CONDITIONAL RANDOM 2 SATISFIABILITY IN DISCRETE HOPFIELD NEURAL NETWORKS FOR ENHANCED LOGIC MINING

NURSHAZNEEM BINTI ROSLAN

UNIVERSITI SAINS MALAYSIA

2025

CONDITIONAL RANDOM 2 SATISFIABILITY IN DISCRETE HOPFIELD NEURAL NETWORKS FOR ENHANCED LOGIC MINING

by

NURSHAZNEEM BINTI ROSLAN

Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

June 2025

ACKNOWLEDGEMENT

In the name of Allah, the Most Gracious and the Most Merciful.

Alhamdulillah. Praises and thanks to Allah, the Almighty, for His blessings throughout my research journey and the successful completion of this thesis. I am deeply grateful to my supervisor, Assoc. Prof. Dr. Saratha Sathasivam, for the opportunity to pursue this research and for her invaluable guidance, support and encouragement. Her empathy and sense of humor made this journey more manageable and meaningful. My sincere thanks also go to all lecturers at the School of Mathematical Sciences, Universiti Sains Malaysia, for their helpful suggestions during seminars throughout my PhD.

I am eternally grateful to my parents, Roslan Arifin and Hasima Osman, for their prayers and sacrifices. My heartfelt thanks to my siblings, Nurazuwa and Nur ‘Aifaa, for their constant support. Special appreciation to my husband, Mohd Khairrul Amry Romli, for his patience, understanding and encouragement throughout this journey. I also extend my thanks to my friends, especially Farah Liyana Azizan, for her early support and patience. My heartfelt appreciation goes to my best friend, Nur ‘Afifah Rusdi, for the laughter, motivation and shared experiences that made this path more meaningful.

I extend my gratitude to the Dean of the School of Mathematical Sciences, Assoc. Prof. Dr. Ahmad Lutfi Amri Bin Ramli and the academic administrative staff for their support. I am especially thankful to Universiti Malaysia Perlis (UniMAP) for the Skim Latihan Akademik Bumiputera (SLAB) scholarship, which provided essential academic and financial support. To all who contributed indirectly to this research, your support is deeply valued. May Allah reward you for your generosity. Thank you very much.

TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

Page

LIST OF SYMBOLS

S_i	Neuron state
N	Set of all neurons
S_i^f	Final neuron state
$S_i^{\max}$	Benchmark neuron state
$(\wedge)$	Logical AND
$(\vee)$	Logical OR
k	Order of the clauses
NN	Number of neurons
NC	Total number of clauses
NT	Number of trials during the retrieval phase
NH	Number of trials during the learning phase
C	Number of neuron combination
P	Logical rule
P_{2SAT}	Logical rule of 2 Satisfiability
$P_{RAN2SAT}$	Logical rule of Random 2 Satisfiability
$P_{YRAN2SAT}$	Logical rule of Y-Type Random 2 Satisfiability
P_{r2SAT}	Logical rule of Weighted Random 2 Satisfiability
$P_{CRAN2SAT}$	Logical rule of Conditional Random 2 Satisfiability
m	Number of second order clause exist in the logical rule
n	Number of first order clauses exist in the logical rule
r	Ratio of negative literals
A_i	Positive literal
$\neg A_i$	Negative literal
Q_{ij}	The inconsistency of logic
E_P	The cost function of logic

W_{ij}	Synaptic weight for second-order clause
W_i	Synaptic weight for first-order clause
$H_P^{\min}$	The minimum energy of the network
H_P	The final energy of the network
h_i	Local field
Tol	Pre-determined tolerance value
$C_i^{(3)}$	Third-order clauses
$C_i^{(2)}$	Second-order clauses
$C_i^{(2)*}$	Second-order clauses for CRAN2SAT
$C_i^{(1)}$	First-order clauses
CR	Clone rate
$Whale(t)$	Candidate search agent
$Whale^{*}(t)$	Best search agent
$Whale_{rand}$	Randomly selected whale from the current population
A	Mathematical coefficients
C	Mathematical coefficients
D,	Distance between the search agent and the best search agent
t	Current iteration
P_{learn}	Learning dataset
P_{test}	Testing dataset
P_{kSAT}^{best}	Best logic of logic mining model
$P_{kSAT}^{induced}$	Induced logic of logic mining model
$P_{best}^{induced}$	Best induced logic of logic mining model
f_D	Fitness for diversification
f_Z	Fitness for global solution
f_{SI}	Fitness in minimizing the similarity indices
$Q\%$	Percentage level of negativity

$\left(S_{Bi^*}^{f}, S_{Di^*}^{f}\right)$	Optimized final neuron state for the second order clause
S_i^{fa}	Optimized final neuron state
$f_{C_i^{(2)*}}$	Fitness of second order clause
$f_{C_i^{(1)}}$	Fitness of first order clause
$n\left(Whale_i^*\left(S_i^f\right)\right)$	The number of duplications
$Pop\left(Whale_i\left(S_i^f\right)\right)$	Whale's population
$Pop^{New}\left(Whale_i\left(S_i^f\right)\right),$	New whale's population
GLI	Gower and Legendre similarity index
JSI	Jaccard similarity index
RT	Rogers-Tanimoto similarity index
$RTFSM$	Rogers-Tanimoto Feature Selection Method
$JFSM$	Jaccard Feature Selection Method
e_{ij}	Attributes of the raw data entries
$\overline{A_i}$	Mean value of each attribute
E_i	Raw data entries with respect to the attributes
P_i	Actual outcome of the dataset
$\left(Unique_{CR2SATRA}\right)$	Unique combinations
$P_{CR2SATRA}^{best}$	Best logic of CR2SATRA
$P_{CR2SATRA}^{induced}$	Induced logic of CR2SATRA
P_{best}^{I}	Best induced logic of CR2SATRA
ρ	Number of best logics trained by DHNN
$RMSE$	Root Mean Square Error
$MAPE$	Mean Absolute Percentage Error
f_{NC}	Maximum fitness
f_i	Current fitness
$NT_{Solution}$	Total number of the solution
$NG_{Solution}$	Number of global solutions

