Second Semester Examination
2023/2024 Academic Session

July / August 2024

EMT 212 – Computational Engineering
(Kejuruteraan Pengkomputeran)

Duration: 3 hours
(Masa: 3 Jam)

Please check that this examination paper consists of FIVE (5) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA (5) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer ALL **FIVE (5)** questions.

*[**Arahan**: Jawab **LIMA (5)** soalan]*

1. (a) State an example of the stopping criterion for the golden section search method so that the final value is correct within the double precision.

(3 marks)

(b) The divergence theorem can be used to derive these relationships in physics because certain physical quantities are being balanced:

(i) The heat equation
(ii) The continuity equation
(iii) Archimedes' principle

State the quantity that is balanced for each relationship.

(3 marks)

(c) Heat spreaders are commonly used to dissipate heat in mobile devices. Explain all possible types of boundary condition that can be specified to model the heat transfer in a heat spreader.

(4 marks)

2. Consider the profit of Toyota from its sales of Corolla Cross hybrid and gasoline-only variants. The profit from selling x unit hybrid and y unit gasoline-only variants is expressed by:

$$f(x, y) = 3x^2 + y^2 - 2000x.$$

Toyota cannot manufacture more than 5000 total units of the Corolla Cross model in each month. Determine the maximum profit the company can make from the sales of the two variants.

(15 marks)

3. Suppose Dell manufactures 3 models of laptop computers: Basic, Business, and Gaming. The models sell for RM 1500, RM 2400, and RM 3400, respectively. The requirements for each model are:

- The Basic model requires 3 hours for circuit board installation and 1 hour to fit the peripheral equipment.
- The Business model requires 1 hour for circuit boards and 5 hours for peripherals.
- The Gaming model requires 3 hours for circuit boards and 2 hours for peripherals.

...3/-

Dell allocates 1200 hours for circuit board work and 600 hours for fitting peripherals. The goal is to determine the quantities for each model Dell should manufacture to maximize the revenue.

(a) Write the mathematical statement of the problem with inequality constraints.

(2 marks)

(b) Write the mathematical statement of the problem with equality constraints.

(2 marks)

(c) Construct the initial tableau. Explain the meaning of the basic variables for the tableau.

(4 marks)

(d) Perform the initial calculation of the simplex method until the Gaussian elimination and explain the next steps to reach the final answer.

(7 marks)

(e) Write the values of the initial tableau as a MATLAB array including the intercept values in the last column.

(5 marks)

4. A rod made of aluminum has a length of 10 cm and an initial temperature of 25°C. The rod is insulated except at both ends. The temperature at the left end side of the rod is 10 °C and 50°C at the right end. The heat generation is absent.

The material properties of aluminum are given as $k = 237\ W\ m^{-1}K^{-1}$, $c_p = 900\ J\ kg^{-1}K^{-1}$, $\rho = 2700\ kg\ m^{-3}$.

(a) Write the mathematical expression of the given problem

(2 marks)

(b) State the initial and boundary conditions of the given problem

(3 marks)

(c) Using $\Delta x = 2.5\ cm$ and $\Delta t = 1\ s$, find the temperature distribution along the rod at $t_n = \{1,2,3,4\}s$ using **explicit** formulation.

(20 marks)

(d) Determine the temperature distribution along the rod when it reaches the **steady state** condition.

(5 marks)

...4/-

(e) **Sketch** the temperature distribution obtained in **Question (c)** and **Question (d)**.

(5 marks)

(f) If the step time were to increase to $\Delta t=5s$, determine the effect of the approximated solution obtained. Justify whether the new value is acceptable. **(Do not solve the problem)**.

(5 marks)

5. Write a complete **MATLAB code** that can be used to produce the temperature distribution for the problem given in **Question 4.** Use **nested FOR** loops to solve for **explicit** formulation**.**

(15 marks)

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APPENDIX 1

1. Formulas for first finite differences

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + O(h)$$
$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$
$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$$

2. Formulas for second finite differences

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O(h)$$
$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2} + O(h)$$
$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} + O(h^2)$$

3. Heat equation

$$-\alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = Q(x,t)$$

4. Convective boundary condition

$$hu + ku' = hu_\infty$$

5. Discrete form of 1D Poisson's equation

$$-k \frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} = f_i$$

6. Explicit and implicit methods for heat equation

$$-\lambda(u_{i+1}^l - 2u_i^l + u_{i-1}^l) = u_i^{l+1} - u_i^l - sf_i^{l+1}$$

$$-\lambda u_{i+1}^{l+1} + (1 + 2\lambda)u_i^{l+1} - \lambda u_{i-1}^{l+1} = u_i^l + sf_i^{l+1}$$

$$\lambda = \frac{\alpha s}{h^2}$$