G-TYPE RANDOM 3 SATISFIABILITY WITH DISCRETE HOPFIELD NEURAL NETWORK

GAO YUAN

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GAO YUAN

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LIST OF SYMBOLS

$P_{GRAN3SAT}$	General formula GRAN3SAT
m_i	Number of the third-order logic clauses
n_i	Number of the second-order logic clauses
k_i	Number of the first-order logic clauses
1	TRUE
-1	FALSE
A_i	Literal
U	Collection of clause numbers
Nc_i	Number of k order clauses
$C_{m_j}^{(3)}$	Third-order logic clause
$C_{n_j}^{(2)}$	Second-order logic clause
$C_{k_j}^{(1)}$	First-order logic clause
S_i	State of neuron i
W_{ij}	Synaptic weight between i and j
W_{ijk}	Synaptic weight between three neurons
$E_{P_{GRAN3SAT}}$	Cost function of the DHNN-GRAN3SAT
θ	Probability value
S_{f_i}	Update neuron state
h_i	Local field
$L_{P_{GRAN3SAT}}$	Lyapunov energy function
$L_{P_{GRAN3SAT}}^{min}$	Minimum energy of the DHNN-GRAN3SAT
$f_z^{\min}$	Number of unsatisfied Clauses

$f_z^{\max}$	Number of satisfied Clauses
T^{+1}	pheromone density of path '+1'
T^{-1}	pheromone density of path '-1'
E^{+1}	Visibility Density of Path '+1'
E^{-1}	Visibility Density of the Path '-1'
P^{+1}	State Transition Probability of Path '+1'
P^{-1}	State Transition Probability of the Path '-1'
$S_{f_i}^t$	Mutated State of Neurons
$C(x, y)$	Set Denoting the Selection of y ($1 \leq y \leq x$) Elements from x, where x Represents the Number of Literals in Each Clause
TP	Number of Samples Correctly Predicted as Positive by the Model
FP	Number of Samples Incorrectly Predicted as Positive by the Model
FN	Number of Samples Incorrectly Predicted as Negative by the Model
TN	Number of Samples Correctly Predicted as Negative by the Model
At_i	i-th Randomly Sampled Attribute
$P^{GRAN3SAT}$	Potential logic set generated based on the GRAN3SAT logical rules
NN	Number of neurons
NC	Number of clauses
$N_{combmax}$	Number of neuron combination
N_{trial}	Number of learning trial
N_l	Current number of learning
NaN	Value is empty
δ_i	Threshold
r	Relaxation rate
N_t	Number of testing trial

Tol	Tolerance value
f_{NC}	Maximum fitness achieved
f_i	Current fitness achieved
W_{WA}	Synaptic weight obtained by Wa method
W_i	Current Synaptic weight
N_w	Number of weights at a time
N_{w_c}	$N_{w_c} = N_w \cdot N_{combmax}$
L_{min}	Minimum energy value
L_f	Final energy value
N_{GLO}	Number of global minimum solutions
N_{LOC}	Number of local minimum solutions
N_{t_c}	$N_{t_c} = N_t \cdot N_{combmax}$
S_i^{ideal}	Ideal neuron state
l	Number of $\{S_i^{\text{ideal}} = 1, S_i = 1\}$
m	Number of $\{S_i^{\text{ideal}} = 1, S_i = -1\}$
n	Number of $\{S_i^{\text{ideal}} = -1, S_i = 1\}$
N_{NC}	Current number of clauses
N_{mean_c}	Average number of clauses
$N_{negative}$	Current number of negative literals
N_{mean_p}	Average number of negative literals
$rand(m_i, n_i, k_i)$	Number of clauses generated randomly
n_i	Number of Iterations Per Algorithm
N_m	Average Number of Iterations
N_{nc}	Number of Neuron Combinations
T_m	Average Execution Time

T_i^{end}	System Time When the Algorithm Ends
T_i^{begin}	System Time When the Algorithm Starts
Nf_i	Number of Function Calls Per Algorithm
Nf_m	Average Number of Algorithm Function Calls
Sg_i	Generated Global Solution
Sg_{i+1}	Current Global Solution
N	Number of Samples Per Group
K	Number of Metaheuristic Algorithms
R_j	Average Rank of the Algorithm
SSA	Sum of Squares Between Groups
SSE	Sum of Squares Within Groups
$\overline{y}$	Average of All Data
$\overline{y_{i.}}$	Average of Samples of Each Algorithm
y_{ij}	Individual Data

LIST OF ABBREVIATIONS

2SAT	2 Satisfiability
2SATRA	2 Satisfiability Reverse Analysis
3SAT	3 Satisfiability
3SATRA	3 satisfiability Reverse Analysis
A2SATRA	Advanced Log-Linear Analysis in 2 Satisfiability Reverse Analysis
ABC	Artificial Bee Colony
ACC	Accuracy
ACO	Ant Colony Optimization
AI	Artificial Intelligence
ANNs	Artificial Neural Networks
AVG	Average
BACO	Binary Ant Colony Optimization
BOSACO	Binary Optimal Solution Ant Colony Optimization
CAM	Content Addressable Memory
CNF	Conjunctive Normal Form
CS	Cuckoo Search
CSA	Clonal Selection Algorithm
DE	Differential Evolution
DHNN	Discrete Hopfield Neural Network
DHNN-2SAT	2 Satisfiability in DHNN
DHNN-GRAN3SAT	GRAN3SAT in DHNN
DHNN-RAN3SAT	RAN3SAT in DHNN

DHNN-SAT	Satisfiability in DHNN
EA	Election Algorithm
EDA	Estimation of Distribution Algorithm
E*k*SATRA	Energy based *k* Satisfiability Reverse Analysis
ES	Exhaustive Search
F1	F1 Score
FGRA	Flexible based G-Type Random 3-Satisfiability Reverse Analysis
G3SATRA	Multi-unit 3-satisfiability-based Reverse Analysis
GA	Genetic Algorithm
GRAN3SAT	G-Type Random 3 Satisfiability
HNN	Hopfield Neural Network
HornSAT	Horn Satisfiability
HTAF	Hyperbolic Tangent Activation Function
IO	Initial Value Optimization
L2SATRA	Log-linear Analysis 2 Satisfiability Reverse Analysis
MAE	Mean Absolute Error
MAJ2SAT	Majority 2 Satisfiability
MAJ3SAT	Majority 3 Satisfiability
MAX	Maximum
MCC	Matthew's Correlation Coefficient
MED	Median
MHNN	Mutated Hopfield Neural Network
MIN	Minimum
MSEM	Multi-attribute Selection Ensemble Method

P2SATRA	Permutation in 2 Satisfiability Based Reverse Analysis
PMV	Proportion of Missing Values
PRE	Precision
PSO	Particle Swarm Optimization
r2SAT	Weighted Random 2 Satisfiability
RA	Reverse Analysis
RAG	Range
RAN2SAT	Random 2 Satisfiability
RAN3SAT	Random 3 Satisfiability
RMSE	Root Mean Square Error
RMSE	Root Mean Square Error
RPSM	Randomized Pairwise-Single Mutation
RWS	Roulette Wheel Selection
S2SATRA	Supervised 2 Satisfiability Reverse Analysis
SAT	Satisfiability
SEN	Sensitivity
SHoRA	Supervised Higher order Reverse Analysis
STD	Standard Deviation
TNV	Total Number of Variations
UR	Updated Rule Enhancement
WA method	Wan Abdullah method