Z_m	Global minima ratio
TV	Total neuron variation
N_D	Number of diversified solutions
N_Z	Number of global solutions
N_G	Number of global minimum ratio
N_{DG}	Number of diversified and global solutions
$NS_{i_D}^{f}$	Number of Diversified solution string
$NS_{i_Z}^{f}$	Number of global solutions strings
S_i^{fB}	Benchmark neuron state initialize by $P_{CRAN2SAT}$
S_i^{fA}	Optimized final neuron state after the optimization
TP	True Positive Classification
TN	True Negative Classification
FP	False Positive Classification
FN	False Negative Classification
ACC	Accuracy
PRE	Precision
SPE	Specificity
MCC	Matthews Correlation Coefficient
MCR	Misclassification Rate
Γ_{best}^{I}	Dependent attribute
$\Gamma_{bestACC}^{I}$	Induced logic with maximum ACC
$\Gamma_{bestPRE}^{I}$	Induced logic with maximum PRE
$\Gamma_{bestSPE}^{I}$	Induced logic with maximum SPE
$\Gamma_{bestMCC}^{I}$	Induced logic with maximum MCC
$\Gamma_{bestMCR}^{I}$	Induced logic with minimum MCR

LIST OF ABBREVIATIONS

IPS	Institut Pengajian Siswazah
USM	Universiti Sains Malaysia
ANNs	Artificial Neural Networks
HNN	Hopfield Neural Network
DHNN	Discrete Hopfield Neural Network
AI	Artificial Intelligence
CAM	Content Addressable Memory
SAT	Satisfiability
WA method	Wan Abdullah method
DHNN-SAT	SAT with DHNN
HORNSAT	Horn Satisfiability
2SAT	2 Satisfiability
3SAT	3 Satisfiability
RAN2SAT	Random 2 Satisfiability
RAN3SAT	Random 3 Satisfiability
MAJ2SAT	Major 2 Satisfiability
YRAN2SAT	Y-Type Random 2 Satisfiability
*r*2SAT	Weighted Random 2 Satisfiability
CRAN2SAT	Conditional Random 2 Satisfiability
HTAF	Hyperbolic Tangent activation function
McP	McCulloch-Pitts function
NA	Non-applicable
CNF	Conjunctive Normal Form
DNF	Disjunctive Normal Form
ES	Exhaustive Search
GA	Genetic Algorithm
CSA	Clonal Selection Algorithm
DEA	Differential Evolution Algorithm
ABC	Artificial Bee Colony Algorithm
EA	Election Algorithm
SCA	Sine Cosine Algorithm

BHA	Black Hole Algorithm
WOA	Whale Optimization Algorithm
BWOA	Binary Whale Optimization Algorithm
HBWOA	Hybrid Binary Whale Optimization Algorithm
RA	Reverse Analysis
2SATRA	2 Satisfiability based Reverse Analysis
3SATRA	3 Satisfiability based Reverse Analysis
E2SATRA	Energy-based 2 Satisfiability Reverse Analysis
P2SATRA	2 Satisfiability based Reverse Analysis with Permutation operator
S2SATRA	Supervised Logic Mining model with Correlation analysis
A2SATRA	Supervised Logic Mining model with Log-linear analysis
CR2SATRA	Conditional Random 2 Satisfiability Reverse Analysis
Avg. Rank	Average Rank
SPSS	Statistical Package for the Social Sciences

LIST OF APPENDICES

2 SATISFIABILITI RAWAK BERSYARAT DALAM RANGKAIAN NEURAL HOPFIELD DISKRET UNTUK MEMPERTINGKATKAN PERLOMBONGAN LOGIK

ABSTRAK

Pemahaman mendalam mengenai mekanisme tambahan dalam Rangkaian Neural Hopfield Diskret penting untuk membangunkan model kecerdasan bagi aplikasi yang lebih luas. Tesis ini memperkenalkan 2 Satisfiabiliti Rawak Bersyarat, peraturan logik baharu yang menjamin sekurang-kurangnya satu operator logik negatif bagi setiap klausa peringkat kedua serta menggabungkan fungsi pengaktifan Smish yang tidak monotonik bagi meningkatkan proses pengemaskinian rangkaian. Selain itu, fungsi multi objektif Algoritma Pengoptimuman Ikan Paus Hibrid Binari digunakan dalam dapatan kembali untuk menghasilkan kepelbagaian penyelesaian neuron dengan penyelesaian global maksimum dan meminimumkan indeks kesamaan. Akhir sekali, tesis ini memperkenalkan pendekatan baru untuk mengenal pasti logik terbaik berdasarkan pelbagai metrik prestasi dalam model perlombongan logik yang dipanggil Analisis Berbalik Terubahsuai Dengan 2 Satisfiabiliti Rawak Bersyarat. Pendekatan ini penting apabila berurusan dengan set data yang tidak seimbang yang membawa kepada peningkatan ruang carian untuk mencari logik teraruh yang optimum. Model perlombongan logik yang dicadangkan menunjukkan keupayaan unggul dalam pengekstrakan pengetahuan sekali gus mempertingkatkan pemahaman set data. Kajian kes menunjukkan nilai purata maksimum 1.0000 bagi *Ketepatan* dan *Kekhususan*. Ini menunjukkan keupayaan model dalam mengklasifikasikan negatif dan positif dengan tepat. Penemuan ini menyumbang kepada pemahaman yang lebih baik tentang model kecerdasan selaras dengan visi Malaysia Madani untuk inovasi dan pembangunan.

CONDITIONAL RANDOM 2 SATISFIABILITY IN DISCRETE HOPFIELD NEURAL NETWORKS FOR ENHANCED LOGIC MINING

ABSTRACT

A deeper understanding of additional mechanisms in Discrete Hopfield Neural Network is essential for developing intelligent model for broader applications. This thesis introduces Conditional Random 2 Satisfiability, a new logical rule guaranteeing the inclusion of at least one negative logic operator for each second-order clause which implements the non-monotonic Smish activation function to enhance the updating process within the network. Furthermore, multi-objective function optimization using a Hybrid Binary Whale Optimization Algorithm is employed during the retrieval phase to generate diversified neuron states with maximum global solutions and minimized similarity indices. Finally, a new approach for identifying the best logic based on various performance metrics in the logic mining model called Conditional Random 2 Satisfiability Reverse Analysis is proposed. This approach is significant when dealing with imbalanced datasets leading to an enhanced search space for finding optimal induced logic. The proposed logic mining model demonstrates superior performance in knowledge extraction leading to a more comprehensive understanding of the datasets. The results of the case study show a maximum average value of 1.0000 for both *Precision* and *Specificity*. This highlights the improved capability of the proposed model in accurately classifying both negative and positive classifications. These findings contribute to a better understanding of intelligent models, aligning with Malaysia Madani vision for innovation and development.

