IMPROVING FORECASTING ACCURACY FOR TIME SERIES DATA USING FUZZY TECHNIQUES AND WAVELET TRANSFORM

ALENEZY ABDULLAH HOBAYKAN A

UNIVERSITI SAINS MALAYSIA

2025

IMPROVING FORECASTING ACCURACY FOR TIME SERIES DATA USING FUZZY TECHNIQUES AND WAVELET TRANSFORM

by

ALENEZY ABDULLAH HOBAYKAN A

Thesis submitted in fulfilment of the requirements
for the degree of
Doctor Of Philosophy

July 2025

ACKNOWLEDGEMENT

I am sincerely grateful to my supervisors, Assoc. Prof. Dr. Mohd. Tahir Ismail and Assoc. Prof. Dr. Sadam Alwadi, for their invaluable guidance and unwavering support throughout my research journey. Their accurate instructions have played a pivotal role in shaping the direction and success of my work. I am also thankful for their constant advice and necessary instructions, which have been instrumental in completing my research.I would like to thank the committee of viva, Dr. Firdaus Mohamad Hamzah, Dr. Wan Muhamad Amir Bin W Ahmad, And Dr. Majid Khan Majahar Ali, for their valuable input and feedback during the defense of my research. Furthermore, I would like to express my gratitude to Dr. Jamil J. Jaber and Dr. Nwaf for their valuable insights and assistance throughout my research. Their expertise has enriched my work and broadened my perspective. Finally, I extend heartfelt thanks to the Ministry of Higher Education and Universiti Sains Malaysia for their support and resources, which have greatly contributed to the quality of my study.

To the greatest people in my life, my uncle, my mother, my wife and my children, the epitome of love and tenderness. They have walked beside me since the beginning. To my soulmate, my kind-hearted wife. And to my children, my support and joy in life. Thank you all.

Lastly, I extend my heartfelt gratitude to all those who have supported me throughout my academic journey, providing me with invaluable experiences and knowledge. I am deeply grateful to everyone who has stood by my side during this critical phase of my life, contributing to my success. May Allah shower His mercy upon this achievement and compensate us for any missed opportunities.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Page

LIST OF SYMBOLS AND ABBREVIATION

ANFIS	Adaptive Network-Based Fuzzy Inference System
ANNs	Arteficianl neutral network
ARCH	Autoregressive Conditional Heteroskedasticity
ARIMA	Autoregressive Integrated Moving Average
BL14	Wavelet filter-Best localization
C6	Wavelet filter-Coiflet
CNNs	Convolutional Neural Networks
CWT	Continuous wavelet transform
D6	Wavelet filter-Daubechies
DWT	Discrete wavelet transform
FI	conventional Fuzzy Inference
FIR.DM	Fuzzy inference rules by descent method
FNNs	Fuzzy Neural Networks
FRBSs	Fuzzzy Rule-Based Systems
FS.HGD	Fuzzy System Heuristic gradient descent
FT	Fourier Transform
GARCH	Generalized Autoregressive conditional heteroskedasticity
GJR-GARCH	Glosten-Jagannathan-Runkle GARCH
HGD	Heuristic gradient descent
HyFIS	Hybrid neural fuzzy inference system

KSA	Kingdom of Saudi Arabia
LA8	Wavelet filter-Least Asymmetric
LSTM	Long Short-Term Memory
MAE	Mean Absolute Error
MAPD	Mean Absolute Percentage Deviation
MAPE	Mean Absolute Percentage Error
MASE	Mean Absolute Scaled Error
ME	Mean Error
MLP	Multi-layer perceptron
MODWT	Maximum Overlap Discrete Wavelet Transform
MPE	Mean Percentage Error
RMSE	Root Mean Squared Error
RNNs	Recurrent Neural Networks
TASI	Tadawul All Share Index
TSEM	Tadawul Stock Exchange Market
TSK	Takagi–Sugeno–Kang
TSK-FRBSs	Takagi-Sugeno-Kang Fuzzy Rule-based Systems
TVP-VAR	The Time-Varying Parameter Vector Autoregression

PENINGKATAN KETEPATAN RAMALAN UNTUK DATA SIRI MASA MENGGUNAKAN TEKNIK KABUR DAN TRANSFORMASI GELOMBANG KECIL

ABSTRAK

Kajian ini memberi tumpuan kepada peningkatan ketepatan ramalan pasaran saham bagi Bursa Saham Arab Saudi (Tadawul) dengan menggunakan teknik pemodelan yang canggih dan pendekatan pembelajaran adaptif. Kajian ini menggunakan transformasi gelombang kecil diskret bertindih maksimum (MODWT) bersama-sama dengan pelbagai fungsi matematik untuk menganalisis data indeks harga saham harian dari Oktober 2011 hingga Disember 2019. Pembolehubah input, termasuk harga minyak dan kadar repo, dipilih dengan teliti berdasarkan analisis korelasi, regresi berganda, dan ujian sebab penyebab Engle dan Granger. Model yang dicadangkan, seperti MODWT-LA8-ANFIS, MODWT-LA8-FS.HGD, MODWT-LA8-HyFIS, dan MODWT-LA8-FIR.DM, menunjukkan prestasi ramalan yang lebih baik berbanding dengan kaedah tradisional seperti ARIMA, ANFIS, FS.HGD, HyFIS, dan FIR.DM. Penilaian prestasi model yang dicadangkan melibatkan pelbagai ukuran statistik, termasuk ralat purata (ME), ralat min kuasa dua purata (RMSE), ralat mutlak min (MAE), dan ralat peratusan purata (MPE). Keputusan menunjukkan keberkesanan model-model ini dalam menganalisis corak pasaran saham dan meramalkan dengan tepat ketidakstabilan harga pasaran saham. Kajian ini menyumbang kepada bidang ramalan pasaran saham dan memberikan pandangan berharga kepada pelabur dan analis kewangan yang beroperasi di Bursa Saham Arab Saudi.

