MUTATION TABU SEARCH USING SYSTEMATIC PROBABILISTIC TWO SATISFIABILITY IN A DISCRETE HOPFIELD NEURAL NETWORK

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by

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LIST OF SYMBOLS

h_i	Local field
$E_{(t)}$	Lyapunov energy function
$C_i^{(k=1)}$	First-order logical clause
$C_i^{(k=2)}$	Second order logical clause
$C_i^{(k=3)}$	Third order logical clause
$p(ch_i)$	Genetic probability
F_{2SAT}	General formula 2SAT
$P_{PRO2SAT}$	General formula PRO2SAT
$C_{PRO2SAT}$	Cost function of PRO2SAT
$f_{\max}$	Total number of clauses
f_i	Number of satisfied clauses
$D_{satisfaction\text{-}learn}$	Logic combination satisfaction of learning phase
$G_{PRO2SAT}$	Number of global minimum energies
SL_{test}	The final state combination of neurons for which the global solution can be found (subject to de-duplication)
S_i	State of neuron *i*
W_{ij}	Synaptic weight between neuron *i* and *j*
$E_{PRO2SAT}$	Energy function of the PRO2SAT
η	Ratio of positive literal
n	Number of the clauses
m	Number of iterations before $f_{\max}=f_i$
a	Number of neuron combinations
b	Number of trials
c	Number of learnings
R	Relaxation rate

p_l	Distributed control probability
θ	DHNN threshold
NN	Number of neurons
CMC	Confusion Matrix of Clause
MAE	Mean Absolute Error
$LRTR$	Logic formula satisfaction Ratio of learning phase
ZM	Global minimum Ratio
SS	Solutio space of retrieval phase
$f_{PPRO2SAT}(S_i)$	Fitness value of logic
$S_{initial}$	Initial solution of Mutation Tabu Search Algorithm
X_{best}	Local optimal solution
G_{best}	Global optimal solution
$NS*$	Candidate solution
TBT	Tabu table
TBS	Tabu operation
L	The length of Tabu table
χ^2	Chi square
df	Degree of freedom
O_{ij}	Observed frequency
E_{ij}	Expected frequency
A_{ij}	Edge distribution of two dimensional table row variables
G^2	Likelihood ratio
α	Confidence interal
$P_{PRO2SAT}^{best}$	Best logic of PRO2SATRA
ACC	Accuragy
$P_{PRO2SAT}^{induce}$	Induced logic of PRO2SATRA
TP	Ture positive

FP	Flase positive
TN	True negative
FN	Flase negative
DC_j	Control error for each logic
s	Number of segments
ns	Number of neighborhood operation
nm	Number of mutated neurons
sr	Selection rate
mr	Mutation rate
mi	Maximum number of iterations
X_α	Head wolf
X_β	Subordinare wolf
X_δ	Common wolf
X_ω	Bottom wolf
S_i^{bs}	Benchmark state
$C_{S_i^{bs},S_i}$	Multiple benchmark state
GLI_{avg}	Mean similarity of consistency interpretations
CT	Computation time
AI	Average iterations
$TV_{similar}$	The average similarity of solutions
F_d	Friedman statistical analysis
EAS	Ensemble attribute selection
NV	Number of Variable
NPV	Negative predictive value
MCC	The matthews correlation coefficient
MAE	Mean Absolute Error
H_0	Null hypothesis

LIST OF ABBREVIATIONS

2SAT	2 Satisfiability in Discrete Hopfield Neural Network
2SATRA	2 Satisfiability Reverse Analysis Method
3SAT	3 Satisfiability in Discrete Hopfield Neural Network
3SATRA	3 Satisfiability Reverse Analysis Method
A2SATRA	Log-linear Analysis 2 Satisfiability Reverse Analysis
ABC	Artificial Bee Colony
ACO	Ant Colony o Optimization
AI	Artificial Intelligence
ANN	Artificial Neural Network
CAM	Content Addressable Memory
CNF	Conjunctive Normal Form
DE	Differential Evolution
DHNN	Discrete Hopfield Neural Network
E2SATRA	Energy-based Reverse Analysis Method with 2 Satisfiability
EA	Election Algorithm
EAS	Ensemble Attribute Selection
EDA	Estimation Distribution Algorithm
ES	Exhaustive Search
G3SATRA	Log-Linear-Based Logic Mining with Multi-Discrete Hopfield Neural Network
GA	Genetic Algorithm
GRAN3SAT	G-type Random 3 Satisfiability
GWO	Grey Wolf Optimization
HEA	Hybrid Election Algorithm
HORN2SAT	Horn 2 Satisfiability in Discrete Hopfield Neural Network
HTAF	Hyperbolic Tangent Activation Function
*k*SAT	*k* Random 2 Satisfiability
L2SATRA	Log-Linear 2 Satisfiability Reverse Analysis Method
MAJ2SAT	Major 2 Satisfiability
MAX2SAT	Maximum 2 Satisfiability
MAX3SAT	Maximum 3 Satisfiability
MAX*k*SAT	Maximum *k* Satisfiability

M-P	McCulloch-Pitts Model
MTS	Mutation Tabu Search algorithm
P2SATRA	Permutation 2 Reverse Analysis Method
PRO2SAT	Probabilistic 2 Satisfiability in Discrete Hopfield Neural Network
PRO2SATRA	Probabilistic 2 Satisfiability Reverse Analysis Method
PSAT	Probabilistic Satisfiability
PSO	Particle Swarm Optimization
*r*2SAT	Weighted Random 2 Satisfiability
RA	Reverse Analysis
RAN2SAT	Random 2 Satisfiability in Discrete Hopfield Neural Network
RAN3SAT	Random 3 Satisfiability in Discrete Hopfield Neural Network
RAN*k*SAT	Random *k* Satisfiability in Discrete Hopfield Neural Network
RWS	Roulette Wheel Selection
S2SATRA	Supervised 2 Satisfiability Reverse Analysis Method
SA	Simulated Annealing Algorithm
SAT	Satisfiability
SHoRA	Modified Supervised 3 Satisfiability Reverse Analysis Method with Multi Unit of Discrete Hopfield Neural Network
TSP	Traveling Salesman Problem
UCI	University of California Irvine
WA	Wan Abdullah Method
YRAN2SAT	Y-type Random 2 Satisfiability in Discrete Hopfield Neural Network