LIST OF APPENDICES

3-SATISFIABILITI RAWAK JENIS-G DALAM RANGKAIAN NEURAL HOPFIELD DISKRET

ABSTRAK

Hubungan logik adalah penting dalam kecerdasan buatan, terutamanya dalam penstrukturan hubungan data, sebagai hubungan logik menyediakan rangka kerja yang berstruktur dan formal. Walau bagaimanapun, model sedia ada sering mengabaikan potensi untuk perwakilan fleksibel dalam peraturan logik satisfiabiliti. Untuk menangani perkara ini, tesis ini mencadangkan peraturan logik yang baharu dipanggil 3 Satisfiabiliti Rawak Jenis-G. Peraturan ini menggabungkan logik perintah tinggi, sistematik, dan bukan sistematik untuk mewujudkan rangka kerja logik dengan fleksibiliti yang tinggi. Apabila dimasukkan ke dalam rangkaian neural Hopfield Diskret, peraturan logik ini nyata sekali meningkatkan ketersesuaian rangkaian dalam memori sekutuan dan pemprosesan tugas logik yang kompleks. Bagi meningkatkan kecekapan latihan, algoritma Pengoptimuman Koloni Semut Penyelesaian Optimum Binari yang baharu diperkenalkan sebagai mekanisme pembelajaran. Dengan pembaikan proses pengawalan dan peraturan kemas kini, algoritma ini nyata sekali mencapai kelajuan penumpuan yang pantas. Untuk fasa perolehan semula, strategi mutasi baharu dipanggil Mutasi Rawak Sepasang-Tunggal dicadangkan, yang meningkatkan kepelbagaian penyelesaian global dan mengoptimumkan kualiti hasil perolehan semula lebih lanjut. Keputusan eksperimen menunjukkan bahawa algoritma pengoptimuman yang dicadangkan mengatasi kaedah sedia ada dalam kedua-dua fasa latihan dan perolehan semula.Akhir sekali, tesis ini memperkenalkan model perlombongan logik berdasarkan analisis berbalik berasaskan 3-Satisfiabiliti Rawak Jenis-G fleksibel, yang menggabungkan Kaedah Ensembel Pemilihan Berbilang-Atribut. Kaedah ini menyesuaikan bilangan atribut dan strategi pemilihan secara adaptif berdasarkan set data yang berbeza. Keputusan eksperimen menunjukkan bahawa model ini bukan sahaja menunjukkan prestasi yang baik dari segi ketepatan, tetapi lebih penting lagi, ia menunjukkan daya adaptasi rentas domain yang kuat dan boleh diaplikasikan secara meluas kepada pelbagai tugas perlombongan data seperti pengelasan dan perlombongan peraturan asosiasi.

G-TYPE RANDOM 3 SATISFIABILITY WITH DISCRETE HOPFIELD NEURAL NETWORK

ABSTRACT

Logical relationships are crucial in artificial intelligence, especially in structuring data relations, as logical relationships provide a structured and formal framework. However, existing models often overlook the potential for flexible representations in satisfiability logical rules. To address this, this thesis proposes a novel logical rule called G-type Random 3 Satisfiability. This rule combines high-order, systematic, and non systematic logic to create a logical framework with high randomness and flexibility. When embedded into Discrete Hopfield neural networks, this logical rule significantly enhances the adaptability of the network in associative memory and complex logical task processing. To improve training efficiency, a novel Binary Optimal Solution Ant Colony Optimization algorithm is introduced as the learning mechanism. By refining the initialization process and update rules, the algorithm achieves a significantly faster convergence speed. For the retrieval phase, a new mutation strategy called Randomized Pairwise-Single Mutation is proposed, which enhances global solution diversity and further optimizes the quality of retrieval results. Experimental results demonstrate that the proposed optimization algorithms outperform existing methods in both the training and retrieval phases. Lastly, the thesis introduces a Flexible base G-type Random 3-Satisfiability reverse analysis logic mining model, which incorporates a Multi-attribute Selection Ensemble Method. This method adaptively adjusts the number of attributes and selection strategies based on the different datasets. The experimental results show that the model not only performs well in accuracy, but more importantly, demonstrates strong cross domain adaptability and can be widely applied to various data mining tasks such as classification and logical rule mining.

CHAPTER 1

INTRODUCTION

1.1 Overview

Artificial Intelligence (AI) models are gaining significant attention for their ability to simulate human intelligence using machine learning. A major breakthrough in AI is the development of Artificial Neural Networks (ANNs), which processes complex data and enables intelligent decision-making. This chapter provides a brief overview of the fundamental concepts of ANNs, forming the foundation for the research in this thesis. This chapter then examines the challenges faced by Discrete Hopfield Neural Network (DHNN) and explores proposed methods to address them. Through detailed analysis, the chapter identifies key challenges in building optimal AI systems, leading to the formulation of problem statements. The research background, problem statements, research questions, research objectives, methodology, and limitations are outlined, followed by an overview of the thesis organization.

1.2 Research Background

In the contemporary technology-driven landscape, AI has emerged as a transformative force, with applications that span a wide range of fields. Among these, ANNs serves as a core technology within AI, playing a key role. ANNs is a computational model that mimics the structure and function of biological neural networks, consisting of many interconnected neural (Hopfield, 1982). Its core principle is to adjust the connection weights between neurons, allowing the network to learn from the input data and perform tasks intelligently (Rusdi et al., 2023). The operation of ANNs involves two main processes: forward propagation and backward propagation. In forward propagation, input data passes through multiple layers of neurons, with activations processed layer by layer until an output is produced. In backward propagation, the network updates connection weights based on the error between the actual output and the expected result, optimizing its performance and accuracy (Rumelhart et al., 1986). ANNs demonstrate broad application potential across various domains. For example, in image recognition and speech recognition, neural network-based deep learning methods have achieved significant advances (LeCun et al., 2015). Furthermore, ANNs have shown strong potential in fields such as natural language processing, medical diagnostics, and financial forecasting (Zamri et al., 2020). However, ANNs also faces several challenges and limitations, one of which is the issue of convergence speed. Convergence speed refers to the time required for network parameters to reach optimal values during training. Due to the typically large number of parameters in neural networks, especially in neural networks, traditional training methods can result in prolonged training times and slower convergence (Abdullah, 1992). This issue significantly affects the efficiency and real-time applicability of neural networks, especially in scenarios requiring rapid responses, presenting a critical limitation that must be addressed. An emerging field, neuro-symbolic methods, aims to accelerate neural network convergence and address the issue of explainability within AI.

1.3 Symbolic Rule in ANN

Current symbolic systems cover both systematic and non-systematic logical rules(Alway et al., 2022), but lack hybrid logical rules, which limits logical expression capabilities when processing diverse datasets. To address this challenge, this thesis proposes the following research directions:

Extending AI symbolism.

