OPTIMIZING DISCRETE HOPFIELD NEURAL NETWORKS USING MAJOR RANDOM 1,3-SATISFIABILITY AND HYBRID ARTIFICIAL BEE COLONY

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by

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LIST OF SYMBOLS

$S_i^{optimal}$	Benchmark neuron state
C_{h_i}	Chromosome
q	Clauses
Q_i^k	Clauses
$\mathbb{C}_{L_{MR1,3SAT}}$	Cost Function of the logical $L_{MR1,3SAT}$
rC	Crossover rate
Γ_i	Current fitness neuron states in DHNN MR 1, 3SAT
$\Psi_{\left(v_i^*\right)}$	Eligibility distance coefficient
n_i	First-order clause
m	First-order clauses
f_{L_i}	Fitness of the candidates
g_i	Gene
G	Generation
Q_{Global}	Global Minimum in the Retrieval Phase
$\max\left[n\left(S_i\right)\right]$	highest frequency of the Neurons
$\tanh\left(h_i\right)$	Hyperbolic tangent function
L_{ij}	Inconsistency of the logical structure
$JFS_{i,j}$	Jaccard Feature Selection
ϕ	Learning iteration
ρ	Literals

$h_i(t)$	Local Field
Q_{Local}	Local Minimum in the Retrieval Phase
P_{3SAT}	Logic 3-Satisfiabiliy
$\varphi_{MAJ2SAT}$	Logic Major 2 Satisfiability
$L_{MR1,3SAT}$	Logic Major Random 3 ,1 Satisfiability
$P_{RAN3SAT}$	Logic Random 3 Satisfiability
$L_{L_{MR1,3SAT}}$	Lyapunov Energy function
$\Gamma_{\max}$	Maximum fitness neuron states in DHNN MR 1, 3SAT
C_i^k	Maximum variables in the Logical Structure
$MAE_{retrieve}$	Mean Absolute Error in the Retrieval Phase
MAE_{learn}	Mean Absolute Error in Learning Phase
$MAPE_{retrieve}$	Mean Absolute Percentage Error in the Retrieval Phase
$MAPE_{retrieve}$	Mean Absolute Percentage Error in Retrieval Phase
$L_{L_{MR1,3SAT}}^{\min}$	Minimum Energy Function
rM	Mutation rate
σ^N	Negative Advisement
S_i	Neuron state
nC_i	Number of Clauses
w	Number of Combination
$G_{L_{MR1,3SAT}}$	Number of Global minima solution
v	Number of Learning
ϑ	Number of learning

$L_{L_{MR1,3SAT}}$	Number of Local Minima Solution
NN	Number of Neurons
φ	Number of Trials
$(+\ /\ -/\ =)$	outperforms, underperforms and equal
P	Population
σ^{P}	Positive advertisement
Tol	Pre-determined value
R	Relaxation rate
$RMSE_{learn}$	Root Mean Square Error in Learning Phase
$RMSE_{retrieve}$	Root Mean Square in the Retrieval Phase
n	Second-order clauses
rS	Selection rate
J_i	Similarity analysis Final Neuron State.
$S_{Jaccard}$	Similarity Jaccard
$Sokal$	Sockel Sneat Index
$SSE_{retrieve}$	Sum Square Error in the Retrieval Phase
SSE_{Energy}	Sum Square Error in the Retrieval Phase
SSE_{learn}	Sum Square Error in Learning Phase
W_{ijk}	Synaptic weight between i ,j and k
W_{ij}	Synaptic weight between i and j
W_i	Synaptic weight of neuron i
P_{learn}	The outcome of the learning data

P_{test}	The outcome of the testing data
m_i	Third Order Clauses
l	Third-order clauses
θ	Threshold constraint of DHNN
TV	Total Neuron Variation
$S_i(t+1)$	Updated neuron state
k	Variables in each clause
v_i^j	Voter

LIST OF ABBREVIATIONS

2SAT	2 Satisfiability
3SAT	3 Satisfiability
Acc	Accuracy
AD	Alzheimer's Disease
AE	Adverse Event
ADNI	Alzheimer's Disease Neuroimaging Initiative
ABC	Artificial Bee Colony
ABCJ	Artificial Bee Colony Algorithm by Jia *et al*.2014
ABCk	Artificial Bee Colony Algorithm by Kasihmuddin *et al*.,2017b
ABCS	Artificial Bee Colony Algorithm by Sidik *et al*.2022
AI	Artificial Intelligence
ANN	Artificial Neural Network
oBABC	Binary Variant Artificial Bee Colony
BHA	Black Hole Algorithm
CSA	Clonal Selection Algorithms
CRAN2SAT	Conditional Random 2 Random
CNF	Conjunctive Normal Form
CAM	Content addressable memory
DEA	Differential Evolution Algorithm
DHNN	Discrete Hopfield Neural Network
EA	Election Algorithm
E2SATRA	Energy based 2 Satisfiability Reverse-based Analysis Method
EDA	Estimation Distribution Algorithm

ES	Exhaustive Search
FN	False Negative
FMI	Fowlkes-Mallow's index
FP	False Positive
GA	Genetic Algorithm
GRAN3SAT	G-Type Random k Satisfiability
HORNSAT	Horn Satisfiability
HES	Hybrid Exhaustive Search
HTAF	Hyperbolic Tangent Activation Function
IPS	Institute Postgraduate Studies
HABC	Hybrid Artificial Bee Colony Algorithm
JFS	Jaccard Feature Selection
*k*SATRA	*k* Satisfiability based Reverse Analysis Method
A2SATRA	Log-linear model in 2SAT
MAJ2SAT	Major 2 Satisfiability
MR 1,3SAT	Major Random 1,3 Satisfiability
MR*k*SAT	Major Random *k*-SAT
MCC	Matthews Correlation Coefficient
MAX*k*SAT	Maximum *k* Satisfiability
MAE	Mean Absolut Error
MAPE	Mean Absolute Percent Error
MNGA	Modified Niche Genetic Algorithm
MR1,3SATRAμ	Multi-unit Major Random 1,3 Satisfiability Reverse Analysis
PSO	Particle Swarm Optimization
P2SATRA	Permutation 2 Satisfiability Reverse Analysis