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PENCARIAN MUTASI TABU MENGGUNAKAN SISTEMATIK 2 SATISFIABILITI BERKEBARANGKALIAN DALAM RANGKAIAN NEURAL HOPFIELD DISKRET

ABSTRAK

Logik satisfiabiliti telah digunakan secara meluas dalam penyelidikan rangkaian kecerdasan buatan. Namun begitu, kajian sedia ada masih belum membangunkan strategi berkesan untuk mengawal taburan literal dalam formulasi. Oleh itu, tesis ini mencadangkan model 2 Satisfiabiliti Kebarangkalian yang mengawal kuantiti dan kedudukan literal positif melalui peraturan positif yang digunakan pada pembolehubah dalam klausa peringkat kedua. Logik ini seterusnya dilaksanakan ke dalam rangkaian neural Hopfield diskret sebagai representasi neuron. Hasil kajian menunjukkan pendekatan kebarangkalian meningkatkan prestasi model dengan 92.3% kebarangkalian untuk mencapai tenaga minimum. Dalam fasa pembelajaran, algoritma Mutasi Carian Tabu Hibrid diperkenalkan bagi memastikan fungsi kos menumpu kepada sifar pada pelbagai nisbah literal positif. Hasil kajian menunjukkan algoritma ini hanya memerlukan purata 10.8 iterasi dengan kerumitan masa yang minimum. Dalam fasa perolehan, peraturan mengemaskini neuron dan strategi mutasi baharu diperkenalkan untuk meningkatkan kepelbagaian penyelesaian, dan hasil kajian menunjukkan bahawa purata kesamaan menurun kepada 0.58. Akhirnya, model perlombongan logik baharu iaitu Analisis Berbalik 2 Satisfiabiliti Kebarangkalian diperkenalkan untuk mengekstrak corak optimum. Model ini mencapai purata kejituan 0.869 dan digunakan dalam kajian kes data Mobile Gold Course. Kajian seterusnya akan meneliti struktur klausa lebih kompleks bagi meningkatkan kebolehsuaian dan kebolehumuman dalam masalah klasifikasi.

MUTATION TABU SEARCH USING SYSTEMATIC PROBABILISTIC TWO SATISFIABILITY IN A DISCRETE HOPFIELD NEURAL NETWORK

ABSTRACT

Satisfiability logic has been extensively applied in artificial neural network research. However, existing studies have not developed effective strategies for controlling the distribution of literals in the formulations. Therefore, this thesis proposes a Probabilistic 2 Satisfiability model that controls the quantity and position of positive literals through a positive rule applied to variables in second-order clauses. This logic is subsequently implemented into a discrete Hopfield neural network as a neuron representation. Results demonstrate that the probabilistic approach improves model performance with a 92.3% probability of reaching the minimum energy. In the learning phase, a Hybrid Mutation Tabu Search algorithm is introduced to ensure that the cost function converges to zero under different positive literal ratios. Results show that the algorithm requires an average of 10.8 iterations with minimal time complexity. In the retrieval phase, a new neuron update rule and mutation strategy are introduced to enhance solution diversity, and the findings indicates that the average similarity decreased to 0.58. Finally, a new logic mining model known as Probabilistic 2 Satisfiability Reverse Analysis is introduced to extracts optimal patterns through induced logic. The model outperformed with an average Accuracy of 0.869 and is applied on a case study of Mobile Gold Course dataset. Future work will explore more complex clause structures to improve the adaptability and generalizability of the model in handling classification problems.

CHAPTER 1

INTRODUCTION

1.1 Overview

In the current technological era, Artificial intelligence (AI) models have garnered widespread attention due to their ability to simulate or rationalize human intelligence through machines. This chapter provides a brief overview of Artificial Neural Networks to align with the context of this thesis. Subsequently, the thesis will address the statement of the research problem, introducing the research questions, objectives, methodology, and its limitations. Finally, the overall structure of the thesis will be outlined.

1.2 Artificial Intelligence

AI is an interdisciplinary field that involves computer science, cognitive science, mathematics, and statistics, aiming to develop human-like intelligence in machines so they can perform tasks that traditionally require human cognitive abilities. The combination of algorithms, data, and computational resources enables the gradual development of perception, learning, reasoning, decision-making, and adaptive capabilities. In recent years, continuous improvements in computational power have led to significant advancements in AI technology, allowing applications to expand into areas such as image recognition, autonomous driving, intelligent medical diagnosis, and machine translation (Chen *et al*., 2023). However, as models become more complex with increasing parameter sizes and nonlinear computations, the decision-making process has gradually evolved into what is commonly referred to as a black box.

The black box phenomenon means that understanding the reasoning behind accurate results remains challenging. This makes determining the presence of biases or hidden factors influencing the outcomes difficult. Addressing this challenge requires a deeper exploration of intelligence to enhance adaptability and decision-making capabilities. Intelligence can be understood as the ability to learn efficiently, apply reasoning effectively, and handle various situations in a specific environment. A deeper understanding of this process helps identify the limitations of AI models and provides a foundation for designing better optimization strategies. The No Free Lunch theorem further demonstrates that no universal optimal model exists, and different problems require different approaches, emphasizing the necessity of understanding intelligence. The implementation of multiple AI methodologies or the refinement of neural network structures and learning mechanisms can enhance the representation ability, learning capacity, and retrieval performance of artificial neural networks (ANN). As AI technology continues to expand across various fields, improving the interpretability of ANN has become a crucial aspect of building reliable and trustworthy AI systems. Among various ANN models, the Discrete Hopfield Neural Network (DHNN) has attracted significant attention in recent years because of the ability to effectively solve discrete optimization problems. The inherent black box characteristics of this network have led to the key considerations explored in this thesis.