CHAPTER 1

INTRODUCTION

1.1 Overview

This chapter introduces the Artificial Neural Networks and Discrete Hopfield Neural Network. By presenting the background of Artificial Neural Networks and addresses the challenges faced by existing Discrete Hopfield Neural Network, this chapter establishes a foundation for understanding the research problem that forms the basis of this thesis. Additionally, this chapter outlines the structure and organization of the subsequent sections. This chapter will provide a roadmap to navigate through the research and gain a comprehensive understanding of the methodology used to address the research problem. Furthermore, this chapter also specifies the research objectives and discusses the methodology and scope of the thesis.

1.2 Fundamentals of Artificial Neural Network

Artificial Neural Networks (ANNs) have become a critical area of research and application in artificial intelligence and machine learning. ANNs are computational models inspired by the structure and functioning of the human brain, which consists an extensive number of neurons that communicate with each other through synaptic connections. Additionally, an ANNs is composed of interconnected nodes known as artificial neurons which organized into input, hidden and output layers. Each neuron performs a weighted sum on the input neuron and subsequently the activation function has been utilized to generate the outputs for the neuron. These artificial neurons perform computations based on the programming and configuration of the ANNs. The ANNs models are designed to process data, learn from it and use the acquired knowledge to make predictions or decisions. Adaptive learning enables ANNs to continuously improve their performance by processing more data. This allows ANNs to effectively adapt to variations in input and optimize the decision-making process.

Due to their adaptability and learning capabilities, ANNs are utilized in numerous fields including speech recognition (Świetlicka et al., 2022), medical natural language processing (Pandey et al., 2022), medical imaging (Kim et al., 2019) and financial forecasting (Wu et al., 2022). However, the training process of an ANNs can be extensive and time-intensive particularly with large datasets. To operate efficiently, ANNs typically require considerable volumes of data because insufficient data can lead to decreased in performance effectiveness. Although ANNs have made significant advancements and are applied in various fields, there are still numerous areas that require further exploration such as training methods, data optimization techniques and the identification of potential application domains. The exploration of research in these areas provides new insights for future research within the field of ANNs.

Understanding the reasoning behind improvements to existing components of ANNs can often be challenging. Although many AI researchers have made significant advancement in enhancing ANNs, difficulties arise when clear justifications are needed to explain why and how the outputs of these models are produced. This challenge primarily arises from the perception of ANNs as black-box models which is a consequence of their characteristic lack of transparency (Si-Ahmed et al., 2023). Therefore, it is crucial to pursue an understanding of how to improve the interpretability of ANNs by developing innovative approaches to explain the reasoning behind additional mechanisms implemented in ANNs models. Recognizing both the potential and possible risk of ANNs emphasizes the importance of *pursuit understanding of intelligence* through enhancements that improve the effectiveness and transparency of ANNs models.

A deep understanding of the mechanisms behind the intelligent system is essential because understanding intelligence requires more than just replication. This requires not only investigating and analyzing how intelligent systems can be replicated but also identifying the mechanisms that determine the effectiveness of the model. Consequently, understanding intelligence requires not only achieving high performance but also being able to explain the reasoning behind those results and provide insights into AI decision-making (Miller, 2019; Si-Ahmed et al., 2023). Achieving this deeper understanding leads to prioritize transparency and encourages the development of ANNs models which illustrate how additional mechanisms within ANNs contribute to optimal performance.

Therefore, a deeper understanding of the additional mechanisms and techniques that can be implemented into existing ANNs models is essential for broader applications. This lead to a more significant contributions to AI research. Additionally, a comprehensive exploration of the structure of intelligence can help researchers identifying and addressing limitations not only in current ANNs models but also in other types of ANNs such as the Discrete Hopfield Neural Network (DHNN). Among the various types of ANNs, the DHNN has been widely applied to solve optimization problems. However, DHNN faces limitations in interpreting network outputs for partitioners during practical applications. Therefore, the following section will discuss in detail the challenges present in existing DHNN.

1.3 Challenges in Discrete Hopfield Neural Network

The Discrete Hopfield Neural Network (DHNN) is a type of recurrent neural network that was popularized by Hopfield & Tank (1985). DHNN has gained significant popularity due to the ability to optimize over discrete time steps making it suitable for various applications. The DHNN consists of neuron states in binary form $\{1,0\}$ or bipolar form $\{-1,1\}$. Each neuron in the DHNN is fully connected to all other neurons within the network. These connections known as synaptic weights (W_{iN}) indicate the strength of the connection between neurons. Figure 1.1 illustrates the general structure of the DHNN which consist of a set of N neurons. In this network, $\{x_1, x_2, ..., x_N\}$ for $i = 1, 2, ...N$ represents the neuron in the network and S_i represents the output of the neuron x_i. In Figure 1.1, the lines are represented in different colours indicating the values of the synaptic weights that represent the connections between neuron states.