Overcoming these expression limitations has become a key driver of ongoing research and innovation. Consequently, researchers have proposed various strategies and recommendations, such as improving network structures (Karim et al., 2021), enhancing the efficiency of training algorithms (Kasihmuddin et al., 2017a), and exploring new learning methods and optimization algorithms (Kasihmuddin et al., 2019). These approaches are expected to further advance the development of ANNs in practical applications and broaden the range of uses of ANNs. In conclusion, as a key technology in the field of AI, ANNs exhibits tremendous potential and wide application prospects. Despite certain limitations, continued research and innovation—particularly those aimed at Extending AI symbolism—hold the promise of overcoming these challenges and further advancing the development and application of ANNs technologies.

1.4 Existing Challenges in Discrete Hopfield Neural Network

In the previous section, the fundamental principles of ANNs and their widespread applications in various fields were explored. However, as artificial intelligence technology advances, certain ANNs models (e.g., DHNN) demonstrate strong capabilities in pattern storage and retrieval. Nevertheless, these models also face significant challenges and limitations (Hopfield, 1982). The DHNN, a classical model of ANNs, is designed based on inspiration from the biological nervous system. The network consists of interconnected discrete neurons linked through weighted connections, forming learning model that facilitate combinatorial optimization processing (Amit et al., 1985). Nevertheless, traditional DHNN models suffer from issues such as lack of interpretability, slow convergence speed, and sensitivity to input data quality, which limit their effectiveness in practical applications. First, the internal workings of DHNN are complex and lack interpretability, making DHNN challenging to apply in critical areas where interpretability and comprehensibility are essential, such as medical diagnosis and financial decision-making (Zamri et al., 2020). To enhance interpretability, symbolic logic modeling can be employed as an effective tool. Using symbolic logic to structure the relationships among DHNN neurons, the internal operations of the network can be made more understandable and interpretable. For example, higher-order logic can be used to represent the structure of DHNN neurons, establishing more interpretable relationships between input and output data (Karim et al., 2021). Second, the convergence issue of DHNN adversely impacts performance in applications. The introduction of symbolic logic not only enhances interpretability, but also helps in understanding how the connection weights between neurons influence the convergence of the network (Abdullah, 1992). Traditional DHNN exhibits low learning efficiency, especially when dealing with large-scale networks. Therefore, researchers have proposed the use of metaheuristic algorithms to optimize the learning process of DHNN. These algorithms, such as genetic algorithms and ant colony optimization, can improve convergence and learning efficiency by exploring the search space more effectively (Kho et al., 2022). Moreover, the sensitivity of DHNN to the quality of input data further restricts the application range of DHNN. Given the fully connected structure of DHNN, where each neuron is linked to every other neuron, the network is highly susceptible to noise and errors in the input data. To address this challenge, data preprocessing and feature engineering techniques can be employed to enhance the quality of training data, thereby improving the robustness of DHNN in practical applications (Manoharam et al., 2023). Through the analysis of these challenges, a broader **philosophical perspective** is proposed:

Creating a flexible symbolic rule.

Introducing new combinations within the existing neurons can contribute to improving the performance and algorithmic efficiency of DHNN (Kasihmuddin et al., 2019). In summary, a thorough analysis of the existing challenges in DHNN and the proposal of improvement strategies will help advance the development of DHNN in combinatorial optimization and other application domains, thereby providing strong support for the progress and application of artificial intelligence technology.

1.5 Problem Statements

The current DHNN faces limitations in performance and adaptability, making DHNN difficult to efficiently handle complex demands in diverse scenarios. Key challenges include constructing flexible logical rules to optimize the training and retrieval processes of DHNN, enhancing the learning efficiency and global solution quality of DHNN, and improving logic mining models for better classification in real-world data. Thus, the big problem is: How can flexible logical rules be created to improve the performance and adaptability of DHNN?

Problem Statement 1: Challenges in Existing Logical Rule Structures and the Need for Flexibility

Explanation of Problem Statement 1: Logical relationships are essential in AI, especially for decision-making, as they provide a structured framework for reasoning and making informed choices. By applying logical rules, AI systems can effectively manage complex situations, evaluate outcomes, and make data-driven decisions. Initially, Kasihmuddin et al. (2017a) and Mansor et al. (2017) studied systematic 2 Satisfiability (2SAT) and 3 Satisfiability (3SAT) logical structures in DHNN. These structures improved the ability of network to find global minima. However, the fixed number of literals in 2SAT and 3SAT clauses limited their expressiveness, restricting the capacity of Content Addressable Memory (CAM) to adapt to complex systems. Inspired by systematic logic, Sathasivam et al. (2020b) introduced non-systematic logical structures by introducing Random 2 Satisfiability (RAN2SAT) into DHNN, which randomly combines first-order and second-order clauses. However, the absence of higher-order clauses limited the ability of RAN2SAT to handle complex logical relationships, particularly those involving intricate variable dependencies, affecting training and retrieval performance. To address this, Karim et al. (2021) proposed Random 3 Satisfiability (RAN3SAT), introducing first-order, second-order, and third-order clauses to improve logical expression. Despite advancements of RAN3SAT, RAN3SAT has limitations. The ratios of different clause types and positive/negative literals are constrained, limiting flexibility in logical rule construction. Additionally, RAN3SAT cannot express systematic logic, leading to suboptimal performance in tasks requiring systematic reasoning. Furthermore, Zamri et al. (2022) introduced Weighted Random 2 Satisfiability (r2SAT) logical rules, which defined the proportion of positive and negative literals in logical structures to improve performance. Similarly, Alway et al. (2022) and Roslan et al. (2023) proposed Majority 2 Satisfiability (MAJ2SAT) and Majority 3 Satisfiability (MAJ3SAT) logical rules by constraining the number of different clause types in logical structures. While these models enhanced the expressiveness of logical formulas through additional constraints, their improvements were limited to specific parameter adjustments. They failed to fully address the flexibility and adaptability of logical rules, making them less effective for complex logical relationships or broader applications. Thus, a more flexible logical rule is needed, introducing higher-order random logic with both systematic and non-systematic logic while allowing flexible constraints on the ratios of clause types and positive or negative literals. This approach aims to maintain randomness and flexibility ensuring that the logical rules when introduced into DHNN form highly adaptive and robust solutions suitable for complex systems.

Problem Statement 2: Challenges in Logical Structures of DHNN and the Need for Flexible Integration

Explanation of Problem Statement 1: DHNN is a fully connected binary neural network with symmetric synaptic weights. The model iteratively updates the state of neurons and can converge to a stable state through the energy formula. To improve the interpretability of the model, Abdullah (1992) proposed introducing logic programming into neural networks. By adding complex logical structures, establish connections between neurons. Building on this, researchers have incorporated more advanced logical structures into DHNN, such as satisfiability logical rules like Horn Satisfiability (HornSAT), to handle complex logical constraints (Kasihmuddin et al., 2019). These hybrid systems are capable of mining data relationships, thereby expanding the application of DHNN in fields such as classification and artificial intelligence. Most existing Satisfiability in DHNN (DHNN-SAT) models (Sathasivam et al., 2020b) use the Wan Abdullah method (WA method) introduced by Abdullah (1992). This method minimizes the cost function by satisfying Satisfiability (SAT), ensuring optimal synaptic weights, effective CAM, and network convergence. Karim et al. (2021) effectively improved the quality and diversity of retrieval results by combining high-order random logical rules with DHNN, but experimental results showed that there is still room for optimization in terms of diversity performance Thus, a key research challenge is introducing flexible logical rules into DHNN while ensuring compatibility with training and retrieval phases. Addressing this will enhance the performance and adaptability of DHNN, providing a strong foundation for optimizing the model and improving the effectiveness of DHNN in complex application scenarios.