IMPROVING FORECASTING ACCURACY FOR TIME SERIES DATA USING FUZZY TECHNIQUES AND WAVELET TRANSFORM

ABSTRACT

This study focuses on improving the accuracy of stock market forecasting for the Saudi Arabia stock exchange (Tadawul) by employing advanced modeling techniques and adaptive learning approaches. The study utilizes the maximum overlapping discrete wavelet transform (MODWT) in conjunction with various mathematical functions to analyze daily stock price indices data from October 2011 to December 2019. Input variables, including oil price and repo rate, are carefully selected based on correlation analysis, multiple regression, and the Engle and Granger Causality test. The proposed models, such as MODWT-LA8-ANFIS, MODWT-LA8-FS.HGD, MODWT-LA8-HyFIS, and MODWT-LA8-FIR.DM, demonstrate superior forecasting performance compared to traditional methods like ARIMA, ANFIS, FS.HGD, HyFIS, and FIR.DM. The performance evaluation of the proposed model involves various statistical measures, including mean error (ME), root mean square error (RMSE), mean absolute error (MAE), and mean percentage error (MPE). The results highlight the effectiveness of these models in decomposing stock market patterns and accurately predicting stock market price volatility. This research contributes to the field of stock market forecasting and offers valuable insights for investors and financial analysts operating in the Saudi Arabia stock exchange.

CHAPTER 1

INTRODUCTION

1.1 Study Background

Understanding the complex dynamics of the connection between stock market behavior and the economy is essential for investors, governments, and market participants to make informed decisions and effectively manage risks. It is widely acknowledged that the stock market serves as a proxy for a country's economic health, reflecting underlying macroeconomic situations in both developed and emerging countries. In developed countries, such as the United States and Western Europe, there has historically been a positive correlation between stock market effectiveness and economic growth. Strong economic indicators, including GDP growth, unemployment rates, and interest rates, often result in improved or weakened company revenues, leading to higher or lower stock prices. Consequently, investors in developed countries can rely on the stock market as a reliable indicator of economic performance, allowing for better asset allocation and risk management decisions. Conversely, in emerging countries, the relationship between stock market performance and the broader economy can be more complex. While periods of economic growth typically bode well for stock markets, they may also be marked by higher volatility due to factors such as political instability, weaker regulatory frameworks, and less mature financial markets.

Time series analysis is essential for understanding and evaluating the movements of stock market indexes. Researchers, investors, and policymakers can acquire significant insights into the historical patterns, trends, and volatility displayed by stock market indexes over time by examining time series data. This understanding is critical for various reasons, including predicting future movements, analyzing market performance, and making informed investment decisions. Time series analysis is also crucial in identifying hidden patterns and trends in stock market indices. It helps investors anticipate potential market turning points and guides investment strategies. Accurate predictions in market indices provide insights into market dynamics, allowing stakeholders to anticipate risks, identify investment opportunities, and optimize portfolio management strategies. This is essential for risk management, as it helps mitigate uncertainties and volatility in financial markets. Accurate forecasts also contribute to market efficiency and stability, fostering investor confidence and capital inflows. Researchers use quantitative models, statistical techniques, and machine learning algorithms to generate reliable predictions.

The study of time series data has a long history, including contributions from noteworthy pioneers. Sir Francis Galton is one such time series analysis luminary (Galton, 1886) . His pioneering studies in the late nineteenth century had a considerable impact on the interpretation of time series data, particularly in the domain of meteorology and weather patterns. He used statistical tools to investigate correlations and patterns in meteorological observations such as temperature and rainfall values. Galton introduced techniques such as regression analysis and correlation to measure relationships between variables in time series data. His work established the foundation for statistical analysis of time series and opened the direction for future developments in the field. Numerous statisticians, mathematicians, and economists have contributed significantly to the advancement of time series analysis methodologies and approaches. Contributions to autoregressive integrated moving average (ARIMA) models, cointegration analysis, and the study of volatility in financial time series by prominent authors such as (Box, Jenkins, Reinsel, & Ljung, 2015), (Granger & Newbold, 1974), and (Robert F Engle, 1982) have advanced the field.