PSAT	Probabilistic Satisfiability Problem
RBFNN	Radial Basis Function Neural Network
RAN2SAT	Random 2 Satisfiability
RAN3SAT	Random 3 Satisfiability
DHNNRAN*k*SAT	RAN*k*SAT in DHNN
RA	Reverse Analysis
RMSE	Root Mean Square Error
RWS	Roulette wheel selection technique
DHNN-SAT	SAT embedded in DHNN
SAT	Satisfiability
SAT	Satisfiability
Sen	Sensitivity
Spec	Specificity
SSE	Sum Square Error
S2SATRA	Supervised 2 Satisfiability Reverse-based Analysis Method
TN	True Negative
TP	True Positive
TSP	Travelling Salesman Problem
TV	Total Neuron Variation
USM	Universiti Sains Malaysia
WA	Wan Abdullah method
*r*SAT	Weighted Random k Satisfiability
*r*2SAT	Weighted Random k Satisfiability
YRAN2SAT	Y-Type Random 2 Satisfiability

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PENGOPTIMUMAN RANGKAIAN NEURAL HOPFIELD DISKRET MENGGUNAKAN MAJOR RANDOM 1,3-SATISFIABILITI DAN KOLONI LEBAH BUATAN HIBRID

ABSTRAK

Satisfiabiliti adalah bahasa simbolik utama dalam rangkaian diskrit neural Hopfield yang menyumbang kepada pembangunan model kecerdasan yang maju. Namun, ketiadaan kombinasi peringkat tinggi satisfiability dalam pembangunan model kecerdasan menghadkan potensi penuh rangkaian tersebut. Oleh itu, tesis ini memperkenalkan pengoptimuman rangkaian neural hopfield diskret menggunakan major random 1,3-satisfiabiliti dan Koloni lebah buatan hybrid untuk menilai tingkah laku neuron dalam rangkaian. Logik yang dicadangkan ini, menunjukkan prestasi yang optimum berbanding logik sedia ada. Untuk meningkatkan kualiti neuron akhir, Algoritma koloni lebah buatan hybrid digunakan semasa fasa perolehan bagi mengoptimumkan neuron akhir dengan memenuhi multi-objektif termasuk maksimum global minima, kepelbagaian keadaan neuron dan minimum indeks persamaan. Peraturan logik dan algoritma baharu menjadi komponen asas model perlombongan logik yang dinamakan Analisis Berbalik Multi-unit Major Random 1,3-Satisfiabiliti yang meningkatkan pengekstrakan pengetahuan dan menyediakan interpretasi yang lebih baik. Tambahan pula, purata nilai Sensitiviti 0.9913 dan nilai Spesifikasi 0.9888 yang diperoleh daripada kajian kes Penyakit Alzheimer Neuroimaging menunjukkan prestasi yang baik dalam mengklasifikasikan kedua-dua klasifikasi benar dan palsu dengan tepat. Penemuan daripada tesis ini secara signifikan menyumbang kepada kemajuan teknologi kecerdasan buatan dalam menangani masalah kehidupan sebenar.

OPTIMIZING DISCRETE HOPFIELD NEURAL NETWORKS USING MAJOR RANDOM 1,3-SATISFIABILITY AND HYBRID ARTIFICIAL BEE COLONY

ABSTRACT

Satisfiability is a key symbolic language in Discrete Hopfield Neural Networks that contributes to the development of advanced Artificial Intelligence models. However, the absence of combined higher-orders satisfiability limits the full potential within the network. Therefore, this thesis introduces a optimizing discrete Hopfield neural networks using major random 1,3-satisfiability and hybrid artificial bee colony to evaluate the neuron behaviours within the network. The proposed logic demonstrates optimal performance compared to existing logical rules. To improve the quality of the final neuron states, the hybrid artificial bee colony algorithm is applied during the retrieval phase to optimize final neuron states by satisfying multi-objective functions including maximizing global minima solutions, promoting diversified neuron states and minimizing the similarity index. This new logical rule and algorithm become as foundational components of the logic mining model, named Multi-unit Major Random 1,3-Satisfiability Reverse Analysis which effectively retrieves optimal induced logic by enhancing knowledge extraction and provides better interpretability. Furthermore, the average Sensitivity value of 0.9913 and Specificity value of 0.9888 obtained from the Alzheimer's Disease Neuroimaging Initiative case study demonstrate the exceptional performance of the proposed model in accurately classifying both true and false classifications. The insights from this thesis significantly contribute to the advancement of Artificial Intelligence technologies in addressing real-life problems.

CHAPTER 1

INTRODUCTION

1.1 Overview

Artificial intelligence requires three multidisciplinary fields which include computer science along with mathematics and engineering principles. The main objective is to develop systems that execute operations that demand human intelligence abilities. This research investigates AI intelligence through both artificial neural network optimization and mechanisms implemented into neural network phases. The primary goal of this chapter to display Artificial Neural Networks (ANN) relevance to AI together with a research problem description and goal. All research analysis along with limitations are included in the chapters methodology section. Finally, an outline of the thesis organization follows to present a complete framework of the thesis structure.

1.2 Artificial Neural Networks in Artificial Intelligence

Artificial Neural Networks (ANN) operates as a computer model that draws its structure from human brain biological neural networks that process information. According to the mathematical representation of ANN, input layers and output layers exist. The individual layers exist as connected neurons utilizing weighted elements (Baba, 2024). ANN features the representation of biological yet artificial neurons, as shown in Figure 1.1. Neuronal processes in biological systems begin when multiple inputs pass through dendrites before the soma uses to generate an output signal through the axon. Both artificial and biological neurons take several inputs which lead to a weighted process before creating new outputs.