Rule-based Understanding of AI

Understanding the behavior and interpretability of neural networks has long been a central topic in AI research (Always *et al*., 2022). In this regard, *rule-based* approaches provide an essential theoretical perspective. Rules play a crucial role in neural networks by influencing model behavior and determining network interpretability (Zamri *et al*., 2022a). Therefore, introducing appropriate rule mechanisms into neural networks is essential for uncovering the internal structure and dynamic relationships of Discrete Hopfield Neural Networks. In this context, symbolic logic rules have been widely applied as a powerful tool for the theoretical analysis of DHNN. These symbolic logic rules not only deepen the understanding of neural network behavior but also enhance interpretability and robustness by incorporating logical reasoning mechanisms. Currently, optimizing neural networks through rule-based approaches has become a research hotspot, attracting significant attention from scholars.

1.3 Symbolic Neural Networks

Symbolic Neural Network are a hybrid model that implements symbolic reasoning with ANN (Achler, 2014). This model enhances the learning process by injecting symbolic knowledge into the neural network, where symbolic reasoning handles logical reasoning and rule representation, while the neural network focuses on feature extraction and pattern recognition. This combination leverages the reasoning power of symbolic logic and the perceptual strengths of ANN, enabling better solutions to complex real-world problems. Typically, the problem to be solved is transformed into a logical expression that is fed into the neural network for resolution. In this context, the DHNN, as an important class of artificial neural network structures, gradually incorporates symbolic logic to improve neuron connectivity and optimize network behavior. Through appropriate symbolic integration, the network can enhance pattern storage and retrieval capabilities while maintaining stability, thereby providing more interpretable and controllable solutions for addressing complex problems.

Satisfiability (SAT) is designed as symbolic representations of neuron connections in the DHNN, also known as logical rules. Logical rules were first introduced into the network by Abdullah (1992). By minimizing the cost function, logical rules were successfully embedded into the neural network. The set of neuron states corresponding to the minimum energy function in the network inherently satisfies the satisfiability logical rules. This helps to better understand the dynamic processes of the network. Based on this, some researchers have embedded other logical rules into the DHNN, such as 2 Satisfiability (Kasihmuddin *et al.*, 2017a), 3 Satisfiability (Mansor *et al.*, 2017), and Random Satisfiability (Sathasivam *et al.*, 2020). As a result of these studies, the black-box characteristics of DHNN is gradually becoming more transparent. This choice allows DHNN to have interpretable inputs and outputs in neuron navigation, providing explainability. However, these rules are relatively constrained, mainly focusing on the structural constraints of clauses, while the positive and negative literals in Satisfiability logical rules are still generated randomly. This randomness somewhat hinders the interpretability of the neural network. Implementing symbolic rules into the neural network improves structural control and reduces randomness in learning, which enhances the consistency, interpretability, and stability of the model's outputs. To further enhance the transparency of DHNN, the research direction of this thesis is as follows:

Expanding the effect of positive-rule in AI based model

In recent years, increasing attention has been paid to the interpretability and robustness of intelligent models. In line with this trend, the expansion of positive-rule mechanisms has emerged as a promising strategy to guide model behavior through structured information (Zamri *et al.*, 2024). In the learning and decision-making process of intelligent models, expanding positive-rule requires integrating positive cases and positive information from multiple dimensions while not excluding the existence of negative cases and negative information. Based on this, the organization and management of positive information and positive cases should follow rules from different dimensions to ensure their effectiveness. By considering positive information from multiple perspectives, the model can analyze problem structures more accurately. The construction of positive rules provides a clear direction for the computational space. To explore how positive rules improve the overall performance of intelligent models, this thesis focuses on the following aspects:

The first perspective optimizes the satisfiability structure by regulating the quantity and distribution of positive literals. Traditional satisfiability logical rules generate positive and negative literals randomly (Sathasivam *et al*,. 2020), increasing diversity but also adding complexity and uncertainty, especially when the number of neurons becomes large. Introducing structured logical rules guides intelligent models into organized search paths, enhancing solving efficiency and interpretability. The second perspective embeds these structured logical rules into DHNN. While studies have implemented satisfiability logical rules into DHNN, forming DHNN-SAT, challenges remain in achieving fast convergence and stable states under different initial neuron state. To address this, the third perspective proposes optimization algorithms to enhance DHNN-SAT performance during learning and retrieval phases. These algorithms ensure logical rules work efficiently with DHNN, minimize the cost function, and lead neurons to the optimal final state. The fourth perspective develops a classification model based on these theories and evaluates real-world datasets to verify feasibility and effectiveness in practical applications.

1.4 Problem Statements

This thesis aims to construct an intelligent model with high interpretability for classification and knowledge extraction. The key challenge lies in the effective implementation and management of positive-rule information to ensure efficient operation and reliability. This section will specifically examine five core issues affecting the development of the proposed intelligent model.

Problem Statement 1: Ineffective dynamical rule to represent logical rule.

