First Semester Examination
2020/2021 Academic Session

February 2021

MSG352 - Linear and Integer Programming

Duration : 2 hours

Please check that this examination paper consists of FOUR (4) pages of printed material before you begin the examination.

Instructions : Answer **THREE (3)** questions.

Question 1

(a) Consider the following Linear Program (LP):

$$\text{maximise} \quad z = 7x_1 + 4x_2 + 6x_3 + 8x_4 + 5x_5$$

subject to

$$\begin{aligned} 4x_1 + 2x_2 + 4x_3 + 5x_4 + 3x_5 &\leq 30 \\ 3x_1 + 2x_2 + 2x_3 + 3x_4 + 2x_5 &\leq 20 \\ x_1, \quad x_2, \quad x_3, \quad x_4, \quad x_5, &\geq 0. \end{aligned}$$

Given that the set of new basic variables determined just before iteration k is $\mathbf{x}_{BV}^k = \{x_4, s_2\}$, continue **solving** the LP using the Revised Simplex Method.

[25 marks]

(b) Consider the following partial tableau at iteration k, obtained when solving a particular Goal Programming problem using the Goal Programming Simplex Method.

Basic	x_1	x_2	x_3	s_1^-	s_1^+	s_2^-	s_2^+	s_3^-	s_3^+	s_4	Soln.
z_1	0	0	0	$-P_1$	0	0	0	0	0	0	0
z_2	0	0	$-50P_2$	$4P_2$	$-4P_2$	0	$-P_2$	0	0	$-350P_2$	$17500P_2$
z_3	0	0	$12P_3$	P_3	$-P_3$	0	0	0	$2P_3$	$-20P_3$	$5000P_3$

Discuss the following:

- How many goals and how many constraints does the Goal Programming problem have? Justify your answer.
- Given that all goals are upper, one-sided goals, what is the objective function of the Goal Programming problem in standard form?
- Which goals have and have not been met? Justify your answer.
- Which variable should be chosen as entering variable? Justify your answer.

[12.5 marks]

Question 2

Amateur car racer Damon Mound has placed second and third in all of his races so far this season. He has determined that improvements must be made to his car to improve its top speed. The five proposed improvements along with their costs (in RM1000s) and potential speed increases are as follows:

Proposed Improvements	Cost (RM1000s)	Speed Increases (km/h)
Alter wing shape	4	4
Fit lighter chassis	11	15
Use better tyres	7	8
Tune up engine	15	17
Alter suspension design	2	1

(a) **Formulate** a Binary Integer Program that increases the potential speed of Damon's car as much as possible, while limiting spending to RM36,000.

[10 marks]

(b) **Solve** Damon's problem using the Implicit Enumeration method. State the improvements Damon makes to his car along with the potential speed increase and cost resulting from those changes.

[25 marks]

Question 3

(a) Consider the following Linear Program:

$$\text{maximise} \quad z = 3x_1 + x_2 + 4x_3$$

subject to

$$\begin{aligned} 6x_1 + 2x_2 + 5x_3 &\leq 25 \\ 3x_1 + 3x_2 + 5x_3 &\leq 20 \\ x_1, \quad x_2, \quad x_3 &\geq 0 \end{aligned}$$

Given that $(x_1^*, x_2^*, x_3^*) = ({}^5/_3, 0, 3)$ and $(y_1^*, y_2^*) = ({}^1/_5, {}^3/_5)$, do the following:

(i) **Obtain** the range of values for the objective function coefficient of x_2 such that the current basis remains optimal.

[5 marks]

(ii) **Compute** whether the current basis remains optimal if a new nonnegative variable x_4 is introduced where $c_4 = 2$ and $\mathbf{a}_4 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

[5 marks]

(b) Consider the following primal Linear Program (LP):

$$\text{minimise } z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to

$$\begin{aligned} x_1 + x_2 + \ldots + x_n &= 1 \\ x_1, \quad x_2, \quad \ldots, \quad x_n &\geq 0. \end{aligned}$$

where

$$c_1, c_2, \ldots, c_n \in \mathbb{R}.$$

(i) **Construct** the dual of the LP.

[2.5 marks]

(ii) **Solve** the dual LP given that $c_1 < c_i, \quad \forall i \in \{2,3,\ldots,n\}$.

[5 marks]

(iii) Use the solution in (b)(ii) to **solve** the primal LP. State all theorems used.

[10 marks]

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