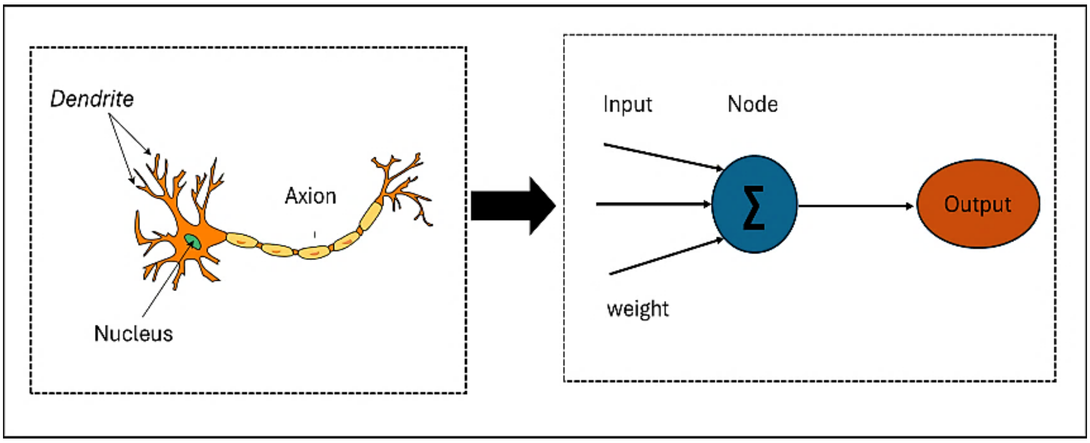

Figure 1.1 A biological neural network (left) and ANN (right) (Makinde & Asuquo, 2012)

The applications of ANN include image and speech recognition through Patel *et al*. (2024) language translation according to Hosseini *et al*. (2024) as well as medical diagnosis research by Sriraman & Samidurai (2024) and stock market prediction through Yasin *et al*. (2024) along with robot control as demonstrated by (Park *et al*., 2024). ANN successfully handles different applications yet people remain unsure about understanding its output interpretation. Such unclearness transforms into a substantial barrier due to restricted information which can be extracted from output values. The development of ANN models requires capabilities to enhance the understanding of output data produced by each neuron.

A necessary strategy is to *define alternative intelligence in AI* models to manage the benefits and risks of ANN. In layman terms, a simple explanation of AI alternative intelligence involves finding novel techniques to improve AI operational efficiency and boost accuracy along with reliability. To do this, need to direct the focus on enhancement methods applicable to ANN choice-making capabilities and data-learning abilities and real-world situation responses. ANN requires new mechanisms and scalable technique which enable better and trustworthy decision-making against changing conditions. The key areas of focus will improve ANN model reliability while making its processes more transparent and reachable to users for trust-based acceptance. By making improvements to ANN, can achieve both explains its choices and guarantees dependable outcomes.

AI model intelligence can be enhanced while its refinement is enabled through additional mechanisms which focus on specific techniques. The new specific method serves as the operational system that modifies multiple elements and parameters of the ANN model. ANN benefit from extra mechanisms during learning and retrieval operations to maximize the output results (Liu *et al.,* 2024). The addition of particular components to the ANN process enables the model to operate at peak performance under various settings when executing specific tasks. To guarantee precise decision-making the different conditions will analyze simulated data and actual real-life data sets. ANN model refinement contributes to better interpretability as well as transparency features. One of the specialized types of ANN is the Discrete Hopfield Neural Network (DHNN) constitutes an ANN specialization which adopts a different operational method through neural network elements that function as fully active or inactive states for efficient pattern storage and recall processes. The development of DHNN models has driven to significant innovations within the broader field of ANN. Therefore, refining the intelligence and capabilities of DHNN models is crucial for improving the ability to solve various complex problems across different fields and applications. The next section will discuss the existing challenges and opportunities for advancing DHNN models.

1.3 Existing Challenges in the Discrete Hopfield Neural Network

Repeated development of ANN technology has prompted different AI applications to evolve while supporting specific model advancement which includes the Discrete Hopfield neural network (DHNN) (Hopfield, 1982). The DHNN represents a distinctive form of recurrent neural networks that uses its pattern-recalling and neuron convergence stabilizing features for numerous fields including pattern recognition (Yao *et al*., 2023) and optimization problems (Guo *et al*., 2024). Generally, the synaptic weight serves to connect all neurons inside a DHNN architecture. This synaptic weight will be stored in the form of Content Addressable Memory (CAM). The network can access sources of memory within the DHNN architecture because of its CAM features. The advancement of DHNN facilities substantial AI field improvements that enhance modeling capabilityThe features of CAM enable the network to access and utilize memories within the DHNN architecture. The development of DHNN has led to significant progress in the field of AI which has improved the capability in modelling DHNN.

Despite these improvements, the main obstacle for DHNN exists in making decisions transparent and understandable particularly when it comes to interpretability. The neural networks "black box" characteristics creates barriers for understanding how decisions operate within the network system (Burrell, 2016). Using the symbolic logical rule as an essential component in DHNN neurons represents an effective method to upgrade black box model capabilities. The symbolic logical rule integration offers a pathway to create better neurons while providing interpretable input-output pairs that strengthens transparent DHNN applications. According to Abdullah (1992), the role of logic in DHNN functions as governing rules that represent neuron principles. The set of neurons minimizes the energy function in network space corresponds directly to the set of SAT logic rule satisfied truth assignment values. SAT integration in DHNN boosted understanding regarding what the network generates as output.

The implementation of SAT logical rules into the DHNN consists of two types: systematic logic according to Kasihmuddin *et al*. (2017a) and non-systematic logic as described by (Sathasivam *et al*. 2020b). The systematic logical structure with a restricted number of clauses has been explored but it lacks the flexibility to effectively model complex relationships. In contrast, the non-systematic logical rules provide more flexibility in the clause structure that allowing of more interconnected connections between neurons. However, these non-systematic logical rules face challenges in terms of the optimal distribution of clause in different combination. The refinement of DHNN models requires implementation of adaptable non-systematic logical rules. These logical rules offer an additional mechanism for encoding complex relationships between neurons, enabling DHNN models to represent and solve a wider range of problems efficiently.

