DEVELOPMENT OF COGNITIVE DIAGNOSTIC ASSESSMENT TO MEASURE THE MASTERY LEVEL OF FACTORISATION OF ALGEBRAIC FRACTIONS AMONG FORM TWO STUDENTS

MOHD AZIZI BIN MOHD NOOR

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by

MOHD AZIZI BIN MOHD NOOR

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LIST OF ABBREVIATIONS

AHM	Attribute hierarchical method
ANN	Artificial neural network
APOS	Action, process, Object and Schema
APT	Aneka pilihan tersusun
CBA	Classroom based assessment
CDA	Cognitive diagnostic assessment
CM	Cognitive model
CDM	Cognitive diagnostic model
CTT	Classical test theory
DINA	Deterministic input noisy output 'AND' gate
DSKP	Dokumen standard Kurikulum dan Pentaksiran
ECD	Evidence-centered design
HCI	Hierarchical consistency index
I-CVI	Item content validity index
IKH	Indeks konsistensi Hierarki
IRT	Item repond theory
JEPeM	Jawatankuasa etika penyelidikan manusia
KSAs	Knowledge, skill, abilities
MCE	Malaysian certificate education
MOE	Ministry of education
OMC	Ordered multiple-choice
PBD	Pentaksiran bilik darjah
PISA	Programme for International Student Assessment

PPPM	Pelan Pembangunan Pendidikan Malaysia
PT3	Pentaksiran tingkatan tiga
RSM	Rule space method
SBA	School based assessment
SPM	Sijil Pelajaran Malaysia
TIMSS	Trends in International Mathematics and Science Study
UASA	Ujian Akhir Sesi Akademik

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PEMBANGUNAN PENTAKSIRAN DIAGNOSTIK KOGNITIF UNTUK MENGUKUR TAHAP PENGUASAAN PELAJAR TERHADAP PEMFAKTORAN PECAHAN ALGEBRA DALAM KALANGAN PELAJAR TINGKATAN DUA

ABSTRAK

Pentaksiran Diagnostik Kognitif (PDK) merupakan salah satu pentaksiran yang disarankan oleh Kementerian Pendidikan Malaysia berdasarkan pada Pelan Pembangunan Pendidikan Malaysia 2013-2025 untuk mengukur perkembangan kognitif pelajar. Pentaksiran ini penting kerana ia memberikan maklumat yang mendalam mengenai keperluan pembelajaran pelajar dan membolehkan guru merancang pengajaran yang lebih berkesan untuk membantu pelajar mencapai matlamat pembelajaran yang ditetapkan. Dengan penekanan pada pembangunan kognitif, pelajar dapat diberi sokongan yang lebih tepat dan sesuai untuk meningkatkan pencapaian mereka dalam bidang ini. Oleh yang demikian, kajian ini bertujuan untuk membangun dan menilai pentaksiran diagnostik kognitif (PDK) yang memfokuskan penguasaan pelajar tingkatan dua terhadap pemfaktoran pecahan algebra dalam matematik. Pembangunan PDK melibatkan dua peringkat iaitu mewujudkan model kognitif pakar dan model kognitif respon pelajar. Guru pakar mata pelajaran Matematik telah membina model kognitif pakar, yang dinilai oleh panel pakar kandungan dan psikometrik. Proses pengesahan menggunakan kriteria yang pelbagai seperti soalan dan atribut yang dibina berdasarkan hierarki taksonomi Bloom, kejelasan soalan, penjajaran dengan dokumen standard kurikulum dan pentaksiran (DSKP) Matematik Tingkatan Dua, serta peratus kandungan topik pemfaktoran algebra yang diuji dalam PDK. Pentaksiran ini merangkumi 30 soalan aneka pilihan berstruktur dan telah diuji ke atas 159 orang pelajar dari Sekolah Menengah Kebangsaan. Pentaksiran itu mendedahkan bahawa model kognitif PDK mempunyai keupayaan diskriminasi yang baik, kecuali soalan yang dianggap terlalu mudah. Namun begitu, setiap soalan mempunyai distraktor yang sesuai, dan versi PDK menunjukkan kebolehpercayaan dari segi konsistensi dan korelasi antara model berasaskan pakar dan berasaskan respons pelajar. Tambahan pula, ujian melibatkan 1073 pelajar Tingkatan Dua, menunjukkan kebolehpercayaan yang konsisten dengan dapatan kajian rintis. Selain itu, terdapat perbezaan pencapaian antara pelajar lelaki dan perempuan menunjukkan wujud perbezaan dalam respon pelajar terhadap pemfaktoran pecahan algebra. Kajian ini juga mengkaji bagaimana faktor seperti meneka dan kecuaian memberi kesan kepada respon pelajar dalam PDK dan mendapati bahawa kecuaian mempunyai kesan yang lebih besar pada atribut yang lebih kompleks, manakala meneka mempengaruhi atribut yang lebih mudah. Sumbangan utama kajian ini ialah pentaksiran ini dapat digunakan oleh guru untuk mendiagnosis kognitif pelajar dari segi tahap penguasaan pemfaktoran pecahan algebra dengan mendalam. Selain itu, pentaksiran ini dapat dijadikan panduan kepada guru untuk mengaplikasikannya dalam mendiagnosis tahap penguasaan pelajar untuk topik lain dalam Matematik. Akhirnya, kajian ini menyerlahkan keberkesanan PDK dalam menganalisis respons pelajar secara menyeluruh. Pengkaji mencadangkan guru menggunakan PDK sebagai alat pentaksiran untuk menganalisis respon pelajar secara terperinci dan menyokong pembelajaran pelajar dalam pemfaktoran pecahan algebra.