Explanation of Problem Statement 1: Due to the "black-box" nature of DHNN, current dynamic rules cannot effectively represent or control logical rules, reducing both interpretability and controllability. Logical rules in DHNN are classified as systematic or non-systematic. Systematic rules, such as 2 Satisfiability (2SAT) proposed by Kasihmuddin *et al*. (2017a), require each clause to contain the same number of literals. Non-systematic rules, such as Random 2 Satisfiability (RAN2SAT) introduced by Sathasivam *et al*. (2020), allow greater structural diversity. Alway *et al*. (2022) proposed Major 2 Satisfiability (MAJ2SAT) to emphasize the dominance of 2SAT clauses. However, neither type can control whether a literal appears in positive or negative form. The quantity and position of literals are generated randomly. This randomness weakens the mapping between logical rules and makes network behavior harder to interpret. Probability Satisfiability, an extended form of logical rules, assigns probabilistic constraints to literals and clauses to model logical relationships under uncertainty. However, existing variants such as Generalized Probability Satisfiability and Weight-based Probability Selection have shown benefits only in standalone satisfiability settings and lack dynamic mechanisms suited for the DHNN framework. This gap prevents probabilistic logical rules from guiding neuron state evolution, slowing convergence and reducing induced logic diversity in logic mining. Therefore, a dynamic rule is needed that can operate stably within DHNN, integrating probability satisfiability with neuron connection updates. This rule should control the distribution of positive literals to better represent logical rules and enhance both controllability and interpretability.

Problem Statement 2: Improper structural organization of symbolic rules hinders DHNN from achieving optimal synaptic connections.

Explanation of Problem Statement 2: The structure of symbolic rules is essential for effective neuron connectivity. In DHNN, improper application of logical rules may lead to imbalanced synaptic weight management and overfitting to a single solution, thereby limiting the diversity of the solution spaces. Therefore, each structural component of satisfiability must be accurately represented in DHNN once encoded as symbolic rules. The combination of DHNN and Satisfiability forms what is known as the DHNN-SAT model. Existing DHNN-SAT models are based on the Wan Abdullah method (WA method) proposed by Abdullah (1992), which achieves the implementation of DHNN and Satisfiability by minimizing the cost function during the learning phase. This approach has been successfully applied in various studies, such as 3SAT (Mansor *et al*., 2017), RAN2SAT (Someetheram *et al*., 2022), and MAJ1,3SAT (Manoharam *et al*., 2024). These studies indicate that the WA method can provide a consistent interpretation for logical rules. However, non-systematic logical rules expose two significant issues in the management of synaptic weights for first order clauses. Compared with second order and third order clauses, first order clauses are more difficult to satisfy and their presence reduces the overall satisfiability of logical rules. This limitation hinders DHNN from obtaining optimal synaptic weights and indirectly leads to local minimum energy states. Therefore, managing the number and position of positive literals within clauses based on 2 Satisfiability logical rules enables effective neuron connections, increasing the probability of obtaining a satisfiable interpretation for logical rules and facilitating the stabilization of DHNN.

Problem Statement 3: Probabilistic learning in DHNN tends to result in more iterations and convergence to local minima.

Explanation of Problem Statement 3: An effective learning phase can enable the network to achieve optimal synaptic weight management. A consistent interpretation of Satisfiability logical rules ensures a zero-cost function for DHNN, leading to optimal synaptic weights compared. As the scale of neurons expands during the learning phase, most DHNN-SAT models employ metaheuristic algorithms as the optimization learning algorithm to find consistent interpretations. Zamri *et al*. (2022a) employed a genetic algorithm for the logic generation and learning phases of RAN2SAT in DHNN, known as r2SAT. Compared to traditional exhaustive search, this model showed superior retrieval performance. However, the mutation operator used was prone to local optima, especially with larger neuron counts, where post-mutation fitness could be worse than pre-mutation. Someetheram *et al*. (2022) proposed a novel unsatisfiable logical rule and embedded an election algorithm, which significantly improved cost function minimization and yielded better learning performance than genetic algorithms and exhaustive search. Currently, most optimization learning algorithms mainly rely on probabilistic methods. However, while these methods help improve learning efficiency, they are easily affected by randomness, which can cause the model to get stuck in local optima, leading to more iterations and higher computation costs. Unfortunately, probabilistic search also lacks stability, meaning the results can vary a lot, making it hard to find a good balance between learning speed and accuracy. To address this issue, it is important to explore more reliable deterministic strategies to enhance the stability of the optimization process, avoid local optima, and reduce the number of iterations.

Problem Statement 4: Neuron bias in retrieval phase limit the diversity of induced logic.

Explanation of Problem Statement 4: The optimal retrieval ability of DHNN that the final neuron states always converge to the global minimum energy. Currently, most DHNN-SAT models also follow the WA method during the retrieval phase proposed by Abdullah (1992). For example, models like *r*2SAT proposed by Zamri *et al*. (2022b), and RAN*k*SAT by Sathasivam *et al*. (2020) have all employed this approach. These models update neuron states using the local field and hyperbolic tangent activation function, and this update mechanism provides good stability during the retrieval phase. However, second or third order clauses cause a positive bias in neuron states during retrieval, leading to highly similar final solutions with limited diversity. As the number of clauses increases, this bias becomes more severe and may even trap the retrieval process in a single pattern of the neuron state. Currently, the MHNN model proposed by Kasihmuddin *et al*. (2019) optimizes the retrieval phase of DHNN using the estimation of distribution algorithm (EDA) to enhance the diversity of global minimum solutions. However, as the number of neurons increases, MHNN falls into a trial-and-error loop, making it difficult for DHNN to converge to the optimal final neuron state. Therefore, a new optimization strategy for the retrieval phase must address both the positive bias of the final neuron state and the trial-and-error dilemma. This requires enhancing the diversity of neuron states after local field computation to ensure that the model generates more diversity solutions, thereby enabling more diverse induced logic for logical mining tasks.

Problem Statement 5: Ineffective multi-attribute selection and unbalanced output handling lead to incomplete rules.

