as Volterra series can be used to identify non-linear behavior in aeroelastic systems. The response for an arbitrary input can be constructed by integrating the nonlinear function or convoluted for linear system. In addition, the non-linear effect such as limit cycle oscillation can only be shown through response and it is not possible to predict the LCO using a purely linear analysis (Sedaghat *et al.,* 2000). For two degrees of freedom flutter, the response will be in two modes, plunging or heaving and pitching. Heave mode response shows the wing section's translational motion with time while the pitch mode response shows the wing section's rotational motion with respect to the elastic axis.

2.6.1 State space

The aeroelastic flutter response of the wing section can be obtained using state space. This method uses the linearized aerodynamic forces developed by Theodorsen to represent the unsteady aerodynamic forces. State space required that the frequency domain aerodynamic forces be converted into the time domain. To use this technique, one must know at what speed to simulate. Therefore, critical flutter speed must be known first. In order to construct the state space model, Fourier transform can be used because the aerodynamic forces are said to be the Fourier transform of the transient load (Olds, 1997). Therefore, the inversed Fourier transform of the forces are the transient loads. The inversed Fourier of transform for Theodorsen Function C(k) are obtained using the convolution Theorem.

When applied, the inversed Fourier transform will yield the Duhamel Integrals D(t) in the lift and moment equations. The integrals in the lift equation are evaluated using the Wagner Function or a numerical approximation (Olds, 1997). State space model can be built when the aerodynamic forces is transformed into the time domain. Then the state space model can be simulated numerically using the integration techniques in order to obtain the flutter response (Bae *et al.*, 2002). The process of obtaining the mathematical formulation using state space is very rigorous and tedious. In addition, state space required the second order system to be reduced to the first order system before simulation can be performed. Furthermore, for the non-linear aeroelastic analysis the complication also arises in the reduction technique (Sedaghat *et al.*, 2000).

2.6.2 Bondgraph

Prof H. M. Paynter gave the revolutionary idea of portraying a dynamic system in terms of power bonds (Breedveld, 2003). These bonds connect the elements of the physical system to the so-called junction structures, which depends on the constraints such as the boundary condition of a system. The power bond technique is a precise tool for capturing the common energy transfer within a system. When compared to mathematical representation, the pictorial representation of a dynamic system increases the understanding of a system behavior. In the graphical form, it produces a clear description of complex systems (Karnopp *et al.*, 1990).

In Bondgraph, a physical system such as flutter can be represented by symbols and lines in order to identify the flow of energy. The lumped parameter elements of Bondgraph such as resistance, capacitance and inertance are interconnected. These elements can be used to represent the component in the physical domain such as spring, mass and inertia effects that are known to exist in most of the dynamic system such as flutter replacing the need for equation of motion. By following a certain rules and procedures, the graphical model can be created to represent the dynamic systems thus preventing mistakes from occuring (Breedveld, 2003).

The graphical representation from Bondgraph can be easily transferred into the computer and simulation can be performed using the existing software such as MATLAB-SIMULINK, ENPORT or CAMP-G without the need to define the state equation to acquired the structural dynamic response. The time step types and size chosen in the simulation must also be properly selected. Fixed or variable time step can be used depending on the computing time available and accuracy required (Karnopp *et al.*, 1990). In this research, Bondgraph for aeroelastic flutter system is obtained using finite mode representation. The concept of Finite mode Bondgraph is similar to the concept of the distributed parameter system representation. Distributed parameter systems are systems, which are represented by partial differential equations. In theory, engineering systems are built from components that behave like inertia, compliance, resistance, etc. Such engineering dynamic system such as flutter can be modeled as a distributed system in order to make it simple to analyze them (Karnopp *et al.*, 1990). Finite mode created a graphical model where many different dynamic systems can be modeled using relatively the same model which really saves time and complication which existed if mathematical modeling is used. The model can be modified and the modification depends on the degree of freedom and the boundary conditions. A minor setback in this technique is the need to obtain the mode shape and modal parameters of the system analytically (Karnopp *et al.*, 1990).

The true appreciation of Bondgraph comes when modeling non-linear systems. The step where the state equation for non-linear system is so rigorous to obtain can be avoided because the non-linearity can be represented using the mathematical block diagram and connected to Bondgraph model. Any forces or effects that cannot be modeled using Bondgraph components can also be represented using mathematical block diagram. This capability has its limitation because not all Bondgraph softwares in the market can perform this task. Any software selected must be carefully checked to have this capability before any model that contains mathematical block diagram can be simulated (Karnopp *et al.*, 1990).

2.7 Experimental Work

There are many experimental works that has been done to study the flutter phenomena. The reason is quite simple; better understanding the behavior of flutter and confirm the finding obtained numerically. Some of the early experimental works done were to determine the aerodynamic forces acting on the oscillating airfoil and compare the measured data with data predicted. Such work was undertaken by Halfman (1951) who concluded that the experimental data measured in his experiment supported the predictions of Theodorsen's theory over a range of reduced frequency k. In the recent years, many works are being done to study the effects of non-linearity in flutter. Such work has been done by Sedaghat, Cooper, Dowell, Thompson and O'Neil to name a few. O'Neil (1998) conducted experiments to investigate the non-linearity found in structural systems that exhibit the effect of spring hardening and softening. The research done by O'Neil (1998) interests this research most because of the linear and non-linear analysis works involved similar with this research. The findings of that research in terms of flutter response predicted are used in this research for validation purposes.

The predicted responses are obtained from analytical model using the non-linear solution package for non-linear case. For experimental part, the test apparatus developed permits linear and nonlinear pitch as well as plunge motion where a pair of cams governs the non-linearity effect. These cams represent the non-linearity effect in the spring. Physical parameters can be easily modified. By using the parameters, the response is first predicted analytically. The response measured is used to validate the predictions. The apparatus for linear and non-linear are executed at the speed 15 m/s, which lies above the flutter speed.

O'Neil (1998) concluded that the predicted analytical result is consistent with the finding from (Lee *et al*., 1986) and (Woolston *et al*., 1957) in term of stable and unstable responses and also their Limit Cycle Oscillation (LCO). The result from the experimental work also shows good agreement with the analytical result. The different between measurements and predictions are most likely caused by the unmodelled non-linearities in damping (O'Neil *et al.*, 1998).