DEVELOPMENT OF COGNITIVE DIAGNOSTIC ASSESSMENT TO MEASURE THE MASTERY LEVEL OF FACTORISATION OF ALGEBRAIC FRACTIONS AMONG FORM TWO STUDENTS

ABSTRACT

Cognitive Diagnostic Assessment (CDA) is one of the assessments recommended by the Ministry of Education of Malaysia based on the Malaysia Education Blueprint 2013-2025 to measure students' cognitive development. This assessment is crucial as it provides in-depth information about students' learning needs and enables teachers to design more effective instruction to help students achieve their learning goals. With a focus on cognitive development, students can be provided with more targeted and appropriate support to enhance their achievement in this area. Thus, this study aims to develop and assess a CDA focusing on form two students' mastery of factorisation of algebraic fractions. The CDA development involved two stages: establishing expert and student response based cognitive models. Expert mathematics teachers constructed the expert-based cognitive models, which were evaluated by a panel of content and psychometric experts. The validation process used different criteria, including items and attributes development based on Bloom's taxonomy hierarchy, items clarity, and alignment with form two standards document curriculum and assessment (SDCA) for Form Two Mathematics. Then, 30 ordered multiple choice (OMC) items were tested on 159 students from National Secondary Schools. The assessment revealed that the CDA's cognitive models had good discrimination ability, except for items deemed too easy. Each item had appropriate distractors, and the CDA versions showed reliability in terms of consistency and correlation between expert-based and student response-based cognitive models. Furthermore, 1073 Form Two

students involved in CDA, showing consistent reliability with pilot study findings. Differences in achievement between male and female students were noted. The study also examined how factors like guessing and slipping impacted responses, exposed that slipping had a greater effect on more complex attributes, while guessing affected easier attributes. The main contribution of this study is that this assessment can be used by teachers to diagnose students' cognitive abilities in terms of their mastery level of algebraic fraction factorization in depth. Additionally, this assessment serves as a guide for teachers to apply it in diagnosing students' mastery levels for other topics in Mathematics. Ultimately, this study highlights the CDA's effectiveness in analysing student responses comprehensively. Therefore, teachers can use the CDA as an assessment tool to offer detailed information towards students' responses and immediate feedback, supporting student learning in factoring algebraic fractions.

CHAPTER 1

INTRODUCTION

1.1 Introduction

Assessment in Malaysia has undergone significant and drastic changes in the education system, becoming the primary concern for the highest stakeholders, specifically the Ministry of Education Malaysia (MOE) (Kementerian Pendidikan Malaysia or KPM). These changes were initiated following the implementation of the Malaysia Education Blueprint 2013-2025, also known as Pelan Pembangunan Pendidikan Malaysia (PPPM) 2013-2025 (MOE, 2013).

One of the key agenda items within this blueprint was the assessment process, which emphasised school-based assessment (SBA) in the teaching and learning process. Furthermore, it was enhanced with classroom-based assessment (CBA), or Pentaksiran Bilik Darjah (PBD), in 2019. SBA is a comprehensive evaluation method that considers various aspects of a student's potential, including cognitive, emotional, and psychomotor abilities. This type of evaluation aligns with the national philosophy of education set forth by the Ministry of Education.