Problem Statement 3:Challenges in the Training Phase of DHNN and the Need for Effective Optimization

Explanation of Problem Statement 1: The training phase in DHNN is critical, as unsatisfied logical rules can lead to local minima in the DHNN. To address learning efficiency, Kho et al. (2022) introduced Ant Colony Optimization (ACO) during the training phase of 2 Satisfiability in DHNN (DHNN-2SAT). The study shows that ACO in DHNN-2SAT performs reliably across various metrics. ACO updates neuron states using pheromone density to guide artificial ants. However, randomness during initialization and increasing neuron numbers make convergence challenging, often causing local convergence and reduced adaptability. When logical rules contain more lower-order clauses, ACO updates may favor one neuron state, leaving first-order clauses unsatisfied. This highlights the importance of initialization and update rules in ensuring global convergence in metaheuristic algorithms. Kasihmuddin et al. (2017b) applied Genetic Algorithm (GA), and Karim et al. (2022) used hybrid Election Algorithm (EA) during DHNN training phase to improve neuron adaptability for complex logic. However, random mutation of GA often leads to local optima, while the positive advertisement operator of EA risks overfitting by favoring a single neuron state. Effective optimization algorithms must balance exploration and exploitation, avoiding local optima and overfitting to enhance model generalizability. Wang (2010) combined DHNN with Estimation of Distribution Algorithm (EDA) to solve the unconstrained binary quadratic programming rule. Their research shows that EDA improves global search of DHNN through probabilistic modeling and sample generation. While probability-based modeling aids initialization and enhances early convergence, heavy reliance on statistical estimation increases computational costs. Thus, training algorithms must balance modeling precision with computational efficiency. To achieve zero-cost function objectives, training algorithms must include effective optimization operations. These algorithms should ensure good initialization, support multidirectional updates, and adapt to complex logical rules. However, while training phase optimization is vital, improving the retrieval phase is equally important to enhance the overall performance of DHNN.

Problem Statement 4:Challenges in the Retrieval Phase of DHNN and the Need for Flexible Mutation Strategies

Explanation of Problem Statement 1: In the retrieval phase, optimizing the diversity and quality of global solutions is crucial for improving the overall performance of DHNN. Expanding the solution space and prioritizing high-quality solutions enhances the robustness and adaptability of DHNN in complex scenarios. Kasihmuddin et al. (2019) proposed the Mutation Hopfield Neural network, which uses a mutation mechanism. While this strategy increases network state diversity, the uncontrollability of mutation operations when dealing with complex logical structures can reduce adaptability and lead to overfitting, affecting solution quality and diversity. This shows that simple mutation mechanisms are insufficient, and further optimization strategies are needed. Guo et al. (2024) proposed introducing mutation points into complex flexible logic, which has inspired researchers to explore combining this approach with higher-order logical mutations in the retrieval phase of DHNN. By designing flexible mutation strategies tailored to different logical rule elements, the adaptability of DHNN and solution quality can be further improved under complex logical rules. As performance improves, effectively mining and extracting logical rules within DHNN to enhance applications in complex systems becomes a critical challenge.

Problem Statement 5: Challenges in Logic Mining Accuracy and the Need for Hybrid Strategies

Explanation of Problem Statement 1: The introduction of logic mining into DHNN is a key method in AI and data mining. The main goal is to extract effective logical rules from datasets, particularly excelling in classification and prediction tasks. Logic mining induces logical formulas from real-world data, providing interpretable solutions for complex problems. Kho et al. (2020) and Zamri et al. (2020) proposed 2 Satisfiability Reverse Analysis (2SATRA) and 3 satisfiability Reverse Analysis (3SATRA), but limitations in attribute positioning and selection reduced the classification accuracy of the model on complex datasets. To enhance 2SATRA, Jamaludin et al. (2022a) proposed Advanced Log-Linear Analysis in 2 Satisfiability Reverse Analysis (A2SATRA) by applying a log-linear model to attribute selection and incorporating permutation operations, while Jamaludin et al. (2020) proposed Energy based k Satisfiability Reverse Analysis (EkSATRA) based on the principle of energy minimization. log-linear model about data structure limit the accuracy of model and adaptability with complex or nonlinear datasets. Kasihmuddin et al. (2022) developed Supervised 2 Satisfiability Reverse Analysis (S2SATRA) by introducing Pearson correlation analysis into 2SATRA. While Pearson correlation effectively measures linear relationships, sensitivity to outliers and inability to capture nonlinear dependencies limit the performance of S2SATRA with noisy or nonlinear datasets. To address these limitations, adopting an introduced strategy that combines multiple methodologies is essential. Leveraging the strengths of different approaches can improve the accuracy and adaptability of logic mining and enhance the robustness of DHNN in complex applications. Introducing multiple algorithms and techniques will enable broader logic pattern extraction and more precise classification and prediction across diverse datasets.

1.6 Research Questions

To ensure the consistency between the problem statements presented in Section 1.5 and the research objectives of this thesis, this section will further detail several key research questions. These questions aim to comprehensively outline various aspects of the proposed study. They not only build on the issues discussed earlier but also explore the research objectives in greater depth. Therefore, this thesis will revolve around the following research questions to ensure a thorough coverage and systematic analysis of the research objectives.

(a) What are higher-order systematic and non-systematic logical rules that balance flexible and randomness while staying adaptable to different questions?

(b) How can higher-order flexible logical rules be introduced into Discrete Hopfield Neural Network, ensuring neuron connections and compatibility with the training and retrieval phases?

(c) How can the ant colony algorithm be optimized being implemented in the Discrete Hopfield Neural Network to enhance learning efficiency and effectively optimize the neuron fitness function?

(d) What mutation strategies can be designed in the retrieval phase of the Discrete Hopfield Neural Network where different higher-order clauses adopt different mutation strategies to ensure diversity of global solutions?

(e) How can a logic mining model based on flexible higher-order logical rules be constructed with adaptable attribute selection and a variable number of attributes, and how can it be applied in a case study using live-streaming e-commerce dataset?

1.7 Research Objectives

The research objectives of this thesis focus on introducing and optimizing flexible logical rules within DHNN. A new logical rule is proposed, combining higher-order random logic with systematic and non-systematic logic while allowing flexible control over the ratio of different clause types and positive/negative literals. To ensure unbiased introduction of the logical rules in DHNN, the training phase focuses on optimizing the neuron fitness objective function, essential for fast convergence and network stability. To meet the diversity requirements of the retrieval phase, a mutation mechanism is introduced. This model ultimately aims to achieve high adaptability and accuracy in classification tasks with real-world datasets. The research obiectives were written based on the Speceasurable, Achievable, Relevant, and Time-bound (SMART) framework. The research objectives are as follows:

(a) To formulate the flexible logical rules, this research introduces G-type random 3-satisfiability, which combines higher-order random logic with both systematic and non-systematic logic. Adaptable constraints will be applied to control the ratio of different clause types and positive/negative literals. This ensures that the developed logical rules maintain both randomness and flexibility.