Applying the learning algorithm during the learning phase helps obtain optimal synaptic weights, which in turn lead to an optimal final neuron state (Sathasivam et al., 2020a). However, the quality of the final neuron state overfits due to the repetitive neuron state (Zamri *et al*., 2022b). In this context, the quality of the final neuron state needs to be optimized by considering the multi-objective function in terms of the diversified solution and global solution with a low similarity index. The quality of the final neuron state stands essential for logic mining operations related to classification tasks. Logic mining involves the extraction of the logical rules from real-life datasets through reverse analysis. The existing logic mining model deals with imbalanced datasets which can lead to an ineffective model in doing classification tasks. Therefore, it is important to have a new approach in dealing with the imbalanced dataset so that the final induced logic that is retrieved by the logic mining model can overcome this issue. Additionally, choosing the best attribute from a dataset is a crucial step since it has the greatest impact on improving the performance of the induced logic. Ideally, the DHNN model can perform well in classification tasks when suitable additional components are applied. These points collectively lead to a broader philosophical standpoint, whereby:

"Increase DHNN intelligence through flexible symbolic-connection with diversity"

This thesis, adopts a philosophical perspective proposing that neurons with the different-order (higher-order and lower-order) of the clauses in the form of the SAT within a DHNN to improve the flexibility. The "*symbolic-connection",* refers to the flexible component with the different order of the clauses. This flexibility makes the logical structure of the DHNN model more precise, helping to tackle SAT-related optimization and logic mining challenges. In this context, the "*with diversity*" highlights the ability of the neurons with multiple quality of the final neuron states with intelligently enhance the DHNN model particularly in improving learning, retrieval processes and implementing novel logic mining techniques. This new achievement can improve the "*DHNN intelligence"* in solving real-world classification tasks with greater efficiency and interpretability. The following section will thoroughly explore these concepts by examining the relevance to the problem statement.

1.4 Problem Statement

The DHNN is great for tackling tough optimization and classification task, but it often operates as a "black box" making the processes difficult to interpret. This lack of clarity can make it harder for DHNN to work well with different datasets and handle real-world problems effectively. In the previous studies, have focused on systematic and non-systematic logical rules, optimization methods and logic mining techniques to improve DHNN performance. However, challenges such as rigid logical structures, limited neuron flexibility and insufficient optimization of learning and retrieval phases still remain.

Problem Statement 1: The limited flexibility of SAT formulations restricts neuron connectivity, reduces the effectiveness of synaptic weight optimization and increases redundancy and overfitting thereby constraining the overall performance of DHNN models.

Explanation Problem statement 1: Neurons in the DHNN model operate as core components of the network by acting as both input and output units. In this context, the structural flexibility of neurons serves as a critical performance factor in DHNN model operations since it allows the network to find stable solutions more efficiently. Significantly, the SAT contains a structured method to arrange neurons since it transforms into a logical structure which the network must satisfy. In this case, the formulation of the SAT is important and must be done carefully to create an effective neural network model. In earlier research, Kasihmuddin *et al.* (2017c) and Mansor *et al*. (2017) focused on formulating SAT based on systematic logical rules such as 2 Satisfiability (2-SAT) and 3 Satisfiability (3-SAT). This SAT formulation is restricted to *k*- literals per clause in the logical structure. However, this restriction limits the connectivity of neurons and reduces the flexibility of the logic structure. To tackle this issue, Sathasivam *et al*. (2020b) introduced Random 2 Satisfiability (RAN2SAT) the first non-systematic SAT logical rule. The RAN2SAT logical rule where *k* is less than and equal to 2 allows for more variation in the range of synaptic weights. However, this formulation consists of first-order and second-order terms without any conditional determination clauses which restricts interactions between neurons and reduces the flexibility of the overall logical structure. Then, the higher-order logical structure known as Random 3 Satisfiability (RAN3SAT) proposed by Karim *et al*. (2021) incorporates first-order, second-order and third-order clauses. The inclusion of higher-order logic in the RAN3SAT structure enhances better neuron connections. However, the equal distribution of different order of clauses can lead to unnecessary repetition in neuron connection which makes the network unclear and causes the overfitting solution. Therefore, selecting clauses within the logical structure should aim for a broader, more balanced and flexible set of neuron connections to improve overall effectiveness. A well-formulated logical structure increases the probability of satisfied interpretations while maintaining a zero-cost function. By minimizing the cost function, the model can better optimize synaptic weights, encourage greater neuron variation and reduce the risk of overfitting.

Problem Statement 2: The restricted combination of SAT in DHNN reduces the flexibility of neuron connections and limits the interpretability of the models output.

Explanation Problem statement 2: Given the 'black box' nature of DHNN, implementing symbolic rules to guide the neural connection schema is crucial for enhancing its interpretability and performance. As an optimization tool, DHNN requires the implementation of logical rules to improve its performance, efficiency and reliability. Current implementations of SAT in DHNN exhibit non-systematic approaches leading to unique development opportunities. Building upon this, Alway *et al*. (2022) introduced Major 2 Satisfiability (MAJ2SAT) with a specific clause ratio and implemented into DHNN to learn logic with a higher proportion of second-order clauses compared to third-order clauses. Although, the MAJ2SAT introduces more variation in logical rules and has a much lower similarity compared to systematic logical rules the lack of first-order logic makes it challenging for the DHNN to classify data effectively. Recently, Gao *et al.* (2022) proposed a flexible higher-order logical structure called G-Random 3 Satisfiability (GRAN3SAT) to be incorporated into the DHNN model. This method prioritizes a flexible and randomized logical structure within the DHNN model. However, the randomized clauses may result in a tendency to produce the repetitive final neuron variation which potentially lead to an overfitted solution in the DHNN model. Similarly, Jiang *et al*. (2024) introduced the J-Type Random 2,3-Satisfiability into the DHNN model by combining second-order and third-order clauses to reduce learning errors and minimize global energy. However, the randomization and combination of higher-order clauses can lead to uncertain satisfied interpretations which potentially affect the global solution and reduce diversity in the final neuron state of the DHNN model. Additionally, this study did not investigate the first-order logic which is very important to the DHNN model that is able to learn lower-order logic to provide a more diversified solution. Therefore, proposing a logical structure with a more structured, flexible and balanced distribution of clauses is essential to ensuring the satisfiability property while enhancing diversity in the DHNN model.