Consequently, four components (psychometrics, classroom-based assessment (CBA), school-based assessment (SBA), and assessment of physical activity, sports, and co-curricular activities) are employed to assess a student’s competency. Among these components, CBA is particularly noteworthy (MOE, 2019b). CBA involves continuous assessment throughout the teaching and learning process within the classroom.

The primary objective of CBA is not to compare students' results but to diagnose their cognitive development, which, in turn, helps teachers refine and strengthen their teaching methods (MOE, 2019b). As such, CBA encompasses various types of assessments highlighted by the Ministry of Education (MOE), including formative, summative, formal, and informal assessments.

Moreover, diagnostic assessment is a recommended component of assessment methods outlined in PPPM (2013-2025) and can be implemented in various assessment types based on the study's objectives.

1.2 Background study

In the field of education, the use of diagnostic assessments, specifically cognitive diagnostic assessments (CDA), serves the purpose of providing comprehensive insights into an individual's cognitive strengths and weaknesses, knowledge state, and error patterns. The primary objective is to identify specific areas of difficulty related to attributes or skills, guiding instructional and remedial interventions accordingly (Leighton & Gierl, 2007c).

Consequently, many researchers, including Toprak and Cakir (2020), Sia et al. (2018),Shanley (2016), Li et al. (2020), Ralston et al. (2018) and Chin (2020), employ CDA to gauge students' cognitive responses, tailored to their research objectives. In various disciplines, such as the medical field, cognitive assessment varies in its definition of diagnosis. In education, teachers utilise this information to adjust instruction by identifying mastered and non-mastered areas among students, with subsequent remedial changes (Geller et al., 2009).

In order to gain a clear understanding of students' mastery levels for specific skills, CDA is an invaluable tool, rooted in learning theories and a psychometric model designed to analyse students' cognitive responses in detail and provide statistical references (Hor, 2015). Moreover, the cognitive diagnostic model (CDM) is employed in CDA to assess students’ responses in algebraic learning based on the objective of study.

For instance, the attribute hierarchical method (AHM) was used to evaluate 286 students’ responses on algebraic skills, revealing common difficulties, particularly in fFactorisation, impacting their mastery of other algebraic attributes (Mafakheri et al., 2020). CDA is premised on the idea that a student’s cognitive response to a task, such as factorizing algebraic fractions, depends on a set of inherent cognitive skills or "attributes." These attributes constitute the cognitive model of cognition, encompassing memory, attention, reasoning, and problem-solving. In essence, diagnostic testing continuously evaluates students’ cognitive processes (Eisenhart, 1991 & Jacob & Parkinson, 2015).

Therefore, this study focuses on the factorisation of algebraic fractions, a skill requiring an understanding of algebraic concepts and their systematic application in problem-solving, denoting procedural skill. Therefore, the functionality of CDA is similar dimension as defined by Leighton and Gierl (2007c), is used in this study to assess students' mastery of factorisation.

Factorisation of algebraic fractions encompasses all the algebraic skills governed by algebraic laws (Ardiansari et al., 2022). It serves as a critical indicator of students' competency in topics related to algebraic laws. Furthermore, it is a significant component of examinations like the *Ujian Akhir Sesi Akademik (UASA)* and the Malaysian Certificate of Education (MCE) or *Sijil Pelajaran Malaysia (SPM).*

For that reason, proficiency in algebra profoundly impacts students' overall success in mathematics on a national level. Generally, algebra deals with mathematical conceptual understanding such as operations and symbols, involving equations, functions, and geometric representations (Kaput et al., 2008). Thus, algebraic concepts are pervasive in mathematics education, spanning from elementary to university levels and intersecting with various fields of science and mathematics (Md Yusoff Daud & Ainun Syakirah Ayub, 2019).

Furthermore, algebra explores the ideas of simplification and solving problems using symbols or letters (Wu, 2001). It holds particular significance in mathematics because it connects to various other topics like geometry, calculus, matrices, trigonometry, statistics, vectors, and other mathematical areas (Jupri et al., 2014 and Makonye & Stepwell, 2016).

For instance, when dealing with algebraic problems like factorisation and algebraic fractions, teachers can approach them geometrically by visualizing functions or equations. Consequently, students need a deep understanding of algebraic concepts, especially in factorisation and algebraic fractions, to succeed in various mathematical disciplines that rely on these concepts.

