STEADY HEAT CONDUCTION SOLUTION USING TRIGONOMETRIC BEZIER FINITE ELEMENT METHOD

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by

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LIST OF ABBREVIATIONS

CAD	Computer Aided Design
CAGD	Computer Aided Geometric Design
FEA	Finite Element Analysis
FEM	Finite Element Method
IGA	Isogeometric Analysis
NURBS	Non Uniform Rational B-Spline
2D	Two-dimensional

LIST OF SYMBOLS

k	Thermal conductivity
T	Temperature
q	Heat source/ heat flux
N_i	Shape function
k_T	Local stiffness matrix
r_T	Local element vector
K_e	Global stiffness matrix
R_e	Global element force vector

PENYELESAIAN KONDUKSI HABA MANTAP MENGGUNAKAN KAEDAH UNSUR TERHINGGA TRIGONOMETRI BEZIER

ABSTRAK

Kaedah Unsur Terhingga (KUT) adalah kaedah berangka yang digunakan dalam kejuruteraan dan pemodelan matematik bagi menyelesaikan beberapa persamaan pembezaan separa. KUT menggunakan poligon asas seperti segi tiga dan segi empat sebagai elemen dalam penyusunan model. Oleh kerana sisi bentuk-bentuk asas ini yang kaku, ia telah menghasilkan tepi yang tajam dan terhad dalam mengendalikan geometri yang tidak sekata atau berlekuk. Untuk mengatasi masalah ini, penghalusan jejaring perlu dilakukan bagi mengekalkan bentuk geometri, tetapi ia menyebabkan peningkatan jumlah elemen dan masa pengiraan. Dalam Analisis Elemen Terhingga (AUT) isogeometri, fungsi splin digunakan sebagai fungsi asas. Analisis Isogeometri (AIG) adalah teknik yang dibangunkan dalam mekanik pengiraan dengan menggabungkan analisis dan proses reka bentuk menjadi satu proses tunggal. Teknik ini berkelebihan dalam mengekalkan ketepatan geometri, demikian dapat mengurangkan jurang antara reka bentuk geometri berbantukan komputer dan AUT. Kebiasaannnya, Splin-B Rasional Tidak Seragam (SBRTS) dan Bernstein-Bézier digunakan sebagai fungsi asas dalam AIG. Walau bagaimanapun, fungsi asas Trigonometri Bézier akan digunakan dalam kajian ini untuk menyelesaikan masalah konduksi haba dalam saluran paip melengkung dua dimensi. Secara ringkasnya, keputusan yang dicatatkan menggunakan kaedah Trigonometri Bézier adalah memberangsangkan. Purata ralat yang dicatatkan adalah rendah dibandingkan dengan kaedah sedia ada seperti Bernstein Bézier.

STEADY HEAT CONDUCTION SOLUTION USING TRIGONOMETRIC BEZIER FINITE ELEMENT METHOD

ABSTRACT

The Finite Element Method (FEM) is a numerical technique used to solve several forms of partial differential equations, which are commonly utilized in engineering and mathematical modelling. Basic polygons such as triangles and quadrilaterals are used as element shapes in FEM. Due to the rigid sides of these basic shapes, they have resulted in sharp edges and are limited in handling irregular or curved geometries. To address this issue, mesh refinement is required to maintain the original geometry of the model, resulting in a larger number of elements and an increase in computational time. The spline functions are used as basis functions in isogeometric Finite Element Analysis(FEA). Isogeometric analysis (IGA) is a technique that recently developed in computational mechanics that offers the possibility of integrating the analysis and the design process into a single and unified process. This technique has the advantage of providing seamless integration of accurate geometry, thus bridging the gap between computer-aided geometric design and finite element analysis. Commonly, nonuniform rational B-splines (NURBS) and Bernstein-Bézier are used as basis functions in IGA. However, in this study, Trigonometric Bézier basis function will be used to solve the heat conduction problem in a two-dimensional curvilinear duct pipe. In summary, the findings indicate that the results obtained using the Trigonometric Bézier method are promising. The mean error recorded is minimal compared to the existing method, namely the Bernstein Bézier.

CHAPTER 1

INTRODUCTION

1.1 Background Study

The definition of heat transfer can be described as the flow of heat within two mediums due to the temperature difference or something that flows from hot objects to cold one (Lienhard, 2005). This process occurs due to molecular interaction, fluid motion, and electromagnetic waves that result from a spatial variation in temperature (Sharma, 2017). There are three modes of transferring heat between two bodies that work relatively to each other, which are: conduction, convection, and thermal radiation. All type can be drawn into specific equation through theorem and laws that fundamentally found in the laws of conservation of energy, momentum, and mass (William Moebs and Sanny, 2019). Alongside with three mentioned modes of heat transfer, heat conduction plays a significant role in various industries and daily life applications, namely cooling fins or extended surfaces, solidification and melting of metals and alloys in metallurgical industries, welding, metal cutting, nuclear heating, periodic temperature variations of the earth's surface, and heating and cooling of buildings (Roos, 2008).

