Second Semester Examination
2022/2023 Academic Session

July / August 2023

EMT212 – Computational Engineering
(*Kejuruteraan Pengkomputeran*)

Duration: 3 hours
(Masa: 3 Jam)

Please check that this examination paper consists of SIX (6) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM (6) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer ALL **FIVE (5)** questions.

*[**Arahan**: Jawab SEMUA **LIMA (5)** soalan]*

1. (a) Oscillating pressure measurements $u(t)$ from a turbine are taken over time t. The data best fits the curve as in **Figure 1**. State ONE advantage and TWO disadvantages of the Golden Section search method if it is used to find the minimum value in the data.

(3 marks)

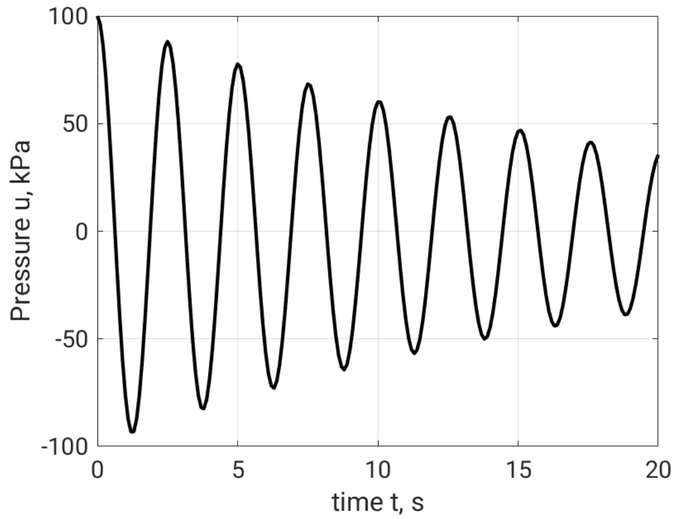

Figure 1

(b) With the aid of a sketch, explain how the Lagrange multiplier can be used to find the shortest distance from the point (x_0, y_0, z_0) to the plane $ax + by + cz = d$. Do not show any calculations.

(2 marks)

(c) Explain why the slack variables are necessary in solving an optimization problem with the simplex method.

(2 marks)

(d) The continuity equation can be applied to describe the flow of crude-oil in a pipeline. With the aid of a sketch, explain why the divergence theorem can have used to derive the continuity equation for this application.

(3 marks)

...3/-

2. Consider a right tetrahedron as shown in **Figure 2** where its vertices are at $(0,0,0)$, $(a,0,0)$, $(0,a,0)$, and $(0,0,a)$. The tetrahedron floats in water such that HALF of its height is in water. Assume its base is parallel to the surface of the water.

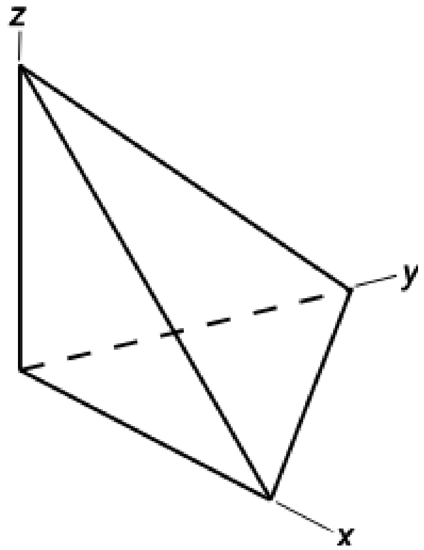

Figure 2

(a) Sketch the vector field $\mathbf{f}(x,y,z)$ of force due to the water pressure on the immersed part of the tetrahedron. You may use a 2-D sketch to simplify the vector field.

(6 marks)

(b) Express the magnitude of the total force $\mathbf{F}$ on the tetrahedron due to the water pressure in terms of the surface integral over the affected area. DO NOT evaluate the integral.

(7 marks)

(c) Use the divergence theorem to express the magnitude of the buoyancy in terms of a and the density of water ρ by evaluating the integrals.

(7 marks)

3. Using the divergence theorem, derive the transient heat equation in the Appendix.

(8 marks)

(a) State at least TWO assumptions that validate the derivation.

(5 marks)

(b) State at least ONE condition to convert the equation into the steady-state equation.