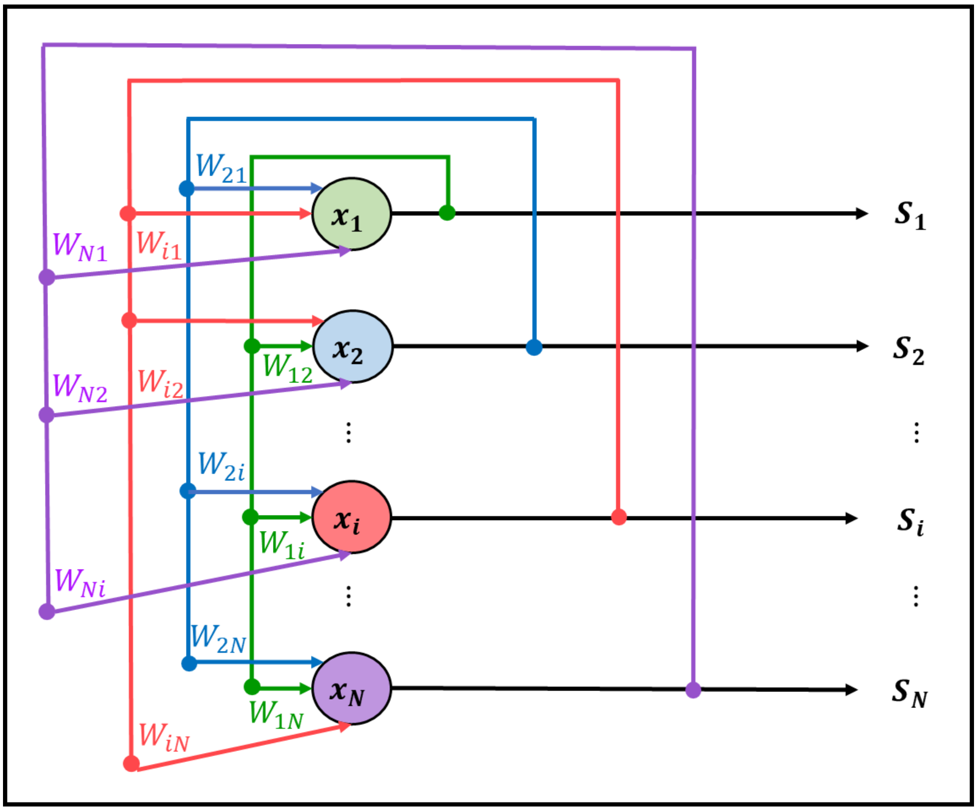

Figure 1.1 General structure of DHNN

In DHNN, the updating process is performed asynchronously indicate that the neurons update their states individually which eliminates the possibility of neuron oscillations. The Lyapunov energy function is a mathematical tool for analyzing the stability of a network. It is based on the concept that the energy of a network tends to decrease over time. The Lyapunov energy function in DHNN is employed to evaluate the updated neuron state based on the interconnected neurons and their corresponding synaptic weights. Thus, the asynchronous update rule ensures that the energy function will eventually reach a minimum value which corresponds to a stable state of the network (Hopfield & Tank, 1985). According to Kasihmuddin et al. (2019), the DHNN converges to a state where the energy cannot be further reduced due to the monotonically decreasing nature of the Lyapunov energy function. A key characteristic of the DHNN is Content Addressable Memory (CAM) which has the ability to act as an associative memory system. This means CAM can store multiple patterns and later retrieve a complete pattern from partial or noisy inputs. Additionally, CAM serves as a dynamic storage system for storing the synaptic weights.

During the learning phase of the DHNN, the network learns patterns by adjusting the synaptic weights between the neurons. The synaptic weights are updated to minimize the energy function of the network which allows the DHNN to learn and store the patterns in its memory. However, DHNN does have its limitations in terms of the limited capacity for storing patterns making it challenging to process large datasets. This limitation can lead to underfitting issues which prevents it from capturing the complexities of the data (Alway et al., 2023). Furthermore, the DHNN tends to become trapped in local minima of the energy function causing the network to retrieved undesired final neuron states. As a result, this can lead to suboptimal neuron states due to the ineffective learning and retrieval phase which limits the ability of the DHNN to retrieve high-quality final neuron states.

The implementation of logic satisfiability in DHNN presents an opportunity for further research and investigation. Developing an optimized DHNN model governed by symbolic logical rules could expand the capabilities of DHNN model beyond solving the Travelling Salesman Problem (Hopfield & Tank, 1985) to applications such as classification tasks. Implementing the symbolic logical rules into the DHNN model can provide a deeper understanding of intelligence leading to a clearer interpretation of the outputs. According to Abdullah (1992), symbolic logic plays a crucial role in DHNN as it governs the representation of the neurons. In this context, the set of solutions that minimize the energy function of the network is equivalent to the set of truth assignments satisfying the Satisfiability (SAT) logical rule. Currently, researchers are exploring various types of logical rules with DHNN (DHNN-SAT) to gain a better understanding of neurons in the network with interpretable inputs and outputs.

Firstly, the existing DHNN-SAT model has limitations in the structure of the logical rules. While various types of logical rules have been proposed within the context of systematic logical rules (Kasihmuddin et al., 2017a; Mansor et al., 2017) and non-systematic logical rules (Guo et al., 2022; Sathasivam et al., 2020; Zamri et al., 2022), these logical rules emphasize the full capability of logical combinations. This consideration can lead to inconsistent interpretations and overlook the role of the negative literals within the logical rules. Consequently, the appearance of the negative literals within the logical rules remains uncertain. Secondly, Hyperbolic Tangent Activation Function (HTAF) is the current activation function in DHNN-SAT model as proposed by Mansor & Sathasivam (2016). The used of HTAF results in overfitted solutions due to the vanishing gradient problem which causes the output values to approach zero (Dubey et al., 2022). As a result, the final neuron state retrieved by the network will be overfitted. Therefore, an activation function that does not suffer from the vanishing gradient problem is needed to enhance the updating rule and produce highly diversified final neuron states.

Thirdly, having a wider range of final neuron states is beneficial for logic mining. According to Zamri et al. (2022), a broader range of final neuron states helps in solving real-world classification tasks. It is essential to systematically investigate the impact of optimization during the learning and retrieval phase of the DHNN to address this issue. An optimal learning phase will ensure that the DHNN model generates optimal value of synaptic weights (Kasihmuddin et al., 2017b; Roslan et al., 2023). However, having an optimal learning phase alone does not guarantee the quality of the retrieved final neuron state. Therefore, optimizing the retrieval phase by considering the diversification of negative states in the final neuron state can produce high-quality solutions facilitate the identification of a wider range of logical patterns within the context of logic mining.

Furthermore, when dealing with imbalanced datasets, implementing a new approach in the logic mining model will lead to a more comprehensive understanding and enable the DHNN-SAT model to retrieve optimal logical patterns. In simple terms, DHNN-SAT model can be considered as a highly intelligent computational model when appropriate mechanisms are implemented into the DHNN-SAT model. By pursuing an understanding of intelligence, additional mechanisms can enhance the DHNN as a robust computational model that can perform effective classification tasks. Thus, this contributes to a broader philosophical standpoint in this thesis which is:

"Enhancing the flexibility of intelligence through symbolic representation"

This philosophical standpoint highlights the use of symbolic representation to improve the capability of AI model and transforming it become a flexible intelligence model. This flexibility allows users to control both inputs and outputs which consequently make the AI model more adaptable in handling different classification tasks across various fields. The symbolic representation offers a promising approach by providing mechanisms for knowledge extraction which can make AI decision-making more transparent and understandable to humans. Implementing additional mechanisms through symbolic representation is particularly relevant for DHNN models which are often considered as a black box models. By implementing symbolic representation, AI model can explore a broader solution space and identify high-quality solutions which similar to how AI model enhances efficiency and decision-making across various fields. Therefore, developing flexible intelligence models is crucial for creating AI model capable of addressing a wider range of applications.