Explanation of Problem Statement 5: Logic mining is a new paradigm in AI that extracts knowledge from real datasets in the form of logical rules. Logic mining was introduced by Sathasivam and Abdullah (2011), using Reverse Analysis (RA) to derive logical rules by computing neuron states at global minimum energy to obtain fixed optimal synaptic weights. However, the RA method produces a large number of logical rules without validation, making it difficult to identify optimal ones that generalize the behavior of the data. To address this issue, Kho *et al*. (2020) introduced a 2 Satisfiability based Reverse Analysis method (2SATRA), which reduces unnecessary learning and selects induced logic based on accuracy. However, the random attribute selection strategy limited its accuracy and classification performance. Kasihmuddin *et al*. (2022) enhanced 2SATRA with correlation analysis (S2SATRA) to extract key attributes, enabling the induced logic to better represent overall data behavior. However, several unresolved issues still limit its broader application and generalization in logic mining. First, although various methods exist for selecting key attributes, previous studies have not evaluated the effectiveness of different strategies. Second, the model does not fully consider information from -1 output data, which may lead to incomplete logical rules. When +1 and -1 outputs coexist, the model tends to favor the majority class, reducing accuracy and robustness in logical mining. Therefore, maximizing true positives (*TP*) and true negatives (*TN*) is necessary to ensure that logical rules comprehensively capture data patterns. At the same time, incorporating a parallel multi-attribute selection approach can accurately identify key attributes of data patterns.

1.5 Research Questions

In order to ensure that the problem statements are in alignment with the thesis objectives, this section outlines five key research questions that provide a comprehensive overview of the aspects to be investigated. These questions are directly derived from the issues identified and discussed in previous sections, ensuring that the focus of the study remains relevant and targeted. The research questions addressed in this study are as follows:

(a) How can the structural components of the systematic logic expression be designed to effectively control the number of positive literals and ensure their even distribution?

(b) What formulation and structure of the systematic logic can control the occurrence of positive literals and effectively manage the connections between neurons in the DHNN?

(c) How can the learning capabilities of the current DHNN be optimized to enable memory capabilities and prevent the generation of repetitive suboptimal neuron states in recent iterations?

(d) How can new neuron update rules and mutation strategies be applied in the retrieval phase of the DHNN to ensure that the global optimal solution can be effectively retrieved regardless of the number of neurons?

(e) How can multiple supervised learning algorithms be implemented for attribute selection in the context of logic mining to ensure that the resulting logic effectively extracts information from the dataset?

1.6 Research Objectives

The research objective was written based on the Specific, Measurable, Achievable, Relevant, and Time-bound (SMART) framework. This study aims to model the DHNN using a novel systematic logic rule that employs probability to control the quantity and distribution of positive literals. In the learning phase of the DHNN, the main goal is to achieve maximum fitness of the objective function by minimizing the cost function, thus ensuring effective weight management. During the retrieval phase of the DHNN, the proposed model will incorporate an intelligent mutation operator to address bias issues and enhance the diversity of solutions generated. Ultimately, the model aims to extract empirical patterns from real-world datasets for classification tasks. The specific research objectives are as follows:

(a) To formulate a novel logical rule namely Probabilistic 2 Satisfiability which consists of the ratio of positive literals in the 2SAT clause. Assigning a probability to each literal allows direct control over the number of positive literals and helps prevent overly concentrated distributions.

(b) To implement Probabilistic 2 Satisfiability into the DHNN as the symbolic rule. Each neuron in the network corresponds to a logical literal, and the synaptic weights are trained by minimizing the cost function, enabling the network to learn and represent the proposed logical rule.

(c) To propose the hybrid mutation tabu search algorithm, designed to minimize the cost function during the DHNN learning phase. This algorithm incorporates mutation and segment-based neighborhood operations to embed tabu search into the systematic logic, enabling consistent interpretations that satisfy the logic rules.

(d) To develop a mutation retrieval phase for the DHNN by introducing a mutation operator.The operator flips literals in unsatisfied clauses to reduce state bias and increase solution diversity, while the mutation retrieval phase guides the network to escape local optima.

(e) To propose a new logic mining model called Probabilistic 2 Satisfiability Reverse Analysis for handling classification tasks with various real-life datasets. The best logic and best induced logic are selected based on the highest average accuracy, capturing the characteristics of true positive and true negative instances.

1.7 Methodologies and Scopes

This thesis aims to develop a classification and knowledge mining model based on the DHNN framework. To achieve this, this section provides an overview of the five methodologies involved in the model's development. The model will first be validated on simulated data to assess its performance before being applied to benchmark datasets for classification tasks.

Methodology for Objective 1: The growing number of satisfiability formulas necessitates selecting an optimal one to manage neuron connections without reducing performance. This thesis proposes Probabilistic 2 Satisfiability, introducing a fixed number of positive literals and using probability to distribute them uniformly across clauses, improving clause satisfiability. The foundation of the proposed satisfiability formula is inspired by the following research. First, the systematic logic inspiration for Probabilistic 2 Satisfiability comes from Kasihmuddin *et al*. (2017a), where each clause is strictly defined to contain two literals, a structure commonly known as 2 Satisfiability. Incorporating 2 Satisfiability into DHNN imparts a more systematic structure to the network. Second, a research direction in propositional satisfiability involves introducing probabilities into propositional formulas. The concept of probability in system logic is inspired by Andersen *et al*. (2001), who proposed

Probability Satisfiability, which focuses on determining the satisfiability of a set of probabilities assigned to propositional formulas. Building on this work, Probabilistic 2 Satisfiability uses the probability assignment method of probability Satisfiability to assign probability values to each literal in the logic, aiming to control the type of logical clauses based on the ratio of positive literals. In this thesis, the proposed DHNN incorporates Probabilistic 2 Satisfiability in both the learning and retrieval phases by minimizing the cost function of the network. The optimal synaptic weights are determined by comparing the cost function with the energy function. These optimal weights then guide the retrieval phase of the network. The stability of the network is evaluated by assessing the difference between the global minimum energy and local minimum energy.