The Wavelet Transform is a sophisticated mathematical technique that has grown in popularity for evaluating signals and time series data due to its ability to capture both time and frequency information simultaneously. The Discrete Wavelet Transform (DWT) is a frequently used version of the Wavelet Transform that decomposes a signal into a series of wavelet coefficients at different scales. The DWT has been used in a variety of applications, including image processing, signal denoising, and feature extraction. However, in the presence of nonstationary signals, the DWT suffers from aliasing. To alleviate this restriction, the Maximum Overlapping Discrete Wavelet Transform (MODWT) was developed. The MODWT extends the capabilities of the DWT by introducing overlapping wavelet coefficients, enabling improved time-frequency localization and enhanced analysis of nonstationary time series data (Percival & Walden, 1993). The MODWT has an advantage over DWT in that it can be used for any sample size and is unaffected by the beginning point of filtering data. The MODWT includes five mathematical functions, namely, Haar, Daubechies (Db), Least Square (LA8), Best localization (BL14), and Coiflet (C6) (Jaber, Yaacob, & Alwadi, 2023).

Researchers and statisticians have developed various methods and techniques to capture the dynamics and characteristics of historical time series data. These models have contributed to advancements in forecasting and the identification of long-term trends and relationships within time series data. In the 1980s, researchers began exploring the application of neural networks for time series prediction. One notable milestone was the work of (Rumelhart, Hinton, & Williams, 1986), which introduced the backpropagation algorithm for training multi-layer perceptron (MLP) networks.

This algorithm enabled neural networks to learn complex patterns in time series data. In the 1990s, recurrent neural networks (RNNs) gained popularity for time series prediction tasks. RNNs are specialized neural network architectures that can model sequential dependencies in data. One influential RNN variant is the Long Short-Term Memory (LSTM) network, introduced by (Hochreiter & Schmidhuber, 1997). LSTMs address the vanishing gradient problem and have shown effectiveness in capturing long-term dependencies in time series data.

During the 2000s, researchers examined hybrid models that integrated numerous neural network designs to improve time series prediction accuracy. These techniques intended to take advantage on the strengths of different models and improve overall performance (G. P. Zhang, 2003). The resurgence of neural networks in the 2010s was driven by improvements in deep learning. Convolutional neural networks (CNNs), which were initially designed for image recognition, have been repurposed for time series analysis by considering time series data as one-dimensional sequences (LeCun, Bengio, & Hinton, 2015).

Fuzzy Neural Networks (FNNs) approach, such as the Adaptive Network-Based Fuzzy Inference System (ANFIS) and the Hybrid Neural Fuzzy Inference System (HyFIS), have emerged as powerful tools in time series analysis for effectively modeling and predicting complex and nonlinear relationships. By integrating fuzzy logic and neural network techniques, FNNs can capture and represent linguistic variables, fuzzy sets, and fuzzy rules, enabling them to handle the inherent uncertainty and imprecision in time series data. ANFIS combines the learning capabilities of neural networks with the linguistic interpretability of fuzzy logic by constructing a fuzzy inference system with adaptive parameters. HyFIS, on the other hand, integrates neural networks and fuzzy systems at multiple levels, allowing for more flexible and accurate modeling of time series data. FNNs excel in time series analysis by leveraging the pattern recognition capabilities of neural networks and the linguistic interpretability of fuzzy logic. They can effectively identify and exploit the underlying patterns and relationships in time series data, making them well-suited for applications such as forecasting and anomaly detection (Jaber, Alkhawaldeh, Alkhawaldeh, Masa’deh, & Alshurideh, 2023; J. Jang, 1993; Kim & Kasabov, 1999).

In addition, gradient descent approach is a widely used optimization algorithm in time series analysis, aiming to minimize a cost or error function by iteratively adjusting model parameters based on the gradient of the error function. It can be applied to models like ARIMA and neural networks to estimate parameters and optimize model performance. Variations include gradient descent learning (FIR.DM) for learning FIR filters' coefficients, which iteratively updates filter coefficients to identify the best configuration for minimizing prediction error, improving accuracy in time series forecasting. Heuristic gradient descent (FS.HGD) combines gradient descent with heuristic search techniques for enhanced optimization in time series modeling (Ishibuchi, Nozaki, Tanaka, Hosaka, & Matsuda, 1993; Nomura, Hayashi, & Wakami, 1992; Pérez Vera & Bermudez Peña, 2022).

1.2 Statement of the Problem

Financial markets have become an important channel for investment in many countries, including the Kingdom of Saudi Arabia, leading to a growing need for forecasting the direction and volatility of stock prices. To study the stock market, investors generally use two analytical approaches: fundamental and technical studies. To make rational decisions on purchasing or selling stocks, fundamental analysis depends on macroeconomic data statistics such as interest rates, money supply, inflation rates, and foreign exchange rates. Technical analysis, on the other hand, makes use of historical financial time series data, which show complex trends shaped by high volatility and non-normal distribution returns.

Volatility prediction is crucial in finance and investment for various reasons. It aids in risk management, investment decision-making, and trading strategies. Accurate predictions help investors assess the risk-reward trade-offs associated with different investment opportunities, enabling informed asset allocation, portfolio diversification, and optimal timing for buying or selling securities. Traders use volatility forecasts to identify optimal entry and exit points, set stop-loss orders, and design risk-adjusted approaches. Volatility also serves as a vital input in pricing models like the Black-Scholes model, enabling accurate valuation of options and other derivative instruments. Finally, volatility prediction contributes to the overall efficiency and liquidity of the stock market, enabling better-informed trading decisions and attracting more market participants, enhancing market liquidity and facilitating smoother operations (Hull, 2023).