This thesis presents an intelligent DHNN model enhanced with symbolic representation to increase the flexibility of intelligent models. This enhancement is achieved through several key implementations. First, by enhancing the structure of non-systematic logical rules to focus on the appearance of the negative literals, the flexibility of the logical rules is acknowledged. Additionally, the effectiveness of the updating rule in the DHNN is significantly improved and thus facilitating multi-objective optimization during the retrieval phase to expand the solution space for optimal solutions attained by the model. This highlights the capability of symbolic representation in navigating a broader solution space contributes to a global solution with a high-quality. Finally, from the perspectives of logic mining model, the multi-unit of DHNN model which emphasizes various confusion matrix illustrates the flexibility of intelligent models in effectively performing classification tasks. The discussion of each of these perspectives will be elaborated in the problem statements section, demonstrating how symbolic representation enhances the flexibility of intelligent models.

1.4 Problem Statements

This thesis aims to develop a flexible and intelligent DHNN model that provides flexibility in performing classification tasks and interpreting the behaviour of datasets across various fields. Developing an optimal classification model capable of performing knowledge extraction presents a significant challenge. Therefore, this section will outline the five main problems that contribute to the development of the flexible and intelligent DHNN model.

Problem Statement 1

Symbolic rules play a crucial role in the study of DHNN, as they define the logical connections between neurons within these networks. The application and exploration of symbolic rules have been extensively investigated in the context of non-systematic second-order logic in DHNN. Significant research has focused on understanding and enhancing the performance of these networks through different logical rules. Currently, most applications of non-systematic second-order logic operate under the unrestricted use of negative literals as seen in Random 2 Satisfiability (RAN2SAT) by Sathasivam et al. (2020) and Y-Type Random 2 Satisfiability (YRAN2SAT) by Guo et al. (2022). Furthermore, Sathasivam et al. (2020) emphasized that the inflexibility of the logical structure in *k*SAT can lead to overfitting solutions causing the network only converges to only one type of solution. Although Weighted Random 2 Satisfiability (*r*2SAT) by Zamri et al. (2022) controls the distribution of the negative literals, there is need for a deeper understanding of how these negative literals within the second-order clauses influence neuron connections in DHNN. Additionally, further investigation is required to explore the impact of the neuron states retrieved by the network. Therefore, it is necessary to identify and implement restrictions within the logical rules to examine the effects of negative literals within the second-order clauses in representing neuron connections in DHNN. Thus, the challenge lies in identifying and implementing a restricted conditions within the existing non-systematic second-order logic to closely examine the effects of negative literals. Establishing these restrictions could potentially provide a more systematic approach for the non-systematic second-order logic and enhance understanding of its impacts in DHNN.

Problem Statement 2

Due to the black box nature of DHNN, a logical rule is essential for governing the neuron connections within the network and ensuring optimal neuron representation. Ineffective implementation of the logical rule in DHNN contributed to poor synaptic weights management leading to a suboptimal final neuron state. The Weighted Random 2 Satisfiability (*r*2SAT) introduced by Zamri et al. (2022) employs a Genetic Algorithm (GA) in the logic phase to determine the distribution of negative literals. However, the proposed logic phase has increased the complexity of the existing DHNN (Romli et al., 2024). To address this complexity, Romli et al. (2024) introduced Weighted Systematic 2 Satisfiability (*rS*2SAT) logical rule which includes features to control the distribution of negative literals. *rS*2SAT aims to enhance the diversification of negative literals within the logical rules and encourages the network to learn negative neuron connections. Despite this improvement, the understanding of how negative literals within second-order clauses in non-systematic logic influences the DHNN when driving towards a global minima solution remains insufficient. As number of neurons increases, the storage capacity in CAM will also increases because more patterns are stored in the network (Alway et al., 2023). However, increases in storage capacity presents challenges due to the complexity of fully capitalizing on the possible combinations of clauses. Due to this issue, DHNN tends to retrieve suboptimal final neuron state. Therefore, in the context of non-systematic logic, it is crucial to understand the influence of negative literals within second-order clauses. This understanding is important because negative neuron connections could potentially improve the capabilities of the network in getting satisfied interpretation leading to an optimal and diversified final neuron state. Thus, implementing a restricted condition within the existing non-systematic second-order logic reduces the storage complexity and subsequently increase neuron connections in DHNN.

Problem Statement 3

Despite the recognized application of DHNN in various fields, the efficiency and effectiveness of the updating rules that utilize conventional activation functions have been insufficiently investigated. The HTAF is a current activation function in logic satisfiability introduced by Mansor & Sathasivam (2016) fails to promotes diversification among the retrieved final neuron states. Additionally, HTAF suffers from the gradient vanishing problem which causes the output values approaches zero (Dubey et al., 2022). From the perspective of logic satisfiability in DHNN, the negative impact of having zero output values during the updating rule leads to a repetitive final neuron state. This consequently results in overfitting of final neuron states. According to Abdeen et al. (2023), a high diversity of final neuron state enables DHNN to locate more global minima solutions in the other solution spaces. However, the repetitive final neuron state will lead to a high similarity between the neuron state and contributes to a low diversification in the retrieved final neuron states by the network. Achieving a high diversification in the retrieved final neuron states is advantageous for logic mining which can be applied across numerous fields (Zamri et al., 2022). To address these challenges, it is crucial to introduced and evaluate a non-monotonic activation function into the DHNN. The proposed activation function should not suffer from the gradient vanishing problem to ensure that the output values will not drastically approach zero values. This enhancement is expected to improve the effectiveness of the updating rule in DHNN resulting a higher variation and diversification among the retrieved final neuron states.