Scope of Methodology Objective 1: While this method provides a feasible implementation of the proposed Probabilistic 2 Satisfiability formula, certain constraints are necessary to ensure the repeatability of the methodology described for Objective 1. First, ensuring that all literals within the clauses remain free from repetition or redundancy is crucial. Repeated literals can lead to redundant information representation and cause the neural network to process more literals and clauses, thereby increasing computational complexity and content addressable memory consumption (Kasihmuddin *et al*., 2017a). More critically, repeated literals may result in an excessively expanded solution space, which could prevent the cost function from reaching zero. Second, the initial minimum number of literals can be set to 10. This setting aims to ensure that the proposed logical rule contains at least one positive literal, and the logical rules generated at different ratios do not repeat or overlap in the number of positive literals (Zamri *et al*., 2022a). Having all positive or all negative literals would disrupt neuron connections, leading to suboptimal synaptic weights. Finally, when using probability to control the distribution of positive literals, a loop control mechanism must be established. As a single code execution may fail to produce the required number of positive literals, executing the loop repeatedly ensures that the desired count is achieved. This process is a critical step in ensuring the correct generation of Probability 2 Satisfiability logical rules.

Methodology for Objective 2: In recent years neural network interpretability has gained significant attention. Symbolic neural networks are an important direction, and DHNN as a feedback-based artificial neural network is used for associative memory and optimization while enhancing interpretability through symbolic language to manage neuron connections. Inspired by this, the Probabilistic 2 Satisfiability formula proposed in this thesis is embedded into DHNN to represent the neurons in the network. The idea of embedding logical rules into DHNN draws inspiration from Abdullah (1992) research. In this study, Abdullah innovatively used the Wan Abdullah method to fix the connection weights between neurons, successfully embedding Horn SAT logic into DHNN. Compared to the traditional Hebbian method, the Wan Abdullah method not only reduced neuron oscillation but also ensured the correct connections between neurons by locating effective synaptic weights, thereby optimizing the DHNN learning process. Furthermore, Abdullah (1992) study emphasized that during the learning phase, the optimal synaptic weights of the neural network can be achieved and stored in the content addressable memory through the satisfiability interpretation of logical rules. In the relaxation phase, the activation function and local field are applied to calculate the final states of neurons, guiding the network to a stable state. To assess the compatibility of the proposed logical rules with DHNN, these logical rules will be compared with existing satisfiability logical rules using various performance metrics. Hyperbolic Activation Function.

Scope of Methodology Objective 2: While this approach serves as an ideal method for successfully embedding Probabilistic 2 Satisfiability into DHNN, certain constraints have been considered to ensure the reproducibility of the methodology described for Objective 2. Firstly, neuron states will be represented in a bipolar form (i.e., 1 and -1). According to Stern and Shea-Brown (2020), the dynamical properties of artificial neural networks based on the Lyapunov energy function are not well-suited to representing information in binary form (1 and 0). This is because some critical coefficients in the energy function might be eliminated or overlooked. Secondly, during the DHNN retrieval phase, the Hyperbolic Activation Function will be used to update the neuron states. HTAF is a differentiable and continuous function, which makes this function suitable for training in DHNN. Compared to linear or symmetric activation functions, This activation function output compression can result in faster convergence rates, reducing retrieval time and decreasing the tendency for neuron state oscillations. Such oscillations could reduce the likelihood of the Hopfield Neural Network converging to the global minimum energy state. Finally, during the DHNN retrieval phase, the Ising spin model, proposed by Sherrington and Kirkpatrick (1975), will be employed for local field calculations. Notably, any errors in local field calculations can undermine the effectiveness of the synaptic weights obtained during the DHNN learning phase, increasing the risk of falling into local optima.

Methodology for Objective 3: In the learning phase of the DHNN-SAT model the increasing number of neurons makes finding consistent interpretations that satisfy logical rules more difficult. The Wan Abdullah method ensures correct synaptic weights by meeting satisfiability constraints. Most existing models adopt metaheuristic algorithms for iterative optimization due to the large scale of the task. However, research on the impact of such algorithms on the cost function under different proportions of positive literals remains limited. This study proposes the Hybrid Mutation Tabu Search Algorithm to optimize DHNN during the learning phase and minimize the cost function. Hybrid Mutation Tabu Search Algorithm consists of three core mechanisms. First, the best solution is selected from 100 randomly generated candidates as the initial solution to accelerate convergence and meet the requirements of Probabilistic 2 Satisfiability. Second, mutation operators, segmentation operations and a tabu list are employed as neighborhood search strategies. The mutation operator, inspired by the work of Kasihmuddin *et al*. (2017a) and Zamri *et al*. (2022a), enables strong global exploration and effective local exploitation. The tabu list prevents short-term repeated searches and guides directional exploration to ensure the cost function converges to zero under varying numbers of positive literals. Finally, a flouting rule releases high-quality solutions from the tabu list to avoid missing the optimal solution. To clearly demonstrate the comparison between the proposed algorithm and existing research results, Hybrid Mutation Tabu Search Algorithm will also be compared and analyzed with existing typical metaheuristic algorithms.

Scope of Methodology Objective 3: Although the proposed metaheuristic algorithm might be the best method for optimizing the efficiency of the learning phase in Objective 2, there are still some scopes to ensure the reproducibility of Objective 3. First, clarifying the objective function of the Hybrid Mutation Tabu Search Algorithm is essential, aiming to achieve the maximum fitness of Probabilistic 2 Satisfiability. Second, the parameter settings for all metaheuristic algorithms used for experimental comparison will be configured based on successful experimental results in existing research. Specifically, for those metaheuristic algorithms that have already been applied to DHNN-SAT models, their experimental parameters will strictly follow the settings in the relevant studies to ensure consistency in experimental methods and reproducibility of the results (i.e. Kasihmuddin *et al*., 2019); Zamri *et al*., 2020; and Guo *et al*., 2024).