Although several publications have been published on the stock price volatility in emerging capital markets (Alshammari et al., 2023; Rashedi et al., 2021), few studies on the Arab capital market in general, and Saudi Arabia in particular, have been conducted. Therefore, the problem statement of this research is to develop a comprehensive mathematical model that effectively measures the volatility of stock prices and enhances the accuracy of predictions. By incorporating the fuzzy neural network approach and the gradient descent approach with MODWT, this research seeks to provide valuable insights for investors in the Kingdom of Saudi Arabia and beyond, facilitating better decision-making in the financial markets.

1.3 Research question

In investigating the modeling of the Saudi Stock Exchange (Tadawul) using Maximal Overlap Discrete Wavelet Transform (MODWT) methods, fuzzy neural networks (ANFIS/HyFIS), and gradient descent approaches (FIR.DM/FS.HGD), this study addresses:

a) How can Adaptive Network-Based Fuzzy Inference System (ANFIS) be effectively integrated with Maximal Overlap Discrete Wavelet Transform (MODWT) using the LA8 filter to enhance the assessment and forecasting accuracy of stock market volatility?

b) How can the integration of the gradient descent approach (specifically Fuzzy Inference Rules by Descent Method - FIR.DM) with MODWT (LA8) and ARIMA models improve the accuracy of stock market volatility measurement?

c) To what extent does combining the Hybrid Neural Fuzzy Inference System (HyFIS) with MODWT (LA8) and ARIMA models enhance the analytical robustness and precision of stock market volatility forecasts?

d) How effectively can the Fuzzy System Heuristic Gradient Descent (FS.HGD) method, integrated with MODWT (LA8) and ARIMA models, estimate and forecast stock market volatility?.

1.4 Research Objectives

The main objective of this study is to model the Saudi Stock Exchange using the maximum overlapping discrete wavelet methods (MODWT). To achieve this, the following objectives will be pursued:

a) To measure stock market volatility by integrating Adaptive Network-Based Fuzzy Inference System (ANFIS) and Maximal Overlap Discrete Wavelet Transform (MODWT) to enhance volatility forecasting..

b) To enhance accuracy in measuring stock market volatility through the integration of the gradient descent approach, Fuzzy inference rules by descent method (FIR.DM), MODWT (LA8), and ARIMA models.

c) To improve stock market volatility analysis by combining Hybrid neural fuzzy inference system (HyFIS), MODWT (LA8), and ARIMA models to leverage the strengths of each approach.

d) To estimate stock market volatility by integrating the gradient descent approach, specifically the Fuzzy System Heuristic gradient descent (FS.HGD), MODWT (LA8), and ARIMA models.

Indeed, This study aims to improve the accuracy of stock price volatility predictions in hybrid models by incorporating the fuzzy neural network approach and the gradient descent approach with MODWT.

1.5 Research Significance

The significance of this research lies in its focus on stock market studies within emerging economies, which have often been overshadowed by research conducted in developed economies. The application of a single model to analyze stock market volatility in different contexts can be problematic and may lead to contentious or inaccurate results. Emerging economies possess unique market indices that are not present in their developed counterparts, adding complexity to the analysis. Moreover, there is a need for an efficient quantitative tool with appropriate measurement tendencies to capture the volatility of stock markets in emerging economies. This research aims to address these challenges by incorporating a volatility model that considers all relevant factors specific to emerging economies. This is particularly important as developing economies offer a wide range of investment opportunities for foreign investors, making accurate volatility analysis crucial for informed decision-making.

1.6 Organization of the thesis

The study is structured to provide a comprehensive analysis of the topic and is organized as follows. The first chapter serves as the introduction, setting the stage for the research and highlighting its significance. It provides an overview of the study's objectives and outlines the research questions that will be addressed throughout the study.

The second chapter delves into an extensive review of related literature, specifically focusing on the fluctuations of stocks and expected returns in financial markets. This chapter addresses controversial issues in the literature, providing clarity on key concepts and theories. It explores how fluctuations in financial markets impact investors' decision-making processes and examines the effects on their portfolios and portfolio liquidity.

The third chapter focuses on the methodology employed in the study. It discusses various neural network models, their characteristics, and the methods used to estimate model parameters. The chapter also describes the careful design of the model and the statistical tests used to assess its validity and effectiveness.

In the fourth chapter presents the results and discussion derived from the analysis of the closing prices of the Saudi stock market over the period of 2011 to 2019. This chapter describes the models used to measure price fluctuations and compares their accuracy in predicting prices. The findings and their implications are thoroughly discussed and analyzed.

In the fifth chapter provides an in-depth exploration of the Tadawul Stock Exchange Market (TSEM) in Saudi Arabia. It offers a historical perspective on the market and discusses the results of the volatility forecasting models applied to the TSEM. The chapter highlights key insights and implications for market participants.

Lastly, the sixth chapter summarizes the conclusions drawn from the research findings. It consolidates the key findings and highlights their significance in the context of the study. Additionally, this chapter offers recommendations for the practical application of the suggested models, providing guidance for investors and market participants.