Problem Statement 4

During the retrieval phase, the final neuron state generated by the DHNN is evaluated to determine whether the solution achieved is global minima solution or local minima solution. During this phase, the retrieved final neuron state is asynchronously updated by applying the value of synaptic weights stored in the CAM. The quality of the final neuron state is often overlooked in most studies of DHNN because the main focus is typically on reaching the final state. It is worth noting that Kasihmuddin et al. (2019) conducted pioneering work to enhance the retrieval phase by implementing an Estimation of Distribution Algorithm (EDA). This implementation aimed to increase the diversified states and thus exploring a wider range of global minimum energy within the search space. However, as the number of neuron increase, the proposed work by Kasihmuddin et al. (2019) is unable to achieved optimal final neuron state. This limitation arises due to the adaptability of Exhaustive Search (ES) as a learning algorithm leading to randomly generated synaptic weights (Sathasivam, 2010). Therefore, the solution will be trapped in the local minima solution. Moreover, this research focusses solely on analyzing the quality of the final neuron state without considering the variability in negative states. Considering the limitations of the previous study, the current approach will struggle to retrieve diversified induced logic which will be beneficial for the logic mining model (Zamri et al., 2022). To address this issue, there is a need for a systematic investigation into the impact of optimization during both the learning and retrieval phases of DHNN. For this reason, the optimal quality of final neuron state can be obtained by acknowledging the appearance of the negative states.

The appearance of the negative states within the final neuron state is crucial as it expand the solution space of optimal solutions attain by the model.

Problem Statement 5

Logic mining is a subset of data mining technique that utilizes logic satisfiability to extract knowledge from complex and large datasets, providing valuable for decision-making across various fields (Sathasivam & Wan Abdullah, 2011). Poor attribute selection can lead to unreliable patterns and inaccurate classifications. As an example, Kho et al. (2020) and Zamri et al. (2020) proposed random attribute selection method that resulted to the inclusion of insignificant attributes. According to Zamri et al. (2024) the proposed logic mining approaches must be able to select the attributes based on the degree of similarity between the attributes and the logical outcomes. Zamri et al. (2024) utilized the Jaccard similarity index as a feature selection method namely as Jaccard Feature Selection Method $(JFSM)$. Unfortunately, $JFSM$ only considers positive attribute entries and ignoring potentially significant negative relationship in the datasets. Thus, a comprehensive approach must utilize both positive and negative entries to better capture dataset relationships. In logic mining model, generating the best logic is crucial because these logical rules are trained by the network to extract the behaviour of the datasets. Existing logic mining model generate the best logic based on highest frequency clauses with positive outcomes (Jamaludin et al., 2021, 2022, 2023; Kasihmuddin et al., 2022; Kho et al., 2020; Sathasivam & Wan Abdullah, 2011; Zamri et al., 2020). Unfortunately, this approach lead to bias solutions when dealing with imbalanced datasets. Additionally, relying on a single induced logic limits the ability of the logic mining model to handle wide range of real-world datasets. To address these limitations, the proposed solution is to develop a new logic mining model based on the Reverse Analysis (RA) method which introduces a new approach for generating the best logic and alternative approach for attribute selection. The aim is to systematically extract knowledge, identify relationships between attributes and outcomes, and improve the overall performance of the logic mining model.

1.5 Research Questions

To ensure consistency between the problem statements outlines in Section 1.4 and the research objectives of this thesis, this section presents several important research questions that address various aspects of the proposed work. These research questions are based on previously identified problems specifically the need to enhance the flexibility of DHNN model to adapt to different data structures. Consequently, this thesis aims to develop a flexible and intelligent DHNN model with symbolic representation to improve both adaptability and performance. The research questions addressed in this thesis are listed as follows:

(a) What is the restricted condition that needs to be embedded with the existing non-systematic second-order logic to examine the effect of negative literals within second-order clauses?

(b) What is the alternative formulation and structure of non-systematic logic that control the appearance of negative literals within the second-order clauses that is compatible with Wan Abdullah method and able to represent the neuron connections in Discrete Hopfield Neural Network?

(c) How does an activation function that overcomes the gradient vanishing problem influence the capability of the updating rule in Discrete Hopfield Neural Network to generate diversified final neuron states?

(d) What is the impact on the quality of the final neuron state when applying nature-inspired optimization algorithms during the retrieval phase of Discrete Hopfield Neural Network with consideration for variations in negative literals?

(e) What approach can be implemented into the current logic mining model to ensure that a set of induced logics obtained from different confusion matrix is effective in doing classification tasks?

1.6 Research Objectives

This thesis aims to propose a flexible and intelligent DHNN model enhanced with symbolic representation. This is achieved through several additional implementations. First, a new non-systematic logical rule is formulated by excluding both positive literals in second-order clauses while maintaining flexibility for first-order clauses. This logical rule is embedded in DHNN to assess the effectiveness of the proposed logical rule during the learning and retrieval phases. To enhance the quality of retrieved final neuron states, a non-monotonic activation function is proposed to improve the updating rule in DHNN. Since the quality of retrieved final neuron states relies on the retrieval phase, nature-inspired optimization algorithms act as symbolic representation enabling multi-objective optimization to expand the solution space and achieve a high-quality final neuron state. Finally, a new approach in generating the best logic in the logic mining model aims to extract patterns from real-life datasets for classification tasks. Consequently, the objectives of this thesis are presented as follows:

(a) To formulate a new non-systematic logical rule namely Conditional Random k Satisfiability, where $k=1,2$ that uses first-order and second-order logic without including both positive literal within the second-order clauses. The proposed Conditional Random 2 Satisfiability will demonstrate the role of negative literal within the second-order clauses.

(b) To implement the proposed Conditional Random *k* Satisfiability into a Discrete Hopfield Neural Network by minimizing the logical inconsistency of the logical rule that corresponds to the zero-cost function. The derived cost function that corresponds to the proposed logic will govern the behaviour of the Discrete Hopfield Neural Network.

(c) To propose a non-monotonic Smish activation function to improve the effectiveness of the updating rule in Discrete Hopfield Neural Network that introduce more variability and diversity in the final neuron states. The performance of the Smish activation function will be evaluated with different types of activation functions during the retrieval phase of Discrete Hopfield Neural Network.