This includes applying the appropriate problem-solving techniques (Alexendra, 2020). In simpler terms, mastering factorisation of algebraic fractions is crucial for excelling in algebra and mathematics as a whole, particularly at higher levels that require advanced problem-solving skills. Therefore, algebra particularly the factorisation of algebraic fractions, is a fundamental prerequisite for higher-order thinking skills and success in advanced mathematics (Alexendra, 2020).

In the Malaysian educational system, algebraic skills, including factorisation of algebraic fractions, play a crucial role from lower secondary (form one to form three) to upper secondary (form four and form five) levels. These skills lay the foundation for learning and mastering algebraic concepts (MOE, 2016b and Piriya Somasundram et al., 2017).

However, students' responses to factorisation of algebraic fractions have often fallen short of expectations, impacting their interest in learning algebra (Sugiarti and Retnawati, 2019 and Mat et al., 2020). Students' engagement during algebraic lessons has been inconsistent (Skilling et al., 2016), and difficulties in mastering algebraic skills, including factorisation, hinder their ability to solve word problems and impede overall algebraic proficiency (Sugiarti & Retnawati, 2019).

Additionally, curriculum transformation can significantly affect students' performance in mathematical thinking processes (Drijvers et al., 2019). In the Malaysian context, the reformation of the secondary school mathematics syllabus, initiated in 2017, has led to changes in the content and approach to algebra learning and affected the attributes arrangement for form one and form two students.

Furthermore, the compressed timeline and modified attributes in learning algebra for lower secondary schools necessitate an updated CDA aligned with the most recent Mathematics KSSM syllabus, which improves the latest instrument CDA developed in this study when compared to previous KBSM modes developed by Hor (2015).

Therefore, in light of these developments, the ideal time to focus on students' mastery of algebraic skills is in form two, where they have acquired foundational attributes in form one and can apply these skills to topics connected to form three and form four KSSM. This includes topics like straight lines (form three), quadratic functions and equations in one variable, linear inequalities in two variables, and graphs of motion.

Hence, the researcher encourages to utilise CDA to diagnose students' mastery of algebraic skills, particularly those related to the factorisation of algebraic fractions among form two students.

1.3 Problem statements

The importance of developing an instrument for cognitive diagnostic assessment (CDA) among teachers cannot be exaggerated, as it is one of the assessment components endorsed by the Ministry of Education (MOE) for classroom-based assessment (CBA) (MOE, 2016a).

Nevertheless, the development of CDA was obstructed by the need to adhere to several criteria. These criteria include the selection of appropriate attributes, ensuring the validity and reliability of CDA, developing items that align with the relevant skills, involving suitable samples, determining valid measurement types for data collection, and assessing the impact of CDA on the teaching and learning process (Chin, 2020; Javidanmehr & Sarab, 2017; Toprak, 2020).

For this reason, few researchers who retrofitted the instruments to measure students' responses on the subject matter (Tan et al., 2019; Wafa et al., 2020; Wu., 2018) in their research. However, the primary concern voiced in CDA was that the measurement attributes in CDA are complicated (Javidanmehr & Sarab, 2017).

The reason for this was that the students' responses on attributes (factorisation of algebraic fractions) was measured based on DSKP (*Dokumen Standard Kurikulum dan Pentaksiran*) or Malaysian standard of Curriculum and Assessement document. As a result, even though the same topics and attributes were assessed, the challengers for development CDA based on other countries are not identical to those in Malaysia.

This is due to cultural, linguistic, and socioeconomic factors in each country (Javidanmehr & Sarab, 2017).

Furthermore, retrofitted instrument CDA from others sources may throw doubt on the validity of the chosen attributes (Javidanmehr & Sarab, 2017). For instance, the attributes selected may outdated along with the changing of the curriculum as such from KBSM to KSSM (MOE, 2015, 2016b, 2017b).

Therefore, the progression of developing CDA influenced the validity and reliability of CDA. The other limitations occur in development of CDA is the selection of attributes synchronised with the Q-Matrix arrangement as a pillar of expert-based cognitive models by Ma and de la Torre (2020) that useful to measure students' cognitive process in learning mathematics specifically in factorisation of algebraic fractions.