Overall, the study follows a logical progression, starting with an introduction and literature review, followed by methodology, results, and discussion, and concluding with key insights and recommendations. This structure ensures a comprehensive analysis of the topic, enriching our understanding of stock fluctuations, expected returns, and volatility forecasting in financial markets.

1.7 Limitations of the Study

This study has certain limitations that should be acknowledged. Firstly, the findings obtained from this research are applicable within the specific time range covered by the study. Unexpected events or exceptional conditions, such as the COVID-19 pandemic, may introduce uncertainties and render the application of the study's results less effective or redundant. Secondly, it is important to recognize that the use of additional time series data beyond the scope of this study may yield varying results in the future. Market dynamics and conditions are subject to change over time, and the inclusion of more recent data could potentially alter the outcomes and conclusions drawn from this research. These limitations highlight the need for continued monitoring and analysis to account for evolving circumstances and ensure the relevance and applicability of research findings in a dynamic and ever-changing environment.

1.8 Summary

The relationship between stock market behavior and the economy is vital for informed decision-making, as markets act as a proxy for economic health. In developed countries, strong economic indicators like GDP growth and unemployment rates result in improved or weakened company revenues, leading to higher or lower stock prices. In emerging countries, the relationship between stock market performance and the broader economy can be more complex, with periods of economic growth often bode well for stock markets but may also be marked by higher volatility due to political instability, weaker regulatory frameworks, and less mature financial markets. Understanding these dynamics requires time series analysis to evaluate movements and identify patterns. To address the challenge of high volatility in stock prices, wavelet analysis provides a valuable tool for measuring volatility over time. New hybrid models, combining wavelet analysis with advanced neural networks, are designed to significantly enhance the accuracy of both measuring and predicting volatility in stock exchange time series, offering decision-makers more reliable insights.

This study primarily aims to model the Saudi Stock Exchange (Tadawul) using MODWT (specifically the LA8 model), focusing on measuring market volatility through approaches like fuzzy neural networks (FNNs) and gradient descent, and to construct a hybrid forecasting model integrating FNNs, gradient descent, ARIMA, and MODWT to enhance the accuracy of volatility analysis and prediction; it seeks to evaluate and validate suitable predictive models for this specific market and contribute to broader volatility research, though its reliance on pre-COVID-19 data may limit the generalizability of findings to typical market conditions.

This study comprises six chapters, each serving a distinct purpose. The first chapter serves as an introduction, presenting the research problem and setting the context for the study. The second chapter conducts a comprehensive review of the relevant literature pertaining to the research subject. In the third chapter, the methodology employed in the study is elaborated upon, outlining the approach and techniques used. The fourth and fifth chapters delve into the presentation and analysis of the results obtained from the research. Finally, the last chapter includes the conclusions drawn from the study and provides recommendations based on the findings.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In the financial market, there are three types of investors: risk-averse, risk-neutral, and risk-loving. Risk-averse investors are inclined to avoid risks as much as possible, prioritizing a lower rate of return with minimal risk over a higher rate with increased risk. They fear incurring losses in their investments and aim to safeguard their capital. On the other hand, risk-neutral investors neither actively seek nor avoid risks but instead take calculated risks to mitigate the potential for significant losses while pursuing moderate to high rates of return. Conversely, risk-loving investors thrive on taking risks, viewing investments as a form of a gamble where they have the opportunity to earn substantial returns or bear significant losses. These individuals are undeterred by the prospect of losses due to their attraction to high rates of return.

The portfolio theory defines risk aversion as a critical factor in making investment decisions. By embracing risk aversion, investors can potentially achieve high returns while minimizing their exposure to risk. Consequently, when investors elect to invest in the stock market, it is critical to use scientific approaches to examine volatility risk in stock prices and model economic phenomena in order to forecast future values. Despite the advancements in financial research, this remains a complex and challenging task.

2.2 Volatility Models

The accurate prediction of stock market volatility is crucial for investors and financial analysts to optimise their investment and risk management strategies. The development of various volatility models and techniques has led to improved predictions of stock market volatility. One of the earliest models for predicting volatility was the autoregressive conditional heteroskedasticity (ARCH) model, introduced by Engle (1982), which estimates the conditional variance of stock returns by considering past variances. The ARCH models are effective in capturing the volatility clustering phenomenon, which refers to the tendency of periods of high volatility to follow periods of high volatility. Bollerslev (1986) introduced the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model as an extension of the ARCH model. The GARCH model suggested in the study has since become a widely used tool in financial econometrics and has played a crucial role in furthering the understanding and modeling of financial time series data, providing as a foundation for the analysis of volatility and risk in financial markets. The GARCH model expands the traditional autoregressive model by enabling the incorporation of lagged values of both the conditional variance and mean. GARCH models have been found to improve the accuracy of volatility predictions, especially in capturing volatility’s long-term persistence.

Black and Scholes (1973) introduced the groundbreaking Black-Scholes model, which revolutionized the field of finance by providing a comprehensive framework for pricing options and corporate liabilities, taking into account the crucial element of volatility. The Black-Scholes model considers various factors, including the underlying asset price, time to expiration, risk-free interest rate, and volatility. The inclusion of volatility as a key input in the model reflects its significant role in option pricing. By incorporating volatility, the Black-Scholes model accounts for the uncertainty and potential fluctuations in the underlying asset's price, which greatly affects the value of options. The consideration of volatility in the model has enhanced the understanding and application of derivatives pricing, enabling more accurate valuation and informed decision-making in financial markets.