(2 marks)

4. A long thin rod with a length of 10 cm, is insulated at all points except at its ends. At $t = 0$, the temperature of the rod is zero, and the left end of the wire is fixed at $60^{\circ}C$ and the right end is fixed at $20^{\circ}C$ for all times. The heat generation in the rod is absent and thermal conductivity of the rod is taken as $k = 0.925\ cm^2/s$.

   (a) Write the mathematical expression, the initial condition, and the boundary conditions for the stated transient heat conduction problem.

   (4 marks)

   (b) For the transient heat problem stated, find the temperature distribution of the rod using finite difference implicit method for 5 spatial and 3 temporal grid points. Let $\Delta x = 2.5$ cm. $\Delta t = 0.1$ s.

   (24 marks)

   (c) Using central finite difference scheme, find the temperature distribution of the rod when it reached steady state conditions. Use 5 grid points for your calculations. Let $\Delta x = 2.5$ cm.

   (8 marks)

   (d) Sketch the temperature distribution in the rod for $t = \{1,2,3\}$ s and steady state condition in the same curve.

   (4 marks)

5. A steady-state heat conduction problem in a 10-cm wire is governed by the following differential equation,

$$-2\frac{\partial^2 u}{\partial x^2} = x$$

   The left end of the wire is fixed at $10^{\circ}C$ and the right end is fixed at $50^{\circ}C$.

   This problem is to be solved using the central finite difference scheme and is discretized into **N** number of grid points. Write a **MATLAB code** to construct the system matrix **A** and vector **b** using **FOR** function**.**

   DO NOT write the complete code to solve the linear system.

   (15 marks)

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APPENDIX 1

1. Newton's Method

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

2. Formulas for first finite differences

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + O(h)$$
$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$
$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$$

3. Formulas for second finite differences

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O(h)$$
$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2} + O(h)$$
$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} + O(h^2)$$

4. Heat equation

$$-\alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = Q(x,t)$$

5. Convective boundary condition

$$hu + ku' = hu_\infty$$

6. Discrete form of 1D Poisson's equation

$$-k \frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} = f_i$$

7. Explicit and implicit methods for heat equation

$$-\lambda(u_{i+1}^l - 2u_i^l + u_{i-1}^l) = u_i^{l+1} - u_i^l - sf_i^{l+1}$$

$$-\lambda u_{i+1}^{l+1} + (1 + 2\lambda)u_i^{l+1} - \lambda u_{i-1}^{l+1} = u_i^l + sf_i^{l+1}$$

$$\lambda = \frac{\alpha s}{h^2}$$

8. Integrals of sine and cosine

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$$
$$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$$

9. Spherical coordinates

$$x = \rho \sin\varphi \cos\theta\,;\;\; y = \rho\sin\varphi\sin\theta\,;\; x = \rho\cos\varphi$$
$$\rho \geq 0; 0 \leq \;\varphi \leq\; \pi$$
$$dV = \rho^2 \sin\varphi\;\; d\rho d\theta d\varphi$$

10. Cylindrical coordinates

$$x = r\cos\theta\,;\;\; y = r\sin\theta\,;\; z = z$$
$$dV = rdzdrd\theta$$

11. Taylor series at point a

$$u(x) = u(a) + u'(a)(x-a) + u''(a)\frac{(x-a)^2}{2!} + u'''(a)\frac{(x-a)^3}{3!} + \cdots$$
$$\ldots + u^{(n)}(a)\frac{(x-a)^n}{n!} + \cdots$$

12. Volumes of selected shapes

Sphere: $\frac{4}{3}\pi r^3$ Tetrahedron: $\frac{a^3}{6\sqrt{2}}$ Cone: $\frac{1}{3}Ah$

13. Miscellaneous

$$\nabla \times \mathbf{u} = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)\mathbf{k}$$

$$\oint_C \; M(x,y)dx + N(x,y)dy = \iint_D \; \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$$\oint_{\partial S} \mathbf{F}(x,y,z)\cdot d\mathbf{r} = \iint_S \; (\nabla \times \mathbf{F})\, dS$$

$$\oint_S \; (\mathbf{F}\cdot\mathbf{n})\, dS = \iiint_V \; \nabla\cdot\mathbf{F}\, dV$$