Initially, this problem related to partial differential equations (PDEs) and can be solved computationally using numerical method. Finite Element Method (FEM) is one of numerical method which defined as a computational technique used to obtain approximate solutions of boundary value problems and most widely used in engineering field. In FEM, it is necessary to use mathematics to clearly understand and quantify the physical phenomena for instance, biological growth cells, thermal transport, wave propagation and all these processes are shown by using Partial Differential Equation (PDEs) which interpreted in governing equation. Generally, numerical method is just a process of converting the governing equation into matrix form, before solving it computationally. The only thing that distinguishes between them is their arguments respectively on solving problems. The fundamental idea of the FEM is to discretise the domain into several subdomains, or finite elements (Chao and Chow, 2002) or usually called elements, which used basic polygon shapes such as triangles and quadrilaterals.

The history of FEM can be traced back to the work of mathematicians and scientists in the mid-19th century. The earliest manuscript on FEM were found in the works of Schellbach and Courant in 1851 and 1943, respectively. However, the development of FEM for structural mechanics problems related to aerospace and civil engineering was independently worked by engineers beginning in the mid-1950s with the papers of Turner et al. (1956), Argyris (1957), and Babuska and Aziz (1972). The books by Strang and Fix (1969) and Zienkiewicz et al. (2005) also laid the foundations for future development in FEM.

Further advancements in FEM have continued to shape the field since its beginning. The introduction of computer technology in the 1960s allowed for more complex and accurate simulations, leading to widespread use of FEM in industries such as automotive and aerospace engineering. Additionally, the development of adaptive meshing techniques has allowed for more efficient and precise modeling of complex geometries. Recent advancements in parallel computing have also significantly increased the speed and scalability of FEM simulations, making it a valuable tool for solving largescale problems in fields such as physics and biology. As research continues to push the boundaries of FEM, it remains a crucial tool for engineers and scientists seeking to understand and optimize real-world systems.

1.2 Problem Statement

Basically, the basic polygon such as triangle and quadrilateral are used as element shapes in standard FEM. On the other hand, it is a debatable topic among researchers between triangular mesh and quadrilateral mesh on their practicality, either triangular or quadrilateral is the best mesh method. Nevertheless, these basic shapes are limited for meshing some irregular geometries due to the stiffer side that produces sharp edges, which require mesh refinement to keep the original shape of the problem model. However, instead of using mesh refinement, a combination of Computer Aided Design (CAD) and FEM can be used to solve the problem, leading to Isogeometric Analysis (IGA) as mentioned by Cottrell et al. (2009). Isogeometric methods reduce the approximation errors in the mesh since the geometry is accurately defined. Instead of using Non-Uniform Rational B-Splines (NURBS), and B-splines, Bernstein-Bezier basis functions can be used as shape functions, as done by Peng et al. (2020), Do et al. (2020), Hackemack (2021) and Song et al. (2023).

1.3 Motivation

Based on the findings of previous research, it has been observed that most studies on IGA have predominantly focused on the use of NURBS, B-splines, and Bernstein basis functions. However, trigonometric basis functions have not been extensively explored. Therefore, the current research aims to investigate the efficiency of trigonometric basis functions in FEM for the solution of steady heat conduction problems. The proposed approach seeks to evaluate whether the inclusion of Trigonometric basis functions in FEM can provide a more accurate solution to the steady heat conduction problem compared to existing methods that rely on other basis functions. The research aims to establish the suitability of trigonometric basis functions in the context of solving heat transfer problems and whether the proposed method can be implemented directly with standard FEM.

Through this study, we seek to contribute to the body of knowledge on IGA and FEM by exploring the effectiveness of trigonometric basis functions as an alternative to NURBS, B-splines, and Bernstein basis functions. The outcomes of the research will provide valuable insights for future works, and the proposed approach may also be used in practical applications related to heat transfer.

1.4 Objectives of the Study

In this study we would like to propose the following objectives:

i) To propose a new basis (shape function) of FEM in solving 2D heat transfer problem.

ii) To solve 2D heat transfer problem using FEM on different types of basis function.

iii) To compare the accuracy and efficiency of existing Bezier FEM method and proposed method with the exact solution.