Problem statement 3: DHNN faces challenges in achieving an efficient optimization mechanism which leading to non-optimal synaptic weights, overfitting and reduced diversity in final neuron states.

Explanation Problem statement 3: DHNN serves regularly as a solution method for combinatorial optimization problems. Both learning and retrieval phases within DHNN require optimization because this leads to better operational efficiency for the system. The effective optimization of the learning phase plays a vital role in reducing the cost function and producing optimal synaptic weights. Similarly, the optimization efforts during the retrieval phase to identify the quality of the final neuron state because it is equally important to the learning phase. In earlier optimization work, Kasihmuddin *et al*. (2019) focused on enhancing the retrieval phase of the DHNN by incorporating a mutation-based approach through the Estimation Distribution Algorithm (EDA) in the *K*-Satisfiability logic. This method aimed to enhance global search capabilities by estimating potential neuron states that correspond to the global minimum energy. However, this approach had several limitations especially, in the learning phase where the use of Exhaustive Search (ES) frequently led to non-optimal synaptic weights. This happened because ES relies on a trial-and-error mechanism to identify satisfied interpretations based on logical rules. Furthermore, this mentioned work only focuses on the single objective function. Another study, by Sathasivam *et al*. (2020a) explored a different approach to optimize DHNN by introducing an Election Algorithm (EA) in the learning phase of the Random *k* SAT (RAN*k*SAT) model. The proposed EA in the learning phase of the RAN*k*SAT to get more optimal synaptic weight and zero-cost function even with a large number of neurons. However, this work focused solely on optimizing the learning phase and did not address improvements in the retrieval phase which remains highly dependent on initial states that can increase the risk of the model getting trapped in local minima and experiencing overfitting. In recent studies, Karim *et al*. (2022) and Alway *et al*. (2023) tackled the challenge of overfitting in DHNN by introducing multi-objective functions during the learning phase. These approaches generated multiple strings of the neuron state that exhibited both high fitness and diversity. To achieve this, both of the works incorporated multiple Content Addressable Memory (CAM) units into the DHNN learning phase. However, the number of CAM units was limited to five which lead the network to often retrieve repetitive final neuron states. Therefore, additional mechanisms need to be developed to optimize both the learning and retrieval phases of the DHNN in order to avoid the tendency towards a repetitive final neuron state. Expanding the diversity of final neuron states while achieving a global minimum solution is essential for enhancing the DHNN as an intelligent optimization model. Implementing a multi-objective function during the retrieval phase can effectively identify final neuron states that exhibit high fitness, diversity and dissimilarity. Once all components of the proposed model are in place, a comprehensive performance evaluation is necessary. Testing the model with real-world datasets, similar to other developing data mining models will provide valuable insights into its practical applicability and effectiveness.

Problem statement 4: Current logic mining models in DHNN rely heavily on frequency-based approaches that prioritize true positives that resulting in biased induced logic and overlooking true negatives which limits accurate knowledge representation and classification effectiveness.

Explanation problem statement 4: In logic mining, logical rules enhance the interpretability of the DHNN model which subsequently boosts the accuracy and effectiveness of the classification model. The final retrieved neuron is translated into a logical rule known as induced logic to strengthen knowledge representation rather than depending on the "black box" approach. Additionally, evaluating the DHNN-SAT ability to correctly identify true positives (*TP*) and true negatives (*TN*) is crucial in the broader context of knowledge extraction as this assessment validates the effectiveness of the logical rules. In the earlier research in logic mining, Sathasivam & Abdullah (2011) introduced the reverse analysis method by implementing HornSAT into Discrete Hopfield Neural Networks. However, the redundant literals produced in HornSAT have limited the knowledge extraction process through induced logic due to the redundant information. To overcome these issues, 2SATRA (Kho *et al*., 2020), 3SATRA (Zamri *et al*., 2020) and E2SATRA (Jamaludin *et al*., 2021b) initiated to improve the existing RA that generates the best induced logic for representing dataset behaviour by using accuracy values as validation metrics. All these mentioned models, rely on a frequency-based approach to select the best logic. The best logic is then determined by considering the highest frequency of 2SAT or 3SAT clauses within the entire learning dataset. Consequently, the retrieved induced logic is biased to only true positive classification. However, focusing solely on true positives in generating the best logic may not lead to an optimal outcome. This approach can create an imbalance especially when the dataset contains more true negatives than true positives. Therefore, considering both *TP* and *TN* is essential in the logic mining model to achieve more accurate results without missing negative cases from real-life datasets. This approach aims to extract the most accurate induced logic, reflecting the overall behaviour of the dataset more effectively without happening any misclassification.

problem statement 5: Existing logic mining models in DHNN often rely on a single induced logic or limited mechanisms in the learning and retrieval phases which restricts diversity, reduces interpretability and prevents the generation of multiple high-quality induced logic for more accurate classification.