(b) To implement models for introducing G-type random 3-satisfiability into the neuron connections of Discrete Hopfield Neural Network while ensuring compatibility with existing training and retrieval processes. This objective aims to improve the performance and adaptability of the Discrete Hopfield Neural Network.

(c) To propose Binary Optimal Solution Ant Colony Optimization algorithms for the training phase of the Discrete Hopfield Neural Network model to maximize learning efficiency and optimize the neuron fitness objective function.

(d) To design Randomized Pairwise-Single Mutation strategies that ensure diversity in retrieval results during the retrieval phase of the Discrete Hopfield Neural Network model and enhance the quality of global solutions.

(e) To construct a Flexible-based G-Type Random 3-Satisfiability logic mining model, where attribute selection strategies and the number of attributes are adjusted based on real-world datasets. The model will be rigorously evaluated through a case study on the Douyin live-streaming e-commerce dataset to ensure adaptability and high accuracy in real-world application.

1.8 Methodology and Scopes

To achieve the research objectives, This thesis proposes the following methodology. First, a new SAT logical rule G-Type Random 3 Satisfiability (GRAN3SAT) is developed to construct highly adaptable and random logical rules. G-Type refers to a generalized type of Random 3-Satisfiability logical rule, where problem structures are extended or modified to incorporate additional logical constraints for enhanced modeling flexibility. Second, the GRAN3SAT logical rules are applied to the DHNN, forming the GRAN3SAT in DHNN (DHNN-GRAN3SAT) model. The Binary Optimal Solution Ant Colony Optimization (BOSACO) algorithm is introduced to optimize the training process. In the retrieval phase, the Randomized Pairwise-Single Mutation (RPSM) strategy is designed to enhance the diversity and quality of retrieval results. Finally, these components are combined to construct the Flexible based G-Type Random 3-Satisfiability Reverse Analysis (FGRA) logic mining model, which is systematically evaluated through experiments across various datasets and logical complexities. The following sections will detail each step.

Methodology for Objective 1: As research on SAT advances, developing more adaptable and flexible logical rules has become a key challenge. To address this, This thesis proposes a new SAT logic called GRAN3SAT. This logical rule combines higher-order random logic with both systematic and non-systematic logic while applying flexible constraints on the ratios of different clause types and the proportion of positive and negative literals. This section details the construction of GRAN3SAT. The design of GRAN3SAT is inspired by Karim et al. (2021) and their work on the RAN3SAT logical structure. Building on RAN3SAT, This thesis expands the randomness of logical rules, removes strict constraints on the logical structure, and introduces a hybrid structure by introducing systematic logic. GRAN3SAT incorporates clauses of varying logical orders (k = 1, 2, 3) and adjusts the ratio of positive and negative literals, creating a highly adaptable framework. To evaluate the flexibility and randomness of GRAN3SAT, systematic experiments were conducted. Flexible constraints on clause types and literal distribution were applied to test the performance of GRAN3SAT across different levels of logical complexity. The metric of clause and literal structure error was introduced to measure changes in logical structure, assessing flexibility based on clause quantity and literal state variations. These experiments ensure that GRAN3SAT maintains logical rule randomness while adapting to different application needs, achieving optimal system performance.

Scope of Methodology 1: This thesis focuses on developing and evaluating the adaptability and flexibility of the GRAN3SAT logical rule, including design of GRAN3SAT, parameter adjustment, structural optimization, and comparison with 3SAT (Mansor et al., 2017), 2SAT (Kasihmuddin et al., 2017a), and RAN3SAT (Karim et al., 2021). Experiments in a standard simulation environment evaluated GRAN3SAT using clause and literal structure errors to measure performance under different complexities. The results demonstrated that GRAN3SAT has advantages in adaptability, but the performance of GRAN3SAT in real-world applications needs further study. This thesis establishes the theoretical foundation for GRAN3SAT.

Methodology for Objective 2: Introducing SAT logical rules into DHNN enhances ability of model to handle complex logical structures, enabling more efficient decision-making and optimization. To address the limitations of the RAN3SAT in DHNN (DHNN-RAN3SAT) model, this thesis incorporates the GRAN3SAT logical rule into DHNN, forming the DHNN-GRAN3SAT model. This model uses GRAN3SAT to initialize neuron states, guide synaptic weight calculations, and optimize the training and retrieval processes for improved performance in complex logical tasks. In the training phase, GRAN3SAT logical rules initialize neuron states, and synaptic weights are calculated using the WA method (Abdullah, 1992). The training process minimizes the cost function by optimizing neuron fitness and analyzing the effects of different learning trials to refine weights and improve learning efficiency. In the retrieval phase, the network uses the optimized weights from GRAN3SAT logical rules to determine final neuron states through local fields and activation functions. Energy distribution analysis evaluates network stability, global solution analysis assesses solution quality, and similarity index calculation ensures diversity in final neuron states, enabling the network to handle diverse input patterns effectively.

Scope of Methodology 2: The scope of this research focuses on developing and evaluating the introduction of the GRAN3SAT logical rule into DHNN (Karim et al., 2021). The model includes neuron state initialization, synaptic weight optimization, and performance analysis during the training and retrieval phases. In the training phase, the study evaluates the impact of different logic structures on neuron fitness and optimizes weight errors. In the retrieval phase, the model assesses the adaptability of DHNN-GRAN3SAT in handling complex logical tasks.

Methodology for Objective 3: In the training phase of the DHNN-SAT model, if the logical rules are not satisfied, the optimal CAM may cause the network to fall into a state of local minimum energy. To address this issue, this thesis introduces a novel Ant Colony Optimization algorithm, BOSACO, to optimize neuron states in the DHNN and align them with GRAN3SAT logic. BOSACO builds on the ACO algorithm (Kho et al., 2022) applied in the DHNN-2SAT training phase and enhances initial efficiency by introducing an EDA selection strategy (Wang, 2010), which optimizes pheromone density and visibility values. By introducing objective function values, BOSACO further improves adaptation to specific problems and accelerates convergence. To evaluate the performance of BOSACO in DHNN-GRAN3SAT, this research measures algorithm complexity through the number of objective function calls and assesses efficiency using the average number of iterations and runtime. Additionally, neuron fitness and weight error formulas are analyzed during the training phase, while energy errors and the proportion of global solutions are evaluated during the retrieval phase, providing a comprehensive assessment of the performance and adaptability of model.

Scope of Methodology 3: The scope of this research focuses on improving the learning and optimization processes in the DHNN-SAT model by introducing the newly proposed BOSACO. This involves designing and optimizing the BOSACO algorithm to handle the convergence efficiency of flexible GRAN3SAT logical rules. BOSACO includes adjusting algorithm parameters by introducing an EDA selection strategy (Wang, 2010) such as pheromone density and visibility and introducing state update strategies to optimize convergence rates and achieve high-quality solutions. Additionally, this research evaluates the performance of BOSACO through computational experiments, measuring iteration times, time complexity, and algorithm complexity. These insights are essential for optimizing neuron states in DHNN and ensuring robust training for complex SAT scenarios.