1.5 Thesis Organization

There are five chapters overall in this thesis. In Chapter 1, heat transfer, FEM and IGA were briefly explained in background study. Problem statement, motivation and objectives are also included in this chapter. In Chapter 2, literature review is presented regarding heat conduction problem, FEM and IGA. In Chapter 3, a new method regarding FEM shape function was proposed and will be utilised on solving two-dimensional heat conduction problem. In Chapter 4, the presentation of the result and data from method used in previous Chapter 3 regarding heat conduction problem are obtained. Finally, Chapter 5 presents the conclusions drawn from the research findings, as well as recommendations for future research in the field. This research organization is designed to provide readers with a comprehensive and logical flow of information, enabling them to understand the research methods, results, and implications in a structured and efficient manner.

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

2.1 Study of Heat Conduction

Heat conduction is a crucial process with numerous applications in various fields such as geological sciences, mechanical engineering, and metallurgical industries. To analyze the thermal stress condition of a material, the temperature distribution must be obtained by solving the heat conduction equation. This equation serves as the starting point for analyzing any phenomena related to heat conduction.

Over the past few years, there have been significant advancements in the understanding and control of heat conduction. Researchers have made progress in both fundamental research and applied research related to heat conduction. For example, Zhang et al. (2008) used a perturbation method to solve the heat transfer of viscoelastic fluids in curved pipes. They made several assumptions, including steady fluid flow, hydrodynamically and thermally fully developed, and negligible viscous dissipation.

Dean (1927) and Mitsunobu and Cheng (1971) earlier research revealed that the convective heat transfer and the Nusselt number in curved pipes are more efficient than those in straight pipes. More researchers are now exploring convective heat transfer in various duct types. In 1998, Garimella et al. investigated forced convective heat transfer in coiled annular ducts through experiments. Same goes with Yang and Ebadian (1993), which investigated the problem of convection heat transfer in an annular sector duct but solved by using a numerical method solution. Chen and Zhang (2003) continued the research by extending Yang and Ebadian's work to a rotating helical pipe. These research contributions have significantly enhanced the understanding of heat conduction and its applications in various fields.

While Kareem et al. (2023) presented new analytical solutions for heat conduction in cylindrical and spherical bodies, which are then validated using explicit and unconditionally stable finite difference methods. The accuracy of these methods is compared to commercial software, and it is shown that the explicit methods are more accurate. Kareem et al. also considered convection and nonlinear radiation on the boundary of the cylinder and demonstrated the accuracy of the explicit numerical methods in reproducing real experimental data. Recent study by Alvarez Hostos et al. (2023) introduces a novel Overset Improved Element-Free Galerkin-Finite Element Method (Ov-IEFG-FEM) for solving transient heat conduction problems with concentrated moving heat sources. Numerical experiments demonstrated the effectiveness of the method in accurately and efficiently solving transient heat conduction problems with concentrated moving heat sources.

2.2 Fundamental of Heat Conduction

The fundamental idea of heat conduction is that heat can be transfered from one body to another through a material medium by means of molecular motion. In other words, when two bodies of different temperatures are brought into contact with each other, heat will flow from the hotter body to the colder body until they reach thermal equilibrium. In the mid-18th century, the French physicist Jean-Baptiste Fourier introduced the mathematical concept of the Fourier series, which can be used to represent any periodic function as a sum of sine and cosine functions. Fourier's work on heat conduction was motivated by his interest in the temperature distribution within solids, and he developed the Fourier law of heat conduction, which describes the rate of heat transfer through a material. Fourier's law was introduced by Fourier in his remarkable book *Théorie Analytique de la Chaleur*, published in 1822, where he began by stating the empirical law known as heat flux, q (W/m^2) which is:

$$q = -k\frac{dT}{dx} \tag{2.1}$$

where k is thermal conductivity and T is temperature and x is the domain of the shape model or spatial coordinate along the direction of heat transfer. Thermal conductivity is a unique property of every medium or substance for conducting the heat flow for instance, the thermal conductivity of pure copper is 399 (W/m K) while pure gold is 317 (W/m K) as shown in Table 2.1.

Table 2.1: Thermal conductivity of medium

Material	Thermal conductivity (W/mK)
Copper (pure)	399
Gold (pure)	317
Aluminum (pure)	237
Iron (pure)	80.2
Carbon steel (1 %)	43
Stainless Steel (18/8)	15.1
Glass	0.81
Plastics	0.2 – 0.3
Wood (shredded/cemented)	0.087
Cork	0.039
Water (liquid)	0.6
Ethylene glycol (liquid)	0.26
Hydrogen (gas)	0.18
Benzene (liquid)	0.159
Air	0.026