Methodology for Objective 4: The efficiency of the retrieval phase in converging the network to the global minimum energy is influenced by several factors, including optimal synaptic weights, the activation function, and the local field. Even under the guidance of optimal synaptic weights, the ability of the retrieval phase to reach the global minimum energy decreases to zero as the number of neurons increases. This is due to the traditional Hyperbolic Tangent Activation Function, which updates the neuron states based on two conditions: when the local field result is greater than or equal to 0, the neuron state is updated to 1; when the local field result is less than 0, the neuron state is updated to -1 (Karlik & Olgac, 2011). This approach biases neuron state changes toward 1, making the retrieval phase prone to local optima and resulting in a high repetition rate of solutions when there are many positive literals. To address this, two improvements are proposed. First, an optimized HTAF activation function is used to keep the neuron state at the next moment consistent with its previous state when the local field equals 0. This adjustment, in conjunction with the optimal synaptic weights, the new neuron update rule, and the local field, enables the Probabilistic 2 Satisfiability to converge to the global minimum energy regardless of the number of neurons. Second, a mutation strategy is introduced during the retrieval phase, where unsatisfied clauses are randomly mutated at any literal to re-establish the final neuron state. The mutation strategy enhances the diversity of solutions while maintaining the number of solutions. To evaluate the variability and diversity of solutions during the retrieval phase after implementing Objective 4, an average similarity measure is proposed. This measure calculates the similarity between non-repetitive solutions and the benchmark solution, providing an assessment of the effectiveness of the retrieval phase.

Scope of Methodology Objective 4: Although this method can be considered the best approach for ensuring optimality in the retrieval phase of DHNN, several limitations must be addressed to ensure the reproducibility of Objective 4. First, a benchmark solution is used to calculate the similarity of solutions. This is because each logic rule corresponds to multiple solutions, and the benchmark solution ensures that every literal is satisfied, making the solution the highest quality solution (Alway *et al*. 2022). Once the logical rules are generated, the benchmark solution remains fixed, which helps maintain the stability of the results. Second, to comprehensively evaluate the performance changes of Probabilistic 2 Satisfiability in the retrieval phase after introducing the new neuron update rule, the benchmark training algorithm ES is also included in the comparison algorithms. This allows for observing the impact of the new neuron update rule on both the quantity and diversity of network solutions.

Methodology for Objective 5: Existing logic mining models can generate optimal induced logic across various datasets. However, these models fall short in thoroughly investigating the role of positive synaptic weights in the generation of induced logic, leading to insufficient exploration of the interpretability of the induced logic. Additionally, these models are not satisfactory when handling imbalanced data. To address these issues, this thesis proposes an improved logic mining model called Probabilistic 2 Satisfiability Reverse Analysis. First, the data is processed into bipolar forms compatible with Probabilistic 2 Satisfiability, involving data format conversion, handling missing values, converting text data into numerical data, data cleaning, and optimal attribute selection. Notably, the optimal attribute selection employs the Ensemble Attribute Selection (EAS) method (Bania & Halder, 2020) to identify the ten attributes most relevant to the dataset. Second, Probabilistic 2 Satisfiability is used as the logical rule for logic mining. With a positive literal ratio of [0.1, 0.9], the variation in positive literals is reflected in the logic rules, aiding the logic mining model in generating more diverse induced logic. Combined with K-fold cross-validation, this approach addresses the challenges of handling imbalanced data by fitting and testing the model on such data. Next, both the learning and retrieval phases use the maximum average accuracy of the dataset to determine the best logic and induced logic. This comprehensive approach aims to identify the best induced logic that avoids overfitting and provides a thorough understanding of the dataset. The proposed logic mining model will be evaluated and studied using 20 repository datasets from well-known databases and a case study of online learning data from top national courses between 2021 and 2023. By introducing symbolic rules to constrain the model structure, the system is able to generalize well even when most combinations have not been seen during training. Although this study does not directly target zero-shot learning, the large combinatorial input space and limited training data constitute a sparse-data scenario. By introducing symbolic rules to constrain the model structure, the system can effectively generalize to unseen combinations. The performance of the proposed model will be compared with all existing logic mining models based on results from Accuracy, Precision, Negative Predictive Value, and Matthews Correlation Coefficient.

Scope of Methodology Objective 5: Although this method may be the best approach for extracting information from real-life datasets, there are still some limitations that need to be overcome to ensure the reproducibility of the thesis objectives. First, when dealing with classification tasks, the K-means clustering method is not applicable, so the data will be directly converted into a bipolar form based on the classification characteristics (Rusdi *et al*., 2023). Second, when generating logical rules using *K*-fold cross-validation, Probabilistic 2 Satisfiability will generate the best logic and induced logic based on the average accuracy of the subsets created by *K*-fold cross-validation. Additionally, the datasets used in this study are primarily sourced from the University of California, Irvine (UCI) Machine Learning Repository. Using these retrieved datasets, the proposed logic mining model will be compared with other existing logic mining models. However, this approach be applied in deffernt environments such as mobile and embedded systems.

1.8 The Organization of the Thesis

Overcoming the black box problem in neural networks requires a deeper, rule-based understanding of their internal logic. Therefore, this thesis focuses on integrating explicit logical rules into the network to make its computational process more transparent and interpretable. The structure of the remaining parts of this thesis is organized as follows. Chapter 2 reviews the literature across the various fields addressed in this thesis, including probabilistic satisfiability, logical in DHNN, optimization algorithms for learning phase, optimization algorithms for retrieval phase, and logic mining with reverse analysis methods. Chapter 3 introduces the research background and foundational knowledge relevant to the topics discussed in this thesis. First, the formulation process of the proposed Probabilistic 2 Satisfiability (PRO2SAT) is outlined, accompanied by examples with different proportions of positive literals. Second, the implementation of PRO2SAT within the DHNN is described. Third, the optimization of learning phase efficiency through the proposed Mutation Tabu Search algorithm is explained. Fourth, the retrieval phase is enhanced by introducing a new neuron update rule and a novel mutation strategy to explore more diverse solutions.