Methodology for Objective 4: During the retrieval phase, improving the diversity and quality of global solutions is essential for enhancing the performance of the DHNN-SAT model. This research proposes a strategy to explore a broad solution space and optimize the selection of high-quality solutions, increasing the adaptability of DHNN in solving complex problems. Inspired by mutation mechanisms in DHNN, which led to the development of Mutated Hopfield Neural Network (MHNN) (Kasihmuddin et al., 2019), this thesis introduces a new method called RPSM to improve global solution quality during retrieval. RPSM builds on the low-order flexible logic mutation strategy of Guo et al. (2024), using both pairwise and single mutations to modify specific neuron states. This approach explores different regions of the solution space while preserving population diversity. By randomly mutating neuron states with a set probability, RPSM increases the diversity of final states and prevents premature convergence by steering toward ideal neuron states. This enhances the search capability of DHNN and improves the quality and diversity of solutions. In this thesis, diversity is evaluated using similarity analysis and Total Number of Variations (TNV). Similarity analysis compares final neuron states with a baseline state to measure diversity, while TNV quantifies the number of global solutions generated by different neuron configurations. These metrics ensure that the DHNN-SAT model maintains solution diversity, which is critical for solving complex problems effectively.

Scope of Methodology 4: The scope of this research focuses on designing and evaluating the application of RPSM in the retrieval phase of the DHNN-SAT model. The thesis aims to enhance the diversity and quality of global solutions (Guo et al., 2024) and evaluates the effectiveness of RPSM through diversity metrics such as similarity analysis and TNV calculation. The objective is to validate the ability of RPSM to improve the adaptability and generalization of the DHNN-SAT model in solving complex problems.

Methodology for Objective 5: The introduction of logic mining techniques into DHNN has become a key approach in AI and data mining, especially for extracting effective logical rules from datasets, achieving success in classification and prediction tasks. Inspired by methodologies like A2SATRA (Jamaludin et al., 2022a), EkSATRA (Jamaludin et al., 2020), and S2SATRA (Kasihmuddin et al., 2022), this thesis proposes the FGRA logic mining model. FGRA is a flexible and intelligent model designed to handle diverse and complex datasets, incorporating key components such as GRAN3SAT, BOSACO, and RPSM. In the FGRA model, the number of neurons is dynamically adjusted based on dataset scale and complexity, optimizing performance. With dynamic feedback, FGRA adjusts the number of attributes, managing high-dimensional data and improving logic mining efficiency. Attribute selection is critical for identifying key variables in datasets (Kasihmuddin et al., 2022), so FGRA uses an Multi-attribute Selection Ensemble Method (MSEM). This approach combines various techniques to select the most relevant attributes, enhancing performance across different datasets. To comprehensively evaluate the model, key performance metrics such as Accuracy, Precision, Sensitivity, F1 Score, and Matthew's Correlation Coefficient are used. These metrics provide insights into the effectiveness of the FGRA model across different scenarios.

Scope of Methodology 5: The scope of this research focuses on developing and evaluating the FGRA logic mining model within DHNN. The FGRA model introduces components like GRAN3SAT, BOSACO, and RPSM, offering high flexibility and adaptability for handling complex and diverse datasets. This research emphasizes dynamically adjusting the neuron count to optimize performance and improving logic mining accuracy and efficiency through MSEM. The performance of model is evaluated using metrics such as Accuracy, Precision, Sensitivity, F1 Score, and Matthew's Correlation Coefficient (Manoharam et al., 2023) across various scenarios.

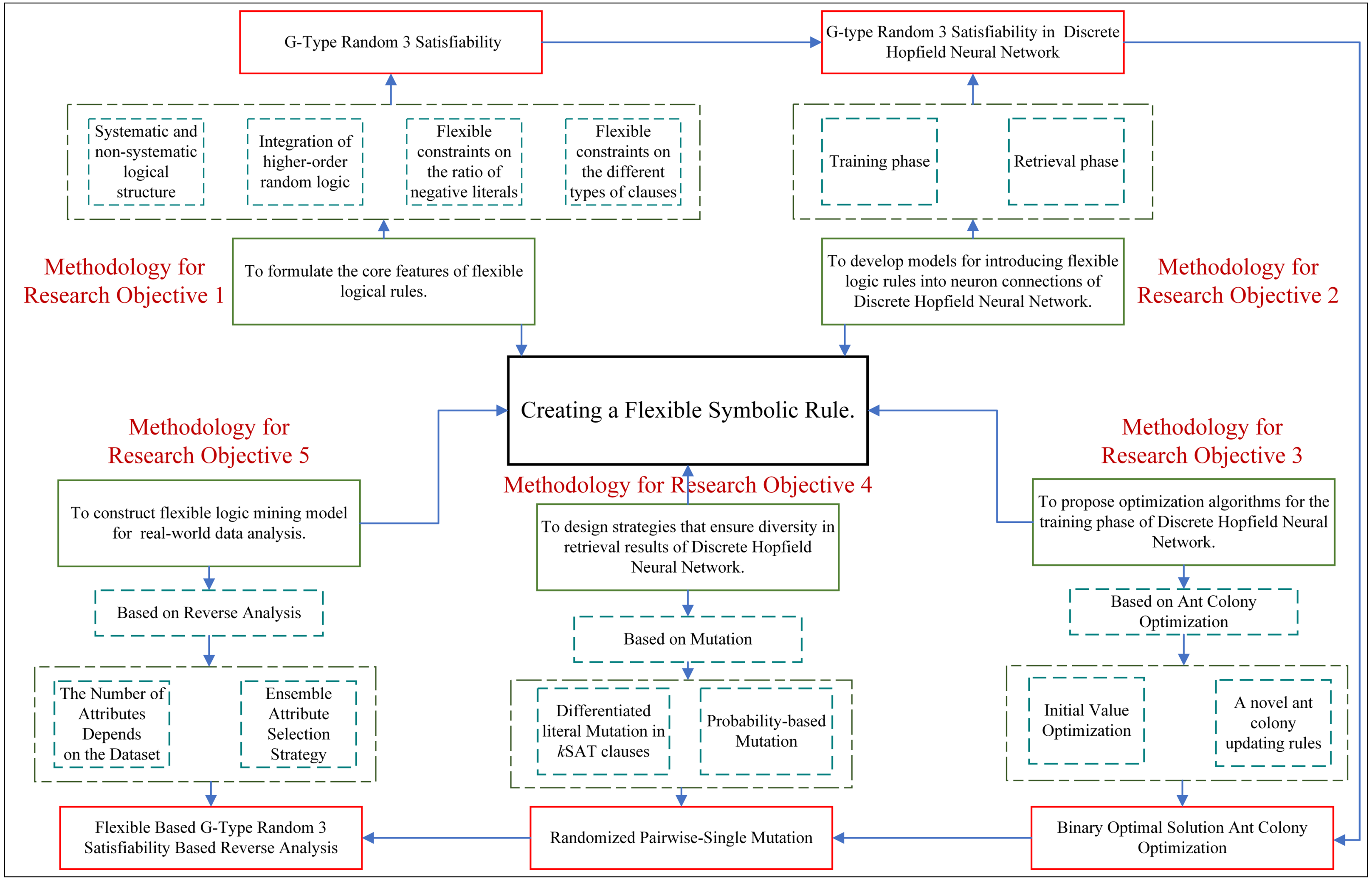

Figure 1.1: The overview of methodology in this thesis.