Derivation of heat equation basically rooted and governed by the principle of conservation of energy. The term conservation means something which does not change. In conservation of thermal energy, the equation is the time rate of change of internal energy is equal to the net heat flowing into the differential element and the derivation of this equation was taken from Yassin et al. (2020) in their textbook as reference whereby,

$$
\begin{aligned}
Q_E &= Q_x + Q_y + Q_h; \\
Q_E &= \rho C \frac{\partial T}{\partial t} dxdy, \\
Q_x &= -\frac{\partial q_x}{\partial x} dxdy, \\
Q_y &= -\frac{\partial q_y}{\partial y} dxdy, \\
Q_h &= q_h dxdy.
\end{aligned}
\tag{2.2}
$$

where,

- Q_E represents the rate of change of internal energy within the material with respect to time. In other words, it describes how the internal energy of the material changes over time due to heat transfer.
- Q_x represents the heat flow in the x-direction. It is calculated as the negative gradient of q_x, the heat flux (heat flow per unit area) in the x-direction, with respect to x. Essentially, it quantifies how heat is flowing in or out of the material in the x-direction.
- Q_y represents the heat flow in the y-direction. It is calculated as the negative

gradient of q_y, the heat flux in the y-direction, with respect to y.

- Q_h represents any heat sources or sinks within the material. It is represented by q_h, the heat generated per unit volume within the material.
- dx represents a very small change in the x-direction.
- dy represents a very small change in the y-direction.

While ρ is the density of substance, C is heat capacity of substance and q is heat flux. By substituting back the equation of Q_E, Q_x, Q_y and Q_h implies:

$$\rho C \frac{\partial T}{\partial t} dxdy = -\frac{\partial q_x}{\partial x} dxdy - \frac{\partial q_y}{\partial y} dxdy + q_h dxdy. \tag{2.3}$$

By simplification, yields

$$\begin{aligned} \rho C \frac{\partial T}{\partial t} &= -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} + q_h; \\ q_x &= -k_x \frac{\partial T}{\partial x}, \\ q_y &= -k_y \frac{\partial T}{\partial y}. \end{aligned} \tag{2.4}$$

So the following general equation of heat equation was obtained:

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + q_h, \tag{2.5}$$

which known as the Fourier's law of heat conduction. Ngarisan (2016) proposed this kind of heat equation in term of heat conduction problem and was presented as

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + Q = 0 \tag{2.6}$$

where Q represents as customization equation for heat source in their study and implemented a steady-state scenario problem which make the equation equals to 0. Heat transfer problem generally can be solved by analytical solution, but for some real life problems it is very difficult to be done because of the complexity of the problems and required tedious work. Instead of analytical solution, heat transfer also can be solved by numerical method and become the main purpose of this study. Conduction problem depends on the nature of the conduction process, all conduction processes are divided broadly into two main categories, which are steady-state condition and unsteady-state condition as stated by Ghoshdastidar (1998). Steady-state condition basically is time independent which means variable related to the conduction problem, temperature and density that can be easily solved using analytical and numerical method.

Unsteady state means that temperature and density are changing with time and has two categories, which are periodic and transient. Transient means that every variable involved (temperature, heat flux, etc.) occur in a short period caused by a sudden change of state. As for periodic, the event happening in cycle or repetitive for instance, the daily variation of earth's temperature due to solar effects. However, the highlight of this study is only for steady-state heat conduction problem.

2.3 Numerical Method in Solving Heat Conduction Problem

As mentioned in previous chapter, the heat conduction problem can be solved using analytical and numerical approaches. One of the numerical approach that can be used is FEM. FEM is a technique used to obtained approximate solution for any differential equation problem, usually applied in engineering and has been approved by most researchers in their papers and textbooks. Hutton (2004) in their textbook stated that FEM is a computational technique to obtained approximate solution of boundary value problem, this statement was supported by Mueller Jr (2005) which stated that FEM is a powerful and versatile tool for practicing engineers that can be used to solved a wide variety of important engineering problems. In research works by Yao et al. (2007), Azmi (2010), Ngarisan (2016), and Papathanasiou et al. (2017), FEM was been used to approximate the differential equation of boundary value problem for heat conduction problem in two-dimensional. Recent research on heat transfer using FEM was also done by Reddy (2020) which presented a numerical study on the heat transfer characteristics of convection in a vertical channel using FEM simulation. The study analyzes various parameters such as velocity, temperature, concentration, the Nusselt number, and the Sherwood number to interpret their behavior in the context of the flow and heat transfer. While Eso et al. (2023) explore the heat flow transfer in different types of materials using an open-source simulation and the FEM. The study discussed the design of material structures with heterogeneities, such as composites, and their thermal behavior and heat flow processes. The results shown that each domain has different temperature values based on the point and time used, indicating the need for further research on other types of heterogeneous materials.