Explanation problem statement 5: In the perspective of the logic mining producing the best logic is essential as it represents the optimal connections between neurons with accurately tuned synaptic weights. High-quality best logic directly impacts the effectiveness of retrieving the induced logic, since it reflects a comprehensive understanding of the dataset overall pattern. In the case of DHNN, having optimal logic enhances the network ability to learn the data efficiently. This increases the likelihood of retrieving high-quality induced logic during the retrieval phase. Initially, Kasihmuddin *et al.* (2022) suggested statistical analysis which is a correlation in the pre-processing phase as an additional component in logic mining to discover the relationships between data set attributes known as supervised logic mining or (S2SATRA). This method obtains optimal attributes that will be presented in the DHNN. Although, S2SATRA was able to increase the significant improvement in the logic mining perceptive the model failed to produce the different optimal induced logic. Continuously, Jamaludin *et al*. (2022) explored another logic mining by incorporating conventional 2SATRA based on a log-linear analysis known as A2SATRA. Even though, the log-linear method was employed to extract the most relevant attributes but the A2SATRA model lacks of permutation operator which limits the exploration of different attribute combinations reduces diversity and potentially resulted in suboptimal performance of the learning phase. Then, Jamaluddin *et al*. (2023) explored logic mining method within the DHNN-SAT model by using 2SAT known as Permutation in 2 Satisfiability Based Reverse Analysis (P2SATRA) which incorporates an additional operator called Permutation. This operator rearranges attributes to form clauses in the proposed logic by expanding the search space for optimal induced logic. However, the P2SATRA did not have any mechanism in the learning phase to minimise the cost function. As a result, the DHNN may lead to improper synaptic weights being stored in CAM. Ultimately, these issues could prevent P2SATRA from achieving the best induced logic. Recently, Alway *et al.* (2023) introduced a learning algorithm within the logic mining model known as 2HESRA, which incorporates a Hybrid Exhaustive Search algorithm in the learning phase. Meanwhile, Zamri *et al*. (2024) proposed another logic mining approach involving non-systematic logic that can tune the best logic into a "super logic" using a Modified Niched Genetic Algorithm (MNGA). This method focuses on second-order and first-order clauses as weighted random 2 Satisfiability Modified Reverse Based Analysis (r2SATMRA). Both logic mining model approaches select the optimal induced logic based on the highest summation of positive and negative outcomes. However, these models did not incorporate additional components in the retrieval phase and relied solely on a single induced logic which can limit the interpretability of the induced logic. Therefore, to enhance the DHNN as an intelligent optimization model it is essential to develop multiple induced logic with both true and false classifications within the learning data. Each of these optimal best logics should prioritize achieving the highest performance metrics relative to the learning dataset. The criteria used in this best logic will directly influence the effectiveness of the induced logic in the retrieval phase. By considering a broad range of best logic possibilities, the extracted induced logic can offer deeper insights into the dataset which can reflect a comprehensive analysis of potential solutions. The multiple induce logic can capture different aspects of the data which leading to more accurate in classification task without missing any information.

1.5 Research Questions

This question addresses a significant challenge in the DHNN which is applied in various domains. The research question is a fundamental step in identifying the research problem and establishing the objectives of this research. These research questions as a guide to direct the exploration address the problem statement within the scope of the research field. In section 1.4, the research problem is outlined by this research question which guides the proper methods and results for this research scope.

(a) What is the formulation of a non-systematic satisfiability that effectively controls clause distribution and highlights the advantage of the high proportion order satisfiability logic?

(b) How can embed a proposed non-systematic logical rule into Discrete Hopfield Neural Networks to improve the transparent of neuron connectivity?

(c) In the context of optimization, which additional operators in learning algorithms can enhance the retrieval phase to ensure high-quality final neuron states by achieving multi-objective function?

(d) In the context of logic mining, how can the best logic be formulated based on the confusion matrix to ensure the best logic effectively extracts information from the dataset?

(e) How to improve in the current logic mining model to generate multiple induced logics from different performance metrics that can classify and extract knowledge effectively?

1.6 Research Objectives

The primary objectives of this thesis focus on modeling a DHNN using a higher-order logical rule with a specific ratio for generating first and third-order clauses. The learning phase of the DHNN aims to minimize the cost function by optimizing synaptic weights. To address the limitations during the learning and retrieval phases the proposed model incorporates a learning algorithm to achieve multi-objective functions. During the retrieval phase, the proposed hybrid operator in the learning algorithm expands to avoid repetitive final neurons. Ultimately, the proposed model will ensure that the model is able to forecast the real-world problem in every situation. The specific objectives of this thesis are as follows:

Objective 1: To formulate a new non-systematic logical rule named Major Random 1,3 Satisfiability by combining third-order and first-order logic into a single logical rule. The proposed logical rule considers higher ratio of third-order clauses as compared to the first-order clauses which will highlight the advantage of the high proportion of third-order satisfiability logic.

Objective 2: To propose the implementation of Major Random 1,3 Satisfiability logic into the Discrete Hopfield Neural Network as symbolic rule. In this approach, each literal in the Major Random 1,3 Satisfiability represents as neuron in the Discrete Hopfield Neural Network. The implementation of this approach will provide transparent observation of the neuron in the network.

Objective 3: To propose a Hybrid Artificial Bee Colony Algorithm in the retrieval phase of the Discrete Hopfield Neural Network. This proposed algorithm includes mutation operator from the Genetic Algorithm. In the context of the optimizing retrieval phase, the final neuron state is to achieve multi-objective functions in terms of global solutions, diversified solutions and dissimilarity.

Objective 4: To construct a new computation method to generate the best logic during the learning phase of logic mining. The optimal best logic will be selected based on the highest values at the selected performance metrics. Selecting the best logic with the highest performance metric enhances the effectiveness of information extraction from the dataset.

Objective 5: To generate a multi-unit Major Random 1,3 Satisfiability Reverse Analysis Method that considers five different best logic as the initial pattern remembered at the learning phase. This approach will extract the multiple induced logic that represents the overall behavior of the dataset and support the accurate recognition of meaningful patterns.