1.9 Organization of Thesis

This thesis consists of eight chapters. Chapter 1 introduces the research background, research questions, and research objectives, laying the foundation for the thesis. Chapter 2 reviews the development of DHNN, the evolution of logical structure rules, the optimization of learning algorithms during the training phase, and the optimization algorithms in the retrieval phase. Chapter 2 also summarizes the development of logic mining models and identifies research gaps. Chapter 3 focuses on logical rules, optimization, and neural network modeling, exploring the application of RAN3SAT logical rules in DHNN and discussing how metaheuristic techniques improve model performance.Chapter 4 proposes the GRAN3SAT logical rule, detailing the structure and principles of GRAN3SAT, and explains introduction into DHNN to address complex logical problems. Additionally, this chapter introduces the BOSACO algorithm to optimize the training process and a mutation mechanism to enhance the diversity of logical rules, forming the FGRA model. Chapter 5 analyzes the performance of GRAN3SAT within DHNN using software simulations, evaluates the impact of parameters on model performance, and compares GRAN3SAT with other logical rules. The overall performance is assessed, and the impact of BOSACO and FGRA on improving the training and retrieval phases is examined. Chapter 6 investigates the application of the FGRA model in real-world scenarios, validating the effectiveness of model using 20 datasets. Metrics such as accuracy, precision, sensitivity, F1 score, and Matthews correlation coefficient demonstrate the adaptability of the FGRA model. Chapter 7 presents a case study on Douyin live-streaming e-commerce, using the FGRA model to analyze the impact of key attributes on the sales of live-streamed products and their interactions, resulting in a classification model. Finally, Chapter 8 summarizes the research findings and contributions, reflects on the achievement of the research objectives, and proposes future research directions and application areas.

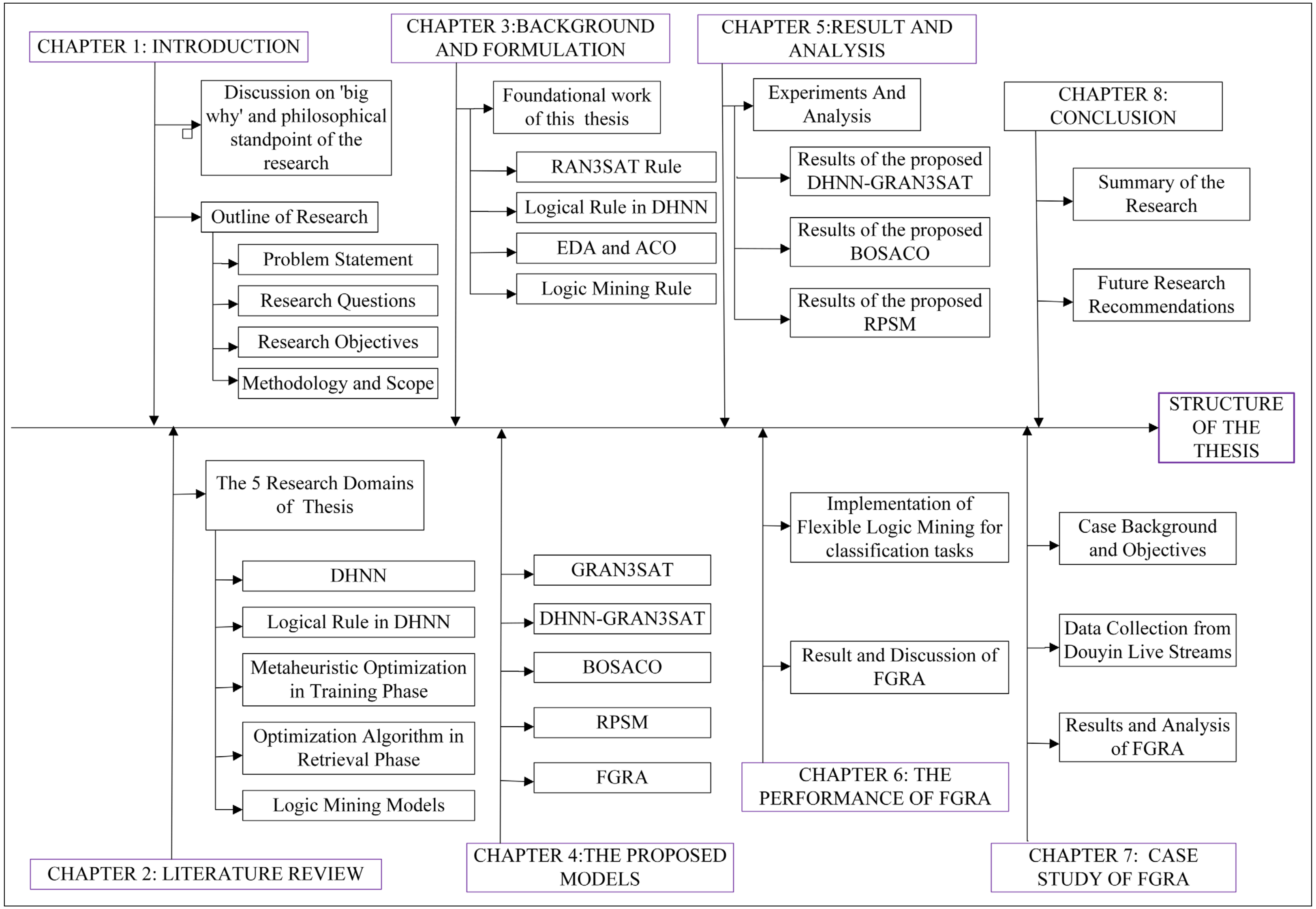

Figure 1.2: The overview of thesis.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter provides a detailed literature review on key advancements related to the proposed model, including DHNN, logical rules in DHNN, optimization algorithms in different phases, and logic mining models. This chapter starts by reviewing the evolution and recent applications of DHNN. Then, this chapter examines developments in SAT research, focusing on its role in representing neurons in DHNN. The chapter also discusses metaheuristic optimization models used in the training phase and algorithms applied in the retrieval phase, emphasizing their impact on improving network performance. Lastly, this chapter introduces logic mining models based on reverse analysis, showcasing their adaptability in interpreting data and extracting logical rules for various applications.