(d) To propose multi-objective function for Hybrid Binary Whale Optimization Algorithm in the retrieval phase of Discrete Hopfield Neural Network. The proposed metaheuristic algorithm will be utilized to optimize the final neuron states with aims of getting more negative states. This approach will improve the capability of retrieved diversified and optimal final neuron states with more negative states which will be good influenced in the perspective of logic mining.

(e) To propose multi-unit Conditional Random 2 Satisfiability Reverse Analysis method that consider five different confusion matrixes in the formulation of the best logic and introduces an unsupervised learning approach for attribute selection in doing classification task. This approach will extract the patterns in the form of induced logic that has the capability to classify and perform knowledge extraction from the datasets.

1.7 Methodology and Scopes

This thesis aims to develop a classification model within the DHNN framework that has the capability in classifying and extracting knowledge from the datasets. In this section, five main methodologies that contribute to the development of the DHNN model will be explained. The performance of the model will be validated using a simulated dataset and the capability of the DHNN model in performing knowledge extraction will be tested with benchmark datasets.

Methodology 1

Research on logic satisfiability in DHNN is currently categorized into systematic and non-systematic logic. The primary distinction between the systematic and non-systematic logical rules lies in the utilization of different combinations of logical clauses which directly impact the connection strength of the logical rules. Many researchers seek to enhance the non-systematic logical rules in different ways. The flexibility of non-systematic logic characterized by the unrestricted logical structure allows for the adjustments and modifications based on specific needs or situations which significantly enhance the capability of the network. The proposed logic in this thesis is inspired by the work of Zamri et al. (2022), who introduced the r2SAT logical rule. This logical rule employs a logic phase to ensure the correct distribution of negative literals within the logical structure which can enhance the neuron connection within the network. However, there is limited research on the role and significance of having high appearance of the negative literals within second-order clauses. From this perspective, this thesis formulates the Conditional Random k Satisfiability which utilizes both first-order clauses and second-order clauses without including both positive literal in the second-order clauses. This formulation highlights the role and significance of having high appearance of the negative literal within the second-order clauses.

Scope Methodology 1

While this methodology has the potential to optimally generate Conditional Random 2-Satisfiability, there are several areas that must be addressed to ensure the reproducibility of the primary objectives of this thesis. To guarantee replicability, certain constraints need to be recognized. Firstly, the formulation of the new logical rule is limited to first-order and second-order clauses $\left(k=1,2\right)$. This constraint facilitates the analysis of the effects of high appearance of the negative literals within second-order clauses. Secondly, redundant literals are excluded from all the clauses in the proposed logical rules. According to Someetheram et al. (2022), the existence of redundant literals in the logical structure can result to inconsistent logic which may lead to a non-satisfiable logic. Finally, the structure of the positive literals within the second-order clauses are comprehensively eliminated from the possible combination of the proposed logical rules. This approach ensures that the proposed logical rule consistently includes at least one negative literal within the second-order clauses.

Methodology 2

The proposed Conditional Random 2 Satisfiability will be embedded as a logical rule within the DHNN. Each literal in the Conditional Random 2 Satisfiability will represent neuron in the network. Within this framework, the connections between the neurons which referred to as synaptic weights will be computed using WA method (Abdullah, 1992). According to WA method, the value of synaptic weight is determined by comparing the coefficient of the cost function associated with the proposed

Conditional Random 2 Satisfiability logical rule to the Lyapunov energy function. The process of logic satisfiability in DHNN consists of two main phases generally referred to as the learning phase and the retrieval phase. During the learning phase, the primary objective is to minimize the inconsistency of the proposed logic which corresponds to achieving a zero-cost function. This process will ensure that a satisfied interpretation is obtained and as a result, optimal value for the synaptic weight will be obtained. The value of the synaptic weights will be stored in CAM as a matrix form. In contrast, during the retrieval phase, the retrieved final neuron state generated by the DHNN is evaluated to determine whether the obtained solution is classified as global minima solution or local minima solution. Consequently, the performance of the proposed logic within DHNN can be examined.

<u>Scope Methodology 2</u>

While this methodology presents an optimal approach for developing DHNN model governed by a logic, several aspects need to be addressed to ensure the reproducibility of the objectives outlined in this thesis. Firstly, the proposed logic excludes positive literals within second-order clauses but imposes no restrictions on first-order clauses. This constraint provides a systematic method for dynamically updating the negative states of neurons within second-order clauses governed by DHNN. Secondly, all clauses for the Conditional Random 2 Satisfiability must be represented in Conjunctive Normal Form (CNF) with the literals within each clause are connected in Disjunctive Normal Form (DNF). If the logic is in DNF form, there will be only one event that makes sense for the interpretation which reduces the interpretability. In addition, the CNF-DNF structure is crucial for employing the WA method (Abdullah, 1992) to calculate synaptic weights. Additionally, the learning phase of the proposed logical rule in the DHNN employs Exhaustive Search (ES) as the learning algorithm to tests the capability of the proposed logical rule in finding a satisfied interpretation by minimizing the cost function. Finally, the final neuron state retrieved by the DHNN is updated based on HTAF (Mansor & Sathasivam, 2016). Thus, the performance evaluation considers ES as the learning algorithm during the learning phase and HTAF as the updating rule in the retrieval phase.

Methodology 3

The activation function in the DHNN plays a critical role in retrieving the final neuron states which significantly influencing the dynamics of the network and its ability to retrieve information. This thesis highlights the importance of utilizing an optimal activation function to enhance the effectiveness of the updating rule in DHNN which consequently leading to a diversified final neuron state. Currently, the HTAF proposed by Mansor & Sathasivam (2016) has been employed for logic satisfiability in DHNN. Although HTAF can achieve a global minima solution, but the quality of the final neuron states was not in terms of diversified states. This limitation arises from the tendency of HTAF suffers from the gradient vanishing problem where output values converge towards zero (Dubey et al., 2022). In the context of logic satisfiability in DHNN, this can lead to overfitting issues as the retrieved final neuron states will converge to a repetitive state. To address this issue, this thesis investigates the potential of the non-monotonic Smish activation function as an updating rule in DHNN aiming to increase the diversification of the retrieved final neuron states. A comparative evaluation will be conducted to measure the performance of the proposed activation function as compared against traditional and benchmark activation functions. Additionally, the Smish activation function will be applied to different logical rules to assess its adaptability. A comprehensive evaluation will be performed to assess the efficiency of the updating rule, the diversification of the final neuron states and the overall impact of implementing the Smish activation function.