1.7 Methodology and Limitations

This section introduces three main methodologies along with the associated limitations to address the research questions outlined in Section 1.3. First, a major higher-order clause logical rule is proposed as a non-systematic logical rule. Second, this higher-order logical rule is embedded into the DHNN to regulate the behavioral responses of higher-order neurons. Third, the Hybrid Artificial Bee Colony (HABC) algorithm is applied to enhance the retrieval phase of DHNN, aiming to achieve global solutions and promote diversity in the final neuron states. Finally, the logic mining approach developed in this thesis integrates all components of Objective 3 to generate a Multi-unit Major Random 1,3 Satisfiability Reverse Analysis method within the DHNN. In detail, the specific methodologies and limitations of this thesis are explained as follows:

Methodology for Objective 1: The fundamental idea of this formulation of the logical structure is based on the previous work by Chang *et al*. (2025) using Boolean algebra and expressed in Conjunctive Normal Form (CNF) where the literals are connected through the OR operator and clauses are connected through the AND operator. Each clause in the proposed SAT will be connected with a disjunction that contributed to the final outcome of the Major Random 1,3 Satisfiability In the context of logic satisfiability in DHNN, the formulation of the logical structure has increased particularly in the non-systematic logical rule which offers a similar distribution of the clauses within the logical structure (Sathasivam *et al.,* 2020b, Karim *et al*., 2021) and thus limits the flexibility of the logical structure. Even though there is a bias distribution within the clauses for second-order clauses and third-order clauses by Alway *et al.* (2022), the advantages of having a high proportion of third-order clauses remain uncertain. Therefore, in this thesis the formulation of the non-systematic Major Random 1,3 Satisfiability was inspired by the proposed work by Alway *et al.* (2022) which highlights the major proportion of second-order clauses within the logical rule. The formulation of the Major Random 1,3 Satisfiability incorporates a high ratio of the third-order clauses compared to the first-order clauses. In this context, the key advantages of the third-order clauses within the logical rule will be highlighted.

Limitation of Methodology 1: Although this methodology might offer the best approach to generate the suggested Major Random 1,3 Satisfiability, there are several limitations have been set to ensure that the first objective can be accomplished. Firstly, the formulation only focuses on the third-order clauses more than 70% compared to the first-order clauses to achieve a higher probability of satisfied interpretation. Secondly, the literal in the clauses does not have redundancy in the Major Random 1,3 Satisfiability and must be in the CNF. The redundant literal leads to inefficiencies and increased complexity during data processing. Furthermore, redundant literals have disadvantages in terms of the optimal synaptic weight. Thirdly, this analysis of higher-order clauses focuses specifically on first- order and third-order clauses excluding second-order clauses from the consideration. Lastly, neurons should be represented in bipolar form as -1 and 1 to prevent the elimination of terms in the Lyapunov energy function within the DHNN model. Representing neuron states as (0, 1) can lead to inaccurate minimum energy due to the value of zero, which is not suitable for this energy function.

Methodology for Objective 2: The method to emphasize objective two where is the Major Random 1,3 Satisfiability embedded into the DHNN model. Therefore, the model needs to evaluate the learning phase by emphasizing the ES Algorithm to get optimal synaptic weight before evaluating the connection between the neurons in the retrieval phase. Secondly, the WA method is used instead of the traditional Hebbian learning strategy to determine the best synaptic weights. This decision is taken to minimize the creation of spurious memories and to decrease oscillations of neurons. Once the optimal synaptic weights of the logical structure are obtained, then the goal is to reach the global minimum energy state within the logical structure. During the retrieval phase in the DHNN, the Hyperbolic Tangent Activation Function (HTAF) is an activation function. The purpose of employing HTAF is to facilitate the convergence of the final neuron states while preventing neuron oscillation, as highlighted by (Abdeen et al., 2023). HTAF is known for the stability and is widely recognized as one of the most reliable activation functions for the implementation of DHNN. This thesis utilized the Sathasivam relaxation rate to introduce a "pause" in the simulation to prevent neuron oscillation.

Limitation of Methodology 2: This thesis exclusively focused on non-systematic logic in the DHNN without any inclusion of hidden layers. This approach may limit the DHNNs capacity to capture complex patterns and relationships within the data that the hidden layers typically play a pivotal role in the learning data structure. Moreover, the research exclusively employed the HTAF activation function in the DHNN, but has certain limitations (Alway e*t al*., 2023). For instance, the DHNN model may suffer from the vanishing gradient problem which can hinder the convergence and learning process. The HTAF defines an appropriate energy function in the DHNN which is often based on the Lyapunov function designed for binary states (-1, 1). In this thesis, a constraint was applied with the relaxation parameter set to a fixed value of 3 in the DHNN as proposed by (Sathasivam *et al.,* 2010). However, the fixed value may not be universally suitable for all datasets or learning tasks. It would have been beneficial to explore different relaxation values to evaluate the impact on network performance.

Methodology for Objective 3: The selection of the search algorithm plays a crucial role in determining the quality of the final neuron. This thesis, utilized an ES as the chosen search algorithm in objective 2. However, it is important to note that the iterative and exploratory nature of ES often referred to as "trial and error" can impact the effectiveness of minimizing the cost function (Kasihmuddin *et al*., 2017c). The number of neurons increases in ES fails to retrieve the optimal synaptic weights. This is which can affect the retrieval phase performance and lead to solutions trapped in local minima. To address issues employed by the EA proposed by Sathasivam *et al*. (2020a) during the learning phase and the Hybrid Artificial Bee Colony (HABC) algorithm during the retrieval phase aim to achieve multi objectives in this thesis. This learning algorithm enables the achievement of the quality final neuron state and surpasses the negated clauses by maximizing the overall neuron variations. This thesis introduces two variants of the Artificial Bee Colony algorithm inspired by the work of Sidik *et al.* (2022), where bitwise operations (NAND) replace arithmetic operations in the update rule from the original ABC algorithm. Once an optimal bitwise operation for the logic is determined, the model performance is compared with ten benchmark metaheuristics in the DHNN such as Genetic Algorithm (GA) Kasihmuddin *et al.* (2017a), Clonal Selection Algorithm (CSA), Evolutionary Differential Algorithm (EDA), EA, Whale Optimizing Algorithm (WOA), Black Hole Algorithm (BHA), and ABCJ by Jia *et al*. (2014), ABCK by Kasihmuddin *et al*. (2017b) and Sine Cosine Algorithm (SCA). In the learning phase of the DHNN, the EA is utilized to optimize the logical structure according to the major third-order clauses. The EA in the learning phase is limited in terms of computing the similar work of EA proposed by (Sathasivam *et al*., 2022a).