Finally, the PRO2SAT reverse analysis method proposed in this study is presented. Chapter 5 evaluates the effectiveness of the proposed model using simulated data and compares the results with existing algorithms in both the learning and retrieval phases. Chapter 6 presents a detailed analysis of the performance of the proposed logic mining model on 20 real-life datasets and compares the results with other existing logic mining models. The effectiveness of the proposed model will be evaluated based on selected performance metrics. Chapter 7 analyzes the best-induced logic extracted by the proposed logic mining model from case study datasets. Chapter 8 summarizes the findings of the thesis and discusses future research directions. Figure 1.1 depicts the overall framework of this research, integrating the problem statements, research objectives, and methodologies.

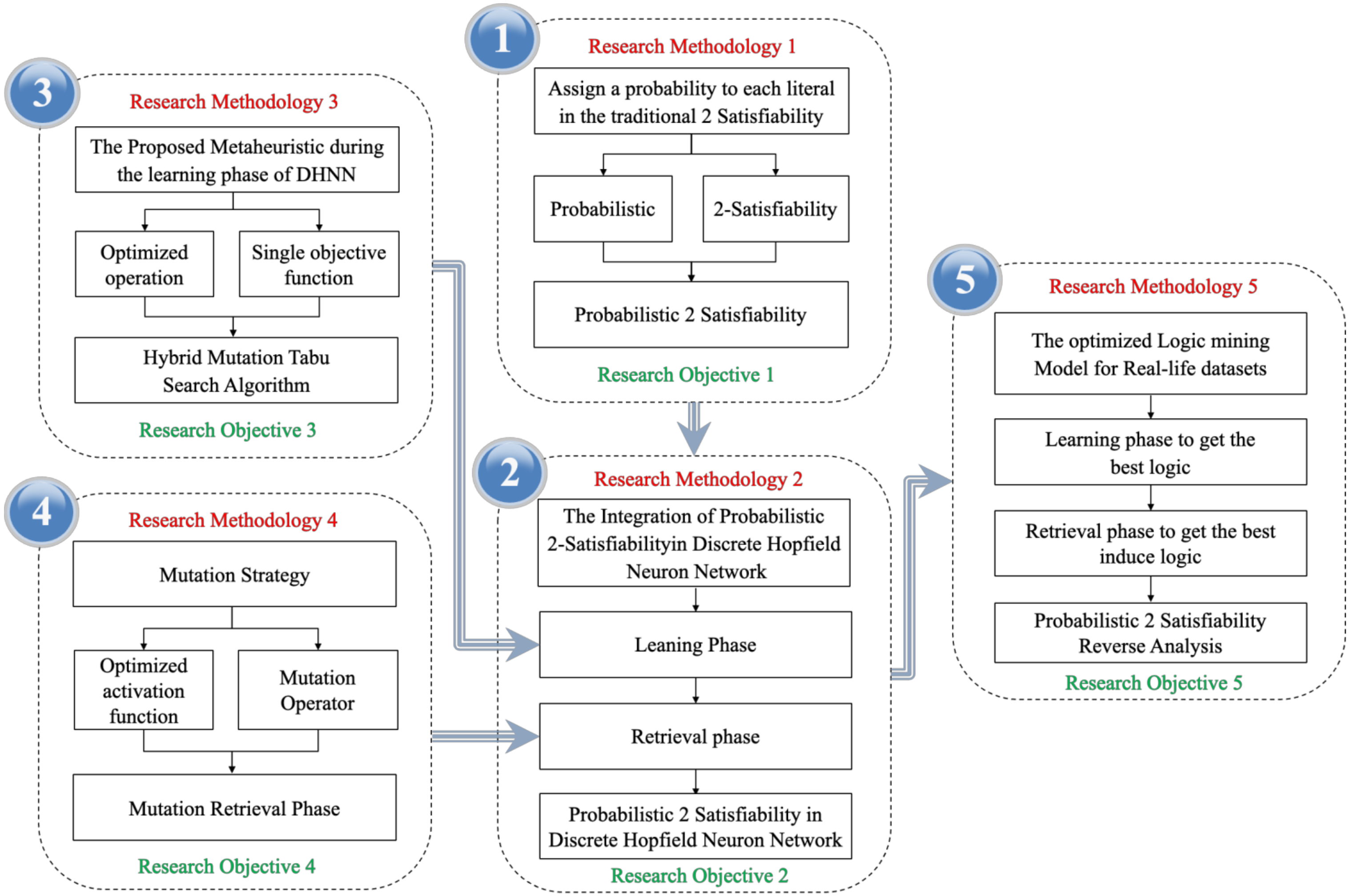

Figure 1.1 Thesis Overview Flowchart

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This section will discuss five important areas related to the proposed model. The first research area will review the development and evolution of Probabilistic Satisfiability over the years, with a particular focus on the structure and function of literal and clause probability. The second part will provide a detailed exploration of the systematic and non-systematic logic in the DHNN-SAT model, both of which are crucial for a deeper understanding of how to effectively embed SAT symbols into the DHNN model. In this section, we will analyze the characteristics of these two types of logic and their practical applications within the model. Next, the third part will introduce methods aimed at enhancing the learning phase of the DHNN-SAT model, primarily focusing on metaheuristic algorithms designed to solve complex optimization problems. The application of these algorithms during the learning phase is expected to significantly improve the performance of the model. The fourth part will concentrate on optimization strategies for the retrieval phase of the DHNN-SAT model, facilitating the generation of the optimal final neuron state. Subsequently, the fifth part will analyze existing logic mining models, placing particular emphasis on the reverse analysis methods they follow. This section will also address the issues and research gaps present in current logic mining models. The final part will provide a comprehensive summary of the discussions above, encapsulating the key findings and theoretical contributions.

2.2 Probabilistic Satisfiability

Probability Satisfiability is an extended logical rule that determines the satisfiability of a set of propositional logic formulas from a probabilistic perspective.