Nelson (1990) focuses on examining the behavior of volatility over time using the GARCH (1,1) model. By exploring the properties of the model, including stationarity and persistence, Nelson gains a deeper understanding of the dynamics of volatility in financial time series data. Nelson (1991) introduces the concept of conditional heteroskedasticity and proposes an exponential GARCH to capture and analyze the changing patterns of volatility in asset returns. This approach provides valuable insights into the relationship between volatility and asset returns, offering a useful tool for understanding and modeling financial market dynamics.

Glosten et al. (1993) investigate the relationship between expected returns and volatility, recognizing the crucial role of volatility in determining risk and return within financial markets. By utilizing the Glosten–Jagannathan–Runkle (GJR-GARCH) model, they analyze how volatility impacts the expected value of nominal excess returns on stocks, providing insights into the interplay between risk and expected returns. This research sheds light on the dynamics between volatility and expected returns, contributing to a deeper understanding of the risk-return relationship in financial markets.

Giot and Laurent (2003) assess and quantify the potential losses associated with commodity market investments by utilizing the value at risk (VaR) approach. The study emphasizes the role of volatility, a significant factor in commodity markets, in understanding and managing market risk. Jacquier et al. (2004) propose a Bayesian analysis of stochastic volatility models with fat-tails and correlated errors. Their objective is to improve the accuracy and flexibility of volatility modeling in financial time series data. By utilizing a Bayesian approach and Markov Chain Monte Carlo

(MCMC) methods, they estimate parameters and generate predictive distributions for volatility. The study demonstrates that the proposed model successfully captures fat-tailed distributions and correlated errors, resulting in enhanced estimates and reliable predictions.

Corsi (2009) introduces the Heterogeneous Autoregressive model of the Realized Volatility (HAR-RV) as a novel approach for volatility forecasting in financial data. The model incorporates the Heterogeneous Market Hypothesis, considering diverse market participants. Despite its simplicity, the HAR-RV model successfully reproduces key empirical characteristics of financial data. Notably, it surpasses traditional models in terms of forecasting performance, indicating its effectiveness in capturing volatility dynamics. Overall, this study highlights the relevance of the HAR-RV model in improving volatility forecasts and understanding the behavior of different market participants.

Dai et al. (2022) investigated the volatility spillover effects and dynamic linkages between crude oil, gold, and Chinese stock markets. The Time-Varying Parameter Vector Autoregression (TVP-VAR) model, based on Diebold and Yilmaz's technique, was used in the study to examine daily data from 2013 to 2020. The results demonstrate a significant level of interdependence between the assets, and volatility spillover increases during major crises. Dai and Zhu (2023) used the quantile VAR model and TVP-VAR model through generalized forecast error variance decomposition. The results show higher risk spillovers under the 0.01 and 0.99 quantiles and that term spread and credit spread significantly predict total return and volatility spillovers.

In connection with the above discussions, the comprehensive modelling of volatility requires an approach that can include information on the behaviour of volatility in both frequency and time models. As a result, examining daily volatility is crucial for gaining insights into market dynamics and identifying patterns and anomalies. Daily volatility analysis is particularly important for active traders, as it allows for optimizing trading strategies and identifying entry and exit points. Overall, studying volatility on a daily basis enhances market understanding and facilitates effective risk management and decision-making. The recent studies on neural network models (Goyal, & Soni, 2025; Zeng, Wu, Abedin & Ahmed, 2025).

2.3 Wavelets History

Wavelets may be traced back to 1807, when Joseph Fourier created the Fourier Series, a unique technique for evaluating time series (Fourier, 1807). The work of Fourier laid the groundwork for the frequency model, which is frequently represented as the Fourier Transform (FT). However, the frequency model and the time model, which focused on the temporal features of a signal, were seen as two distinct methods in time series, each with a unique set of limitations. This meant that using one method excludes using the other, and there was no unified methodology capable of properly utilizing both the frequency and time models at the same time.

The breakthrough in bridging this gap came with the emergence of wavelets. The pivotal contribution was made by Alfred Haar in 1910 when he proposed an orthogonal system of functions defined in the interval [0, 1]. Haar's groundbreaking work led to the discovery of the simplest wavelet known as the Haar Wavelet (Haar). By using this wavelet, Haar demonstrated the possibility of constructing a mathematical framework that could integrate both time and frequency information in signal analysis (Alfred, 1910).

The Haar Wavelet gave important insight into the concept of multiresolution analysis, opening the door for additional advances in wavelet theory. In the late 1970s and early 1980s, scholars such as Jean Morlet and Alex Grossman improved on Haar's work and created the concept of continuous wavelet transform (CWT). This transforming advancement enabled the analysis of signals at various scales, capturing localized information in both the time and frequency domains (Grossmann & Morlet, 1984). The evolution of wavelet theory gained further momentum with the contributions of Yves Meyer in the early 1980s. Meyer's introduction of multiresolution analysis (MRA) provided a comprehensive framework for decomposing signals into different frequency bands with varying resolutions. This decomposition technique formed the basis for the development of discrete wavelet transform (DWT), a computationally efficient version of the CWT (Meyer, 1985).