2.4 General Procedure for FEM

Hutton (2004) proposed the procedure for FEM into three steps, which are pre-processing, solution and post-processing. In the pre-processing step, Hutton generally describes and defines the model into examples, which are

- Problem of geometric domain.
- The element type(s) to be used.
- Material properties of the elements.
- Geometric properties of the elements (length, area, etc).
- Element connectivities (model meshing).
- Physical constraints (boundary conditions).
- Loadings.

During processing or solution phase, finite element software like MATLAB, Mathematica, ANSYS and Abaqus is used to assemble governing algebraic equations into matrix form and compute the unknown values of the primary field variables. The computed values are then used by back substitution to compute additional, derived variables such as reaction forces, element stresses and heat flow. While in the post-processing step involved the analysis and evaluation of the solution results for example:

- Sort element stresses in order of magnitude.
- Check equilibrium.
- Calculate factors of safety.
- Plot deformed structural shape.
- Animate dynamic model behaviour.

- Procedure color-coded temperature plots.

Following by these steps, Yao et al. (2007) stated the pre-processing step in their study by defining and discretising the shape elements into finite element mesh then generated model material and properties. Moving on to the solution phase, the loading history and predetermined parameters governing the solution are established using model function tools. This allows for the creation of loads or material properties as needed. In post-processing step, initial time step size and total number of time steps by using file analyse operation to specify the total solution time in the analysis control tools. In Ngarisan (2016), the general procedure of FEM was presented in the flowchart as shown in Figure 2.1.

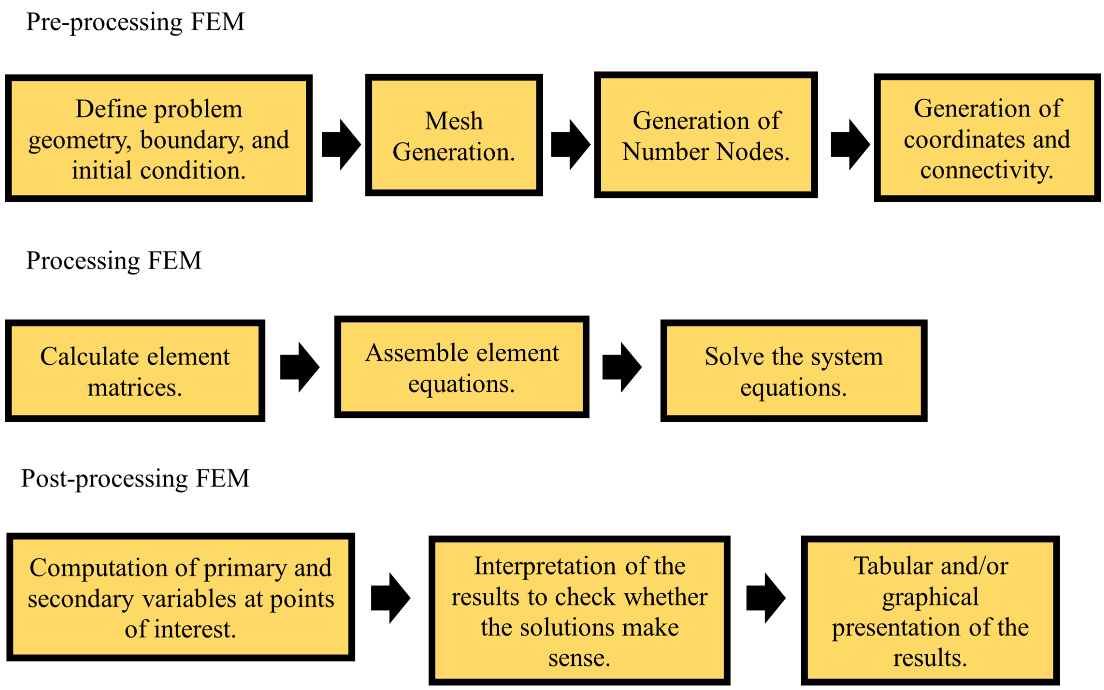

Figure 2.1: Flowchart of pre-processing, processing and post-processing of FEM by Ngarisan (2016)

For a better view regarding the procedure, the basic steps in FEM for solving any

PDEs problems will be briefly explained .

a) Element discretization

In this process, the geometry of the problem is model by an assemblage of small area regions (finite element), to form a finite elements mesh. These elements have nodes defined on the element boundaries or within the element. Next, the shape and geometry properties of the element then will be determined.