2.2 Discrete Hopfield Neural Network

The Hopfield Neural Network (HNN) has been widely used as a key tool for handling complex tasks such as optimization rules, logic programming, and reasoning. Initially introduced by Hopfield (1982), HNN was designed as a content-addressable memory system and is known for the use of binary threshold nodes. The discrete variant DHNN has gained significant attention for the efficiency in handling optimization rules. Hopfield and Tank (1985) demonstrated the application in combinatorial optimization, showing that DHNN could map problem constraints into the energy function of network to find effective solutions. This established DHNN as a practical tool for computational tasks. However, early studies also noted that DHNN could become stuck in local minima, leading to suboptimal results. As research progressed, scholars refined the structure and functionality of DHNN. Li et al. (2009) analyzed the stability of DHNN using a dynamic weight function matrix and found that the network could achieve stability after multiple iterations. This dynamic adjustment improved the flexibility of DHNN, making DHNN more stable for handling complex tasks. Building on this, Guo et al. (2008) developed a DHNN model for energy optimization in real-time operating systems by redefining the energy function and operational equations. This model transformed the power optimization rule into a stable state search, enabling DHNN applications in real-time computing. In addition to the strong performance of DHNN in optimization rules, the DHNN has shown significant learning capabilities by dynamically adjusting internal parameters, such as weights and thresholds, based on input data. Abdullah (1992) introduced a logic programming method for HNN, demonstrating that the network could learn logical rules through Hebbian learning. This reduced logical inconsistencies in data interpretation, making DHNN more adaptable for use in dynamic systems. However, while Hebbian learning was effective in rule acquisition and synaptic strengthening, the model struggled with complex logic programming tasks. To address this, Kasihmuddin et al. (2017a) proposed an improved logic programming approach. This method expanded the adaptability of network, enabling better learning and execution of complex logical rules. By refining the dynamic behavior of the network, WA method greatly improved the efficiency of DHNN and flexibility in logic programming. In summary, the DHNN has become a key tool for handling complex tasks like optimization, logic programming, and reasoning. With continuous innovations, DHNN demonstrates strong computation, learning ability, and adaptability, excelling in dynamic environments. Introducing complex logical structures has further broadened applications of DHNN in logic programming. However, despite the significant achievements of DHNN, the interpretability of DHNN remains a major challenge in current research. Although existing research attempts to improve model interpretability by establishing associations between neurons and logical rules, there is still a lack of a systematic approach that can flexibly regulate neuron connectivity while balancing training and retrieval stages when dealing with complex logical structures. This limitation highlights the importance of continuing to develop DHNN.

2.3 Logical Rules in Discrete Hopfield Neural Network

Introducing Propositional Satisfiability (SAT) in DHNN has become a promising approach to advance artificial intelligence systems. This introduction represents complex logical relationships within neural networks, leveraging their parallel processing, robustness, and optimization capabilities to enable efficient reasoning and system scalability. Sathasivam (2010) introduced HornSAT into HNN, linking logical variables to neurons and determining synaptic weights by comparing cost functions. This improved convergence speed and stability, allowing the network to handle complex reasoning tasks and broadening application in intelligent systems. Building on this foundation, Kasihmuddin et al. (2017a) explored 2SAT structures in DHNN to enhance optimization and global minima identification, addressing overfitting issues. Mansor et al. (2017) extended this by introducing 3SAT, increasing the complexity of neuron representation and enabling the network to process more intricate logical relationships. Though limited by fixed literals in clauses, 2SAT and 3SAT expanded the expressive capacity of model and paved the way for more complex intelligent systems. Researchers later focused on non-systematic logical structures to increase flexibility. Sathasivam et al. (2020b) introduced RAN2SAT into DHNN, creating random rules for first and second-order clauses to handle complex random logic. Karim et al. (2021) introduced RAN3SAT, incorporating first, second, and third-order clauses, improving the ability to find global minima and addressing challenges in simpler clauses. Guo et al. (2024) further enhanced flexibility with the hybrid YRAN2SAT model, combining systematic and non-systematic logical rules to improve the ability of DHNN to handle complex rules. Additionally, researchers have enhanced the logical expressiveness and reasoning capabilities of models by adding constraints to logical formulas. Zamri et al. (2022) proposed the r2SAT model, which explicitly defines positive and negative literals in logic structures, considering how the proportion of negative literals affects outcomes. Similarly, Alway et al. (2022) and Roslan et al. (2023) introduced MAJ2SAT and MAJ3SAT, imposing constraints on the number of different types of clauses in logic structures. These models utilized varying clause numbers to increase the complexity of logical expressions, improving the reasoning and optimization capabilities of DHNN. In conclusion, introducing SAT into DHNN has shown great potential in optimization and logical reasoning. By adding complex logical structures and optimization methods, researchers have improved the flexibility and expressiveness of DHNN, providing a strong foundation for building efficient and scalable intelligent systems. These advancements also guide further exploration of SAT and DHNN in complex applications. However, there are still limitations in combining high-order random logic, systematic and non-systematic logic, as well as flexible constraint clauses and literals. Model optimization is also crucial for improving performance and scalability. Current research should focus on improving the efficiency and reliability of DHNN for complex reasoning tasks through advanced optimization algorithms.

2.4 Metaheuristic Algorithm as Optimization Model in Training Phase

In computer science, many practical problems can be transformed into optimization rules. Due to the diversity and complexity of optimization rules, developing efficient algorithms to handle optimization rules is a key challenge. Metaheuristic algorithms are powerful tools for handling large-scale optimization rules. They are not problem-specific and have broad applicability. By simulating natural evolution or group behavior, these algorithms avoid local optima and improve solution efficiency. ACO is an effective method for handling high-dimensional combinatorial rules. Dorigo and Gambardella (1997) explains that ACO mimics the foraging behavior of ants, which find the shortest path between nest and food by depositing and evaporating pheromones. This indirect communication helps ants converge on optimal solutions. Dorigo et al. (1999) further highlights that ACO effectively balances exploration and exploitation, making ACO a versatile technique for handling both discrete and continuous optimization rules. In addition to ACO, EDA has also gained attention for the ability of the model to handle complex optimization rules. Instead of relying on crossover and mutation, EDA uses probabilistic models to guide the search, improving global search capabilities. Wang (2010) noted that EDA is particularly suited for high-dimensional rules. During the training phase of DHNN, metaheuristic algorithms are used to find satisfactory interpretations of SAT. If logical rule interpretations are suboptimal, the optimized CAM can lead to local minimum energy. To address this, various methods have been developed to reduce learning errors. Kho et al. (2022) implemented the ACO algorithm in DHNN-2SAT, which performed reliably across most performance metrics by using pheromone density to guide neuron state updates. Kasihmuddin et al. (2017b) introduced GA to enhance neuron fitness through crossover and mutation, reducing the cost function. To address initial convergence issues, Kasihmuddin et al. (2017a) proposed the Artificial Bee Colony (ABC) algorithm, using bitwise operators to update neuron states. Simulations showed ABC achieved optimal neuron states. Later, Sidik et al. (2022) introduced a Binary ABC, replacing bitwise operators to reduce iterative errors and improve r2SAT logic generation. In another advancement, Bazuhair et al. (2021) proposed an EA that enhanced neuron fitness using positive and negative campaigning and alliances, optimizing RAN3SAT logic. Karim et al. (2022) proposed an improved EA, achieving global maxima during the learning phase and optimal results in retrieval phase. Despite the success of EA in minimizing training errors, Wang (2010) suggested introducing EDA in DHNN to leverage the global search capability of model for better optimization. In summary, metaheuristic algorithms play a crucial role in handling optimization rules in computer science. From ACO to GA, EA, and EDA, various algorithms provide effective means for solving complex optimization problems by simulating natural behavior and evolutionary mechanisms. However, as the scale of neural networks grows and logical rules become more complex, developing an optimization algorithm that balances efficient learning with rule flexibility remains a key challenge in the learning phase of DHNN. The limitations of traditional algorithms in computational efficiency and adaptability call for further research and innovation.

2.5 Optimization Algorithm in Retrieval Phase

In the process of optimizing neural networks, generating optimal and diversified neuron states is crucial for the overall performance of the network. An effective retrieval