The culmination of these advancements occurred when Ingrid Daubechies, inspired by the work of Meyer, developed a family of wavelets known as Daubechies wavelets. These wavelets possessed desirable properties such as orthogonality and compact support, making them well-suited for practical applications. Daubechies' contributions revolutionized the field of wavelet analysis and paved the way for its widespread adoption in various disciplines, including signal and image processing (Ingrid Daubechies, 1988).

As an extension of the wavelet transform, the concept of wavelet packets developed in the early 1990s. Wavelet packets enabled more flexible signal decomposition, allowing for greater control over frequency and time resolution. This development gave a richer representation of signals and opened up new options for signal analysis and processing. Mallat used filter banks to establish the concept of the discrete wavelet transform (DWT) in 1989 (Mallat, 1989). This method enabled the effective implementation of the wavelet transform, which resulted in significant advances in signal and image compression methods. In the late 1990s and early 2000s, wavelet-based denoising techniques gained significant attention. Researchers proposed various algorithms and methods leveraging the properties of wavelets to effectively remove noise from signals and images while preserving important features. These denoising techniques found applications in fields such as biomedical signal processing, audio restoration, and image denoising (Cohen, Daubechies, &Vial, 1993; Yu & Acton, 2002).

The utilization of wavelet analysis in financial analysis is rooted in its capacity to capture temporal and frequency variations, which is particularly valuable in dealing with the dynamic nature of financial variables. This characteristic becomes crucial as financial variables often exhibit distinct behaviors over time. Jaber et al. (2017) utilize a combination of AutoRegressive Integrated Moving Average (ARIMA) and wavelet analysis techniques, known as ARIMA-WT, to achieve more accurate volatility risk predictions. The integration of these two methods aims to capture both long-term trends and short-term fluctuations in volatility, enhancing the accuracy of the volatility risk forecasts for the Jordanian banking sector. The study likely applies the ARIMA-WT approach to analyze historical data, identify volatility patterns, and develop forecasting models to assist in managing and mitigating volatility risk in the banking industry in Jordan.

Yaacob et al. (2021) explores the use of wavelet filters in analyzing mortality data and improving mortality rate forecasting. Different wavelet filters, including least asymmetric (LA8), Coiflet (C6), and best-localized (BL14) filters, are discussed for their accuracy in fitting the data. The study evaluates the performance of the MODWT-ARIMA model with various wavelet filters and finds that the BL14 filter consistently outperforms its counterparts in terms of forecast accuracy. The study highlights the effectiveness of wavelet-based methods in capturing variations, magnitudes, and phases of the data. Overall, the hybrid model combining the Lee-Carter model with the MODWT using the BL14 filter shows improved forecast accuracy for mortality rates. Jaber et al. (2023) focuses on the application of wavelet types in mortality forecasting. Specifically, the study proposes a hybrid model, LC-WT-ANFIS, which combines wavelet functions and adaptive network-based fuzzy inference system (ANFIS) to improve the accuracy of the Lee-Carter (LC) model. Different wavelet filters, such as least squares (la8), best localised (BL14), and Coiflet (C6), are utilized in the model. The results demonstrate that the LC-WT-ANFIS model outperforms the traditional LC model, indicating the effectiveness of incorporating wavelet techniques in mortality forecasting.

Furthermore, wavelet analysis offers the advantage of not requiring excessive assumptions to initiate the analysis, which is advantageous considering the complex nature of financial analysis. Another key benefit of wavelet analysis is its ability to identify discontinuities or shifts in data, which aids researchers in determining significant turning points or changes in the data series.

2.4 Neural Network History

Neural networks have emerged as strong tools for a variety of financial applications, with the potential to enhance decision-making, risk assessment, and prediction precision. The initial development of neural networks may be traced back to Frank Rosenblatt's fundamental work in 1958, when he developed the idea of the perceptron and proved its capacity to learn linearly separable patterns (Rosenblatt, 1958). This marked the start of neural network research and set the framework for later subsequent developments.

In 1986, the authors Rumelhart et al. (1986) established the backpropagation algorithm, which was a significant development that allowed for the effective training of neural networks with numerous layers. This approach transformed the discipline by allowing nonlinear patterns to be learned and supporting the training of deep neural networks. In 1989, the pioneering work of Moody and Darken, who used neural networks to anticipate stock market fluctuations, neural networks acquired popularity in finance (Moody & Darken, 1989). Their findings proved neural networks' ability to capture complicated patterns and predict financial time series. In 1997, the authors Hochreiter and Schmidhuber (1997) invention of Recurrent Neural Networks (RNNs) was another significant milestone. RNNs added the capacity to represent sequential and temporal relationships, making them ideal for applications like speech recognition, natural language processing, and time series analysis.