b) Element equation

For computational purposes, the variation formulation of the problem was constructed in the subsequent step. Based on discretization, shape function can be determined (triangular, rectangular, quadrilateral etc). Different function will lead to the different type of weighted residual methods which is used to obtain the local stiffness matrix in the FEM. Galerkin's method is a famous method as it is quite simple to use and readily adaptable to the finite element method. Assemblage of the element will produce the equation in the form, which

$$K^{(e)}T^{(e)} = F^{(e)}; \tag{2.7}$$

where $K^{(e)}$ is element stiffness matrix, $T^{(e)}$ is element displacement vector, $F^{(e)}$ is element vector, and (e) is the element.

c) Global equation

All element equation will be combined into equation to form global equation. The combination of the equation applied using principle of matrix superposition property.

d) Boundary conditions

The existed or given boundary conditions is applied and will modify the equation into the more simple equation.

e) Solve the global equation

The global equations will be obtained before it can be solved. By solving these equations, the displacement at all nodes will be determined. The quantities such as stress and strain can be determined from these nodal values.

2.5 FEM Modification

The FEM modification are basically the changes of standard FEM function by adding or changing some features in term of basis or shape function, mesh refinement or integration with another approach which yield to a new method. Based on the classical finite element method, some scholars have proposed several new methods to solve partial differential equations (PDEs) problems numerically. Baumann and Oden (1999) presented a discontinuous hp finite element method (escalating the polynomial degree and elements size) that is suitable for solving convection-diffusion problems in convective dominance. The hp-version Finite Element Method (FEM) is an advanced numerical technique that combines the benefits of both the h-version and p-version FEM. In hp-FEM, both the mesh size (h) and the polynomial order (p) of the basis functions are varied to achieve higher accuracy and efficiency in solving PDEs and approximating solutions over a domain. Baumann and Oden presented a new method that combines the best features of finite volume and finite element techniques, with special attention given to conservation, flexible accuracy, and stability. A priori error estimates and numerical experiments indicate that the method is robust and capable of delivering high accuracy.

Adjerid and Klauser (2005) proposed a local discontinuous Galerkin method for solving one-dimensional transient convection-diffusion problems, which was accompanied by an asymptotically posterior error estimation. Maleknejad and Mirzaee (2005) introduced stable numerical algorithms for heat convection and integral equations. Liu et al. (2008) studied a mixed time discontinuous space-time finite element method that lowered the order of equations for solving convection-diffusion equation. Jin et al. (2016) proposed a Petrov-Galerkin finite element method that combined "shifted" fractional powers with continuous piecewise linear elements to construct the test and trial function spaces.

Recently, Mirzaee et al. (2021) proposed a new meshless method that uses a proposed basis function to solve 2D time fractional Tricomi equations and stochastic time-fractional sine-Gordon equations. While a study from Alvarez Hostos et al. (2023) proposed the method combines the Improved Element-Free Galerkin (IEFG) technique with the Finite Element Method (FEM) to accurately capture thermal gradients near the heat source using a separate set of overlapping nodes (patch nodes) that move with the heat source, while solving the thermal problem outside the heat source area using FEM on a coarse finite element mesh.

2.6 Splines and FEM Relation

The use of spline basis functions has become increasingly popular in the field of finite element analysis due to their ability to provide better numerical solutions. There are various types of spline basis functions that can be utilized to construct function spaces for finite element analysis, including Bernstein, B-spline, and NURBS functions.

Bernstein basis functions are often used to construct polynomial space and can be used to create function spaces for finite element analysis. Over the years, researchers have focused on developing spline functions with excellent geometrical properties. One such example is the spline finite element method, which was introduced by Shi in 1979 to solve the equilibrium problem of plate-beam composite elastic structures in regular regions. Shi derived a unified computation scheme for various boundary conditions using spline basis functions.

In 2005, Hughes et al. proposed a new spline finite element method using NURBS basis functions for finite element analysis. This approach introduced the concept of Isogeometric Analysis (IGA), which aimed to bridge the gap between computer-aided design (CAD) and finite element analysis. However, Sun and Su (2022) stated that NURBS basis functions fail to express transcendental curves such as the cycloid and helix, and their rational form is unstable. This increases the complexity of computing integrals and derivatives. Besides, Bhatti and Bracken (2007) proposed a method that combines Bernstein basis functions and the Galerkin method to achieve better results in solving partial differential equations.

To overcome disadvantages of rational Bernstein basis functions, scholars have made significant efforts to develop new basis functions. In 2003, Lü et al. proposed an integral approach to construct C-Bézier basis functions for the space T = span$\{1, u, \ldots, u^{n-2}, \sin u, \cos u\}$, which extended the spaces of mixed algebra and trigonometric polynomial. Recently, Sun and Su (2022) combined C-Bézier and H-Bézier basis functions with the Galerkin finite element method to solve the convection-diffusion equation. This approach demonstrated excellent numerical performance and robustness, making it a promising technique for solving partial differential equations.