Scope Methodology 3

While this methodology presents an optimal approach for generating diversified final neuron states, several considerations must be addressed to ensure the reproducibility of the objectives outlined in this thesis. First, the proposed non-monotonic Smish activation is embedded in the retrieval phase of the DHNN as an updating rule to retrieve diversified final neuron states. However, no single activation function is universally suitable for all neural networks or problems across different domains. Second, this thesis only compares the proposed non-monotonic Smish activation with the existing activation function that has been utilized in the context of logic satisfiability in DHNN, such as the McCulloch Pitt function used by Abdullah (1992), and the Elliot Symmetric activation function and HTAF were employed by Mansor & Sathasivam (2016). Additionally, other activation functions with similar properties to the non-monotonic Smish activation function and HTAF have been included for comparison. In this context, the efficiency of the proposed activation is evaluated by comparing its performance with these alternative activation functions.

Methodology 4

As the number of neurons increases, the logic satisfiability in DHNN becomes ineffective due to the difficulty of obtaining a satisfied interpretation during the learning phase leading to suboptimal states. To address this, this thesis proposes using metaheuristic algorithms to optimize both learning and retrieval phases, aims to enhance satisfied interpretations and generating diversified final neuron states. Election Algorithm (EA) is employed in the learning phase to minimize the cost function resulting an optimal synaptic weight stored in the CAM. Sathasivam et al. (2020) introduced binary EA by utilizing a binary operator to align with fitness function of DHNN. Additionally, the Hybrid Binary Whale Optimization Algorithm (HBWOA) is employed during the retrieval phase to improve the capability of retrieve state that diversified, global and minimum similarity index. The HBWOA proposed in this thesis is an improved version of the Binary Whale Optimization Algorithm (BWOA) proposed by Hussien et al. (2020). HBWOA implements cloning operator from the Clonal Selection Algorithm (CSA) (Zamri et al., 2020) and a mutation mechanism from the Genetic Algorithm (GA) (Kasihmuddin et al., 2017a) to further enhance the performance and effectiveness in achieving multi-objective optimization. The cloning operator is used to replicate the best solutions in the population enabling the algorithm to explore a wider solution space and discover better outcomes. While, a mutation mechanism from GA is introduced where solutions that do not meet fitness criteria will undergo greedy mutation. Therefore, by utilizing both cloning and mutation operators, the proposed HBWOA optimizes the retrieval phase of DHNN by generating high-quality of final neuron states. To evaluate the quality of the final neuron states, a diversity analysis will be conducted before and after applying the metaheuristic. Additionally, the proposed HBWOA will be compared with evolutionary and swarm-based algorithms to demonstrate the improvements of HBWOA.

Scope Methodology 4

Although this methodology is an effective strategy for achieving diverse final neuron states, several aspects must be addressed to ensure the reproducibility of the objectives outlined in this thesis. First, the learning phase of DHNN utilizes the Election Algorithm (EA) by Sathasivam et al. (2020) due to the capability of EA in minimizing the cost function and generate global minima solutions with a maximum number of neurons. In this context, the binary operator was applied to modify several functions in the EA to align with the fitness function of DHNN. Additionally, unlike the approach by Karim et al. (2022) that optimize the learning phase of DHNN, the HBWOA is employed to optimize the final neuron state with the aim of getting more negative states. This approach will improve the capability of retrieved diversified and high-quality neuron states. Finally, to preserve the optimal solutions obtained before optimization, the first-order clauses of the logical rule are excluded from the updating process of HBWOA. This ensures that the optimal solutions achieved during the learning phase remain unaffected while the HBWOA focuses solely on improving solutions for the second-order clauses. Since there are no trade-offs between the multi-objective optimization, this thesis is not associated with Pareto optimality. By addressing these scopes, this research ensures that the proposed method consistent with the defined methodology.

Methodology 5

In the field of logic mining, the objective is to retrieve an optimal induced logic that accurately describes the relationship between input features and the output variable with specific focus on classification tasks. In this thesis, a novel logic mining model called as Conditional Random 2 Satisfiability Reverse Analysis method (CR2SATRA) is proposed to address the limitations in previous logic mining models (Jamaludin et al., 2021, 2022, 2023; Kasihmuddin et al., 2022; Kho et al., 2020; Sathasivam & Wan Abdullah, 2011; Zamri et al., 2020). The aim of CR2SATRA is to enhance the overall performance and effectiveness of the previous logic mining models by introducing several modifications in the pre-processing phase, learning phase and retrieval phase. First, during pre-processing phase, CR2SATRA employs Rogers-Tanimoto Feature Selection Method $\left(RTFSM\right)$ which acts as an attributes selection method to eliminate irrelevant attributes that have low similarity to the class of the dataset. Next, CR2SATRA use *k*-fold cross-validation to help the logic mining model to explore a broader search space (Alway et al., 2023). Additionally, CR2SATRA generates only unique logical combinations of CRAN2SAT to expand the search space while avoiding learn the redundant logical rules. To address the limitation of existing logic mining models which often identifying the best logic based on the highest frequency of clauses associated with a positive outcome, this thesis introduces new approach of generating the best logic. The best logic is now generated based on five different confusion matrix such as Accuracy, Precision, Specificity, Matthews Correlation Coefficient and Misclassification Rate. As a result, CR2SATRA produces a set on induce logic based on different confusion matrixes. For example, if the best logic is generated based on the highest value of Accuracy, then the optimal induce logic that is produce by the logic mining model is based on the metric Accuracy. This approach expands the search space offering flexibility in doing classification tasks by producing different sets of induced logic. Additionally, CR2SATRA implements logical permutation as proposed by Jamaludin et al. (2023) to further enhance the search space and increase the probability of identifying the most optimal induced logic.

<u>Scope Methodology 5</u>

While this methodology offers an optimal approach for extracting logic from real-world datasets, several considerations must be addressed to ensure the reproducibility of the objectives outlined in this thesis. First, the dataset used in this thesis were obtained from UC Irvine Machine Learning Repository and Kaggle Machine Learning Repository which did not require any pilot study. It is assumed that the selected datasets are appropriate for classification tasks. Second, the criteria of the dataset focus on the dataset with more than 100 instances and between 11 to 40 attributes. These criterions