In 2006, Deep Belief Networks (DBNs) were proposed by Hinton et al. (2006) as a generative model built of many layers of constrained Boltzmann machines. DBNs use a layer-wise pre-training approach followed by backpropagation fine-tuning. In 2013, Deep Reinforcement Learning (DRL) were proposed by Mnih et al. (2013). The model is a subfield of machine learning that combines deep learning techniques with reinforcement learning algorithms to enable agents to learn and make decisions in complex environments. This method enabled efficient deep neural network training and demonstrated enhanced performance in a variety of applications. Khan et al. (2023) explored the use of DRL for predicting stock market movements. Their framework incorporated a deep neural network trained with the Q-learning algorithm to make optimal trading decisions based on historical data. DRL showed promise in capturing complex patterns and sudden shifts in stock prices compared to traditional statistical and technical approaches. These are just a few significant milestones in the history of neural network models.

In 2017, the Transformer was introduced as a groundbreaking neural network architecture for natural language processing tasks (Vaswani et al., 2017). It revolutionized the field by using self-attention mechanisms to efficiently capture long-range dependencies. The Transformer's parallel processing capability accelerated training and became the state-of-the-art model for machine translation. It has since been adapted to other domains, including time series forecasting, with models like Temporal Fusion Transformers (TFT) demonstrating effective multi-horizon forecasting (Lim, Arık, Loeff, & Pfister, 2021). The recent studies on neural network models (Jaber, Alkhawaldeh, & Khatatbeh, 2025; Hamadneh, Jaber & Sathasivam, 2024).

2.5 Related Literature to Fuzzzy Rule-Based Systems (FRBSs)

Fuzzy Rule-Based Systems (FRBSs) have evolved significantly since their launch, contributing to a wide range of applications dealing with imprecise and uncertain information. The academic history of FRBSs may be traced back to the mid-1960s, when Lotfi A. Zadeh proposed fuzzy logic as a mathematical framework for dealing with imprecision. In 1965, Zadeh's significant work on "Fuzzy Sets" lay the foundation for FRBSs (Lotfi A Zadeh, 1965). In 1974, Mamdani presents the application of fuzzy algorithms for controlling a basic dynamic plant. The study investigates the concept of fuzzy logic and its application to control issues with imprecise inputs and outputs. Mamdani provides a fuzzy control system that makes control decisions using linguistic criteria, demonstrating its effectiveness in a case study. The study concludes by emphasizing fuzzy logic's possibilities in solving control situations when accurate mathematical models are unavailable or hallenging to construct (Mamdani, 1974).

In 1975, Mamdani and Assilian propose an innovative experiment in linguistic rule synthesis for a fuzzy logic controller. The study investigates the application of fuzzy logic in the control of a simulated steam engine system. Mamdani and Assilian present a fuzzy control system that makes control decisions based on imprecise inputs and outputs using linguistic criteria. They illustrate the usefulness of the fuzzy logic controller in regulating the steam engine system through their experiment. The study concludes by emphasizing fuzzy logic's capacity to handle complicated control tasks and to give human-understandable explanations for decision-making processes. In other words, they propose the Mamdani fuzzy inference system, a pioneering work that defined the architecture and inference mechanism of FRBSs. Mamdani's system integrated fuzzy sets, fuzzy rules, and expert knowledge to perform reasoning and decision-making tasks (Mamdani & Assilian, 1975).

In 1985, Takagi and Sugeno present a fuzzy identification approach for modeling and control applications. Based on input-output data, the paper proposes a fuzzy inference system that can identify and model complicated systems. Takagi and Sugeno propose using fuzzy rules and membership functions to express and approximate the behavior of the system. Several case studies, including system identification and control tasks, are used to show the effectiveness of their approach. The study concludes by emphasizing the benefits of the fuzzy identification approach, such as its capacity to handle non-linear and uncertain systems, as well as its potential applications in a variety of fields. In other words, they enhanced the FRBS framework by incorporating the Sugeno fuzzy model, commonly known as the Takagi-Sugeno-

Kang (TSK) model. This model replaced Mamdani's system's linguistic output variable with a crisp function, allowing for precise modeling and control (Takagi & Sugeno, 1985).

FRBSs were hybridized with other computational intelligence approaches by researchers in the 1990s. Hybrid models that combine fuzzy logic with neural networks, genetic algorithms, and expert systems emerged, improving the capabilities and application of FRBSs. Jang (1991) introduces a novel fuzzy inference system that combines generalized neural networks (GNNs), the gradient descent algorithm, and the Kalman filter algorithm. The study presents the concept of GNNs and develops a supervised learning procedure for updating parameters. The results highlight the superior accuracy and flexibility of the proposed fuzzy modeling approach, demonstrating its potential in various applications. This innovative methodology integrates neural networks, fuzzy logic, and the Kalman filter algorithm to enable effective system identification and prediction.

Yager and Filev (1994) integrated FRBSs with genetic algorithms. Yager and DeMaria suggested a genetic algorithm-based method for evolution of fuzzy rule-based systems in 1995. This method enabled the automatic development and optimization of fuzzy rule sets based on evolutionary ideas, enhancing adaptability improving performance of FRBSs. Gegov and Frank (1995) introduce a distributed fuzzy control method for effectively controlling multivariable systems. The authors propose a decomposition technique to break down complex systems into smaller subsystems, enabling independent control using fuzzy logic. The study evaluates the approach through simulation studies and compares it to centralized control strategies. The results indicate that the distributed fuzzy control method achieves comparable or even superior control performance in comparison to