2.7 Isogeometric Analysis Review

Computer Aided Design (CAD) and Finite Element Analysis (FEA) are frequently utilised in engineering courses. FEA was developed with the aim of enhancing the accuracy and reliability of engineering analyses. On the other hand, CAD was developed to streamline the design process and improve its efficiency. The evolution of FEA can be traced back to the 1940s, while CAD reached maturity as a field in the 1970s. This phenomenon can be attributed to the utilisation of various mathematical models to depict a single model. The present study employs trivariate polynomials of low order, typically one or two, to approximate the solid object in the context of FEA. Conversely, the same model is represented by non-uniform rational B-splines (NURBS). The transfer of a CAD model to a FEA model necessitates the use of mesh generators, which are a technology that converts the CAD model into a finite element (FE) mesh that is appropriate for FEA, owing to the disparity in geometric representation. The process of meshing intricate structures is known to be a time-intensive task, often surpassing the duration of the analysis itself. Furthermore, in the event that there is a need to alter the geometry of the object, it is necessary to repeat the laborious process of meshing.

The first work that attempted to link CAD and FEA was the work of Kagan et al.

in 1999 where B-splines were utilized to represent the solid geometry in the FE model. Therefore, both CAD and FEA models employ the same technology B-splines to construct the desired model. Along this line of research, another notable contribution was made by Cirak et al. (2000) in which subdivision surface, which is a CAD technology extensively used in computer animation, was used in a finite element thin shell model. The idea then was generalized, and a new field was emerged called IGA by Hughes et al. in 2005 where NURBS were adopted in FE solid, structural and fluid mechanics models. Isogeometric analysis (IGA) introduced technique that employs the Computer Aided Design (CAD) concept of Non-uniform Rational B-splines (NURBS) tool to bridge the substantial bottleneck between the CAD and finite element analysis (FEA) fields (Agrawal and Gautam, 2018). According to Guo et al. (2018), they stated that isogeometric analysis can eliminate geometry cleanup and mesh healing procedures while achieving the same (or better) accuracy as standard FEA. IGA not only reduces the gap between CAD and FEA, but also triggered a new spline research after a quiet period, for instance the locally refined splines and the polynomial splines over hierarchical T-meshes (PHT). There are some advantages of IGA which are:

1. IGA is closely link to CAD data (important for optimization problems) and exact geometry representation (important for shell problems, fluids etc)

2. B-splines/NURBS are very smooth functions with easily obtained high order continuity (facilitates the construction of C1 plate/shell elements or PDEs with high order derivatives)

3. Easy to be implemented into existing FE codes by using the Bezier extraction. It is available in LS-Dyna and Abaqus. Bezier extraction-based IGA code can

be parallelized using the standard domain decomposition methods that being applied for FEM.

It should be emphasized that IGA is not restricted to B-splines or NURBS only, some other splines are also successfully employed in the analysis. Researchers have developed new approaches for solving boundary problems with IGA. Koo et al. (2013) proposes an isogeometric shape design sensitivity analysis method that incorporates a mixed transformation approach to handle essential boundary conditions effectively. While, Natarajan et al. (2015) proposed method combines isogeometric analysis with the scaled boundary finite element method, allowing for the use of n-sided polygonal domains and the modeling of stress and strain singularities without enrichment. In some studies, researchers have improved the theoretical aspects of IGA to make it more efficient and accurate. A study by Phung-Van et al. (2015), presents a formulation based on Isogeometric Analysis (IGA) and Higher-order Shear Deformation Theory (HSDT) to investigate the static, free vibration, and dynamic control of piezoelectric composite plates integrated with sensors and actuators. The proposed method achieves accurate and reliable numerical predictions, as verified by comparing them with other available numerical approaches. A paper by Mantzaflaris and Jüttler (2014) proposes a new approach called Integration by Interpolation and Lookup (IIL) to overcome the bottleneck of matrix assembly in isogeometric analysis, by using spline interpolation and pre-computed look-up tables for evaluating integrals. The IIL method proposed in the paper demonstrates its ability to maintain the overall approximation order of the Galerkin discretization, provided that the spline interpolation is sufficiently accurate .

CHAPTER 3

RESEARCH METHODOLOGY

In this chapter, a new FEM basis function is proposed and will be implemented in this study. Two study on heat conduction was carried out in this paper. First study was a heat conduction problem with analytical solution on 2D rectangular plate and it will be used as a validation purpose of this method. Another study was done on a complex geometry shape which referred based on previous study by Azmi (2010) and the approximated solution will be compared with their existing result. In this study a steady-state heat conduction problem will be considered. All calculations and simulations regarding this study were done using MATLAB programming software. The coding steps and algorithms are shown in the Appendix section.