Limitation of Methodology 3: However, there are limitations to ensuring reproducibility such as randomizing the value of literals to obtain accurate synaptic weights and focusing on only two bitwise operations proposed by Sidik *et al.* (2022). This methodology provides an optimal approach to studying different bitwise operations in search space but there are a few limitations regarding the reproducibility of the objectives of this thesis. Firstly, the ABC algorithm operators emphasize exploitation more than exploration particularly in the early stages to avoid lower fitness solutions (Chang *et al.* 2025). Secondly, this thesis only considers two bitwise operations for improving the solution space in the retrieval phase which results in only finding wider search spaces. The HABC rate of generation is fixed at 1000. This generation rate in the HABC exploits a more satisfied interpretation of the proposed logical structure. Most importantly, the intelligent mutation operator only involves the third-order clauses to make the unsatisfied clauses a to be satisfied clauses. Lastly, the diversity function of the negativity in the final neuron states cannot be less than 50% of the total clauses because the impact of the negation cannot be seen (Karim *et al*., 2022).

Methodology for Objective 4: Logic mining is a key focus of this thesis that aims to get a systematic structure of the logic to represent information from datasets. Firstly, the logic mining method involves three layers including the pre-processing, learning phase and retrieval phase in the DHNN model. In the pre-processing layer, the dataset is split into a learning data set and a test set (Guo *et al*., 2025). The logic phase utilizes

Major Random 1,3 Satisfiability-based Reverse Analysis (MR1,3SATRA) to extract information from the dataset and generate logical rules based on correct classifications. Secondly, the logic mining model of the MR1,3SATRA method includes the permutation operator in the learning phases while using EA in the learning phase and HABC in the retrieval phase. This proposed logic mining model evaluates the real-life data set for the UCI Machine Learning Repository (UCI) and Kaggle websites with random selection only. The induced logic from the final neuron state is compared with the testing dataset and then the best-induced logic is selected based on performance evaluation using confusion metrics. The proposed model is compared with existing logic mining models. The pre-processing stage ends with randomly selecting attributes. Next, the proposed logic mining model introduces a new way of embedding the learning dataset into the logical rule. To evaluate this, the binary confusion matrix classification is checked based on how the dependent attribute in the learning dataset matches the outcome of the logical rule after embedding. Afterward, the logic with the highest accuracy is selected as the best induced logic. The proposed logic mining model is tested using 20 datasets from trusted repositories. It will then be compared to other existing models based on accuracy, sensitivity, specificity, Fowlkes-mallows index and Matthew correlation coefficient.

Limitation of Methodology 4: The limitation of objective four is the use of HTAF in the DHNN and not comparing the EA with other metaheuristics. However, this methodology provides an optimal approach to logic extraction from real-life datasets. In the validation of the data set, the multi-variant data set should contain more than 15 attributes to avoid missing insignificant attributes during the pre-processing phase. The assortment of data needs to be carefully considered. If the data is categorical, *k*-mean clustering cannot be used directly. It needs to be processed manually and change according to the nature of the information (Rusdi *et al*., 2025). This will lead to the execution of changing to bipolar forms using the frequency approach. The same train-test splitting and k-fold cross-validation ratio will be used for all models and held as consistent in fair comparison and reproducibility (Zamri *et al.,* 2024).

Methodology for Objective 5: Introducing the attribute selection method in the pre-processing phase which is Jaccard high similarity (JHS) to find significant attributes to enhance the learning process for the best MR1,3SAT in the DHNN. The JHS attribute selection method is the same as the work proposed by Zamri *et al.* (2024). The new logic mining model is enhanced from the element of Objective 4, known as multi-unit Major Random 1,3 Satisfiability Reverse Analysis. This method introduces multiple "best logics" where each one is associated with different performance metrics. Importantly, each of these best logics corresponds to a distinct CAM (Content Addressable Memory) in DHNN. The model utilizes several CAM units that offer multiple local field computations which results in several high-quality induced logics (Alway *et al*., 2023). This approach expands the search space and helps identify induced logics that avoid overfitting.

Limitation of Methodology 5: The initial limitation of objective 5 is that the process of selecting multi-unit induced logical rules relies on the choice of values for true positives and true negatives. Depending on a single set, the values of accuracy may not be sufficient for the complex task in the dataset which could result in suboptimal solutions. Additionally, the utilization of multi-unit optimal logical rules can lead to a sustainable extension of the search space within the DHNN. While this expanded search space is beneficial, especially for the complexity of the datasets it may also increase the challenge due to the increased computational task. Objective 5 only focusing the JHS approach to selecting significant attributes. Firstly, insignificant attributes from the dataset should be removed before translation into higher-order logical rules. Then, the logical structure of the Major Random 1,3 Satisfiability must be 10 literals from the attribute selection through JHS. The attribute selection is based only on the highest mean positive instance that is related to the predicted value. The complexity of the logic mining process can lead to faster and more efficiency in the Major Random 1,3 Satisfiability-based Reverse Analysis model in the learning phase.

1.8 Organization of Thesis

This thesis is organized into eight chapters. Chapter 1 introduces the study and provides an overview. Chapter 2 reviews literature on DHNN, Boolean Non-Systematic Satisfiability, Artificial Bee Colony Algorithms, optimization in DHNN retrieval phase and logic mining which is highlighting the research gaps. Chapter 3 explains the theoretical foundation while Chapter 4 details the implementation of Major Random *k*-Satisfiability logic in DHNN. Chapter 5 presents results through tables and graphs by comparing the proposed model with existing logical structure and algorithms. Chapter 6 uses reverse analysis method to achieve objectives four and five, evaluating the logic mining model with performance metrics. Chapter 7 applies the model to ADNI data for clinical outcome prediction, and Chapter 8 concludes with findings and future directions. Figures 1.2 and figure 1.3 illustrate the overall flow and key methods of the thesis.