3.1 Types of FEM Mesh

Meshing is one of the important steps in performing an accurate simulation using FEM. A mesh is made up of elements which contain nodes (coordinate locations in space that can vary based on type of element) that represent the shape of the geometry. Solving PDEs problem using FEM is not an easy task for irregular shapes, but it is much easier with common polygon shapes like rectangle and square. There are different types of meshes commonly used in FEM, namely triangular mesh, and quadrilateral (quad) mesh for standard FEM. Recently, B-splines, Bernstein and Non-Uniform Rational B-splines (NURBS) are being used as a surface product for discretizing and preserving the model of irregular geometry especially with curvilinear design, since the common mesh need for refinement to keep the exact shape representation of the model. In this study, 2 types of mesh will be implemented which are:

1. Triangular mesh.

2. Quadrilateral mesh.

3.2 Method of Solving Heat Conduction Problems

Partial Differential Equations (PDEs) such as heat conduction has various type of solution namely analytical or exact solution using separational variables method Pinsky (2011) and Green function method Churchill and Brown (1963). On the other hand, numerical method such as Finite Difference Method (FDM) and Finite Element Method (FEM) also can be an alternative solution and more practical in complex problem. In this chapter, heat conduction problem will be solved using 3 different method which are:

1. Standard or Classical FEM.

2. Bernstein-Bezier FEM.

3. Trigonometric-Bezier FEM.

3.3 Flowchart of Study and Basic Steps in Finite Element Method

All steps in this study were simplified in the following flowchart as shown in Figure 3.1. Begining with the pre-processing part, then the chart will follow the whole process until the results is obtained. The general flowchart in Figure 3.1 is modelled based on the Galerkin Finite Element Method approach.

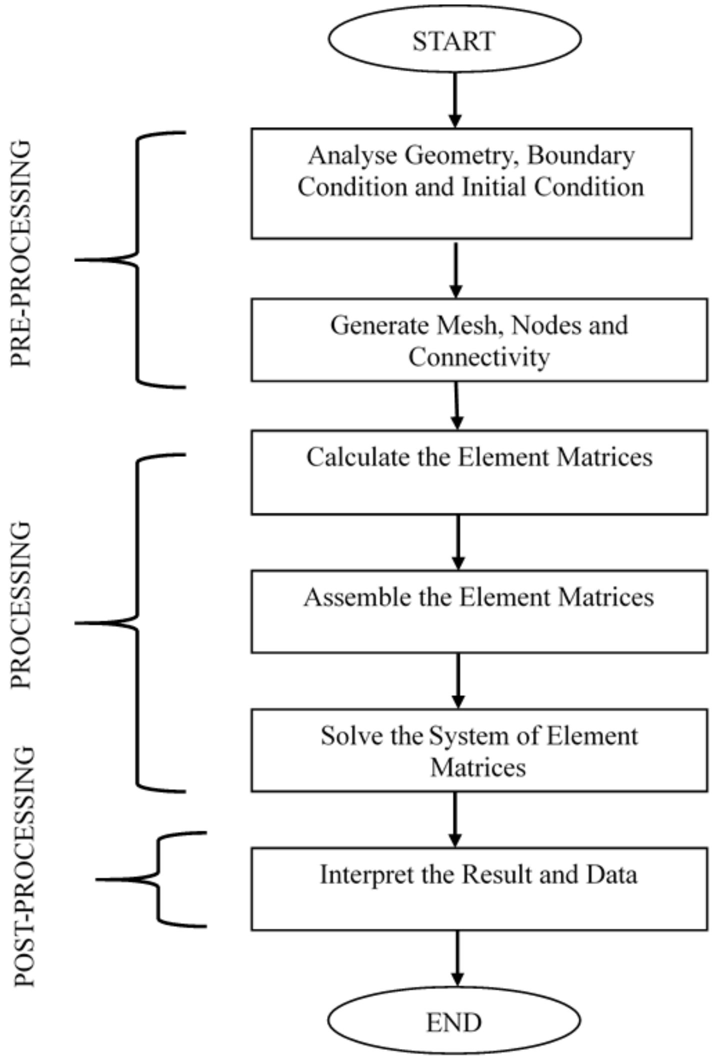

Figure 3.1: General Flowchart for Finite Element Method

3.4 Pre-processing

Pre-processing part is the initial step in solving a problem using FEM. Following are the steps that have been used to determine the solution to the problem.

a) Analyse Geometry, Boundary and Initial Conditions

Considering from previous research by Azmi (2010), and the complexity of the problem model, which doesn't mention any exact solution in their study, a simple heat conduction problem with exact solution was conducted on simple 2D