Owing to the previous reason, as the primary component of CDA analysis, the Q-matrix is produced based on the specified attributes as Ma and de la Torre (2020) retrieved from theories in the literature, the test's design, the expert panel's judgements in selecting the attributes to develop the expert-based cognitive models, and students' think-alouds; each approach alone or in combination (Javidanmehr & Sarab, 2017).

As humans, the data sources for Q-matrix construction, may pose a danger to the validity of the results (Messick, 1995). As a result, to improve the quality of Q-Matrix, the Boolean Matrix is used to increase the possibility that a plausible attribute exists in lieu of the selection of the attributes involved from the expert content. This establishes the relationship between the attributes as proposed by Gierl et al., (2000, 2007) and Leighton et al., (2004).

In addition, the measurement of attribute arrangement and consistency of fit model through hierarchical consistency index (HCI) may add values to increase the validity and reliability of CDA, then the development of students' cognitive processes.

Since the attributes became main consideration of the quality CDA, the selection of attributes in factorisation of algebraic fractions covered all the attributes involved for algebraic laws in two stages: conceptual understanding and procedural skill stage (Ardiansari et al., 2022).

Unfortunately, students' responses on mastering this topic in Malaysia raised with an issue that students have a problem of conceptual understanding and concluded with poor performance in algebraic skill especially at the higher-level stage (Nurmaizatul Hazir ah Mustaffa, 2017).

This is parallel with the result of TIMSS 2019 analysis. Example, the items categorised as highest level for item 2 and 3 of assessment in TIMSS 2019, where Malaysia student was the bottom 3 with students respond correctly. This happened due to students do not have strong foundation in algebra especially involving the factorisation and algebraic fractions. They are unable to understand the concept of variables, applying the arithmetic operations and algebraic expressions (Jupri & Drijvers, 2016).

Because of this, the trajectory of those results in TIMSS 2019 yielded the program for international student assessment (PISA) 2022 exposed Malaysia achievement in mathematics literacy decreased from 2018 to 2020 respectively and still under average provided by Organisation for Economic Co-operation and Development (OECD) (MOE, 2022).

The consequences of this problems, students have difficulties to understand the keywords or main idea to develop equations from problem solving questions (Blanton et al., 2015). Therefore it affected students performances in learning algebra especially in factorisation of algebraic fractions, demotivate and end up with isolating themselves from lesson activities (Skilling et al., 2020).

Hence, identification of students' mastery level for each attribute through CDA will give a clear picture about students' performance entirely in algebra. For that reason, CDA may help teacher to diagnose students' correctly and incorrectly respond to attributes mastery and non-mastery level in factorisation of algebraic fractions.

Furthermore, teachers are able to plan for effective instructional to enhance students' understanding and strategised their teaching better in algebra specifically in factorisation of algebraic fractions. Students' patterns of response in factorisation of algebraic fractions are very important to be highlighted in each item and attribute involved.

With thoroughly observing patterns of students' responses, the errors of students' understanding can be identified specifically and it can give the advantages to the teachers to curb the students' misconception by providing the appropriate remedial class (Chin, 2020). Hence, the advantages of identifying the mastery level of students' responses are also able to be identified students' misconception and errors simultaneously.

In addition, difficulties in understanding the conceptual knowledge stage made troubles in the next stage of algebraic procedural skill especially in word problem solving (Ling et al., 2019) and affected overall of students' score. Hence, it is so important for students to master the conceptual understanding involving the basic laws and operations of algebra to execute the procedural skill of solutions clear and precise because these stages were correlated each other (Al-Mutawah et al., 2018).

When going deeper into issues in factorisation of algebraic fractions, the errors that students encountered were the result of the process of simplifying algebraic fractions. Student’s difficulties caused by cancellation errors, factors errors, trinomial factorisation errors, lowest common denominator errors, careless, and achieving correct answers using wrong mathematical processes(Makonye & Khanyile, 2015).

Furthermore, long-term students were experiencing difficulties while learning mathematics at a higher level (matriculation) (Khairul Azuad, 2017). Those shortcomings done by students will explain why students tend to do errors on algorithm such as common factors, cross sectional processes, and also poor performance in fractions and integers (Md Yusoff Daud & Ainun Syakirah Ayub, 2019).

On the other hand, gender performance in learning algebra is also interesting to be explored since these issues frequently raised by TIMSS report (2019). With identifying the performance of these groups male and female students will clearly explain the gaps of understanding the algebraic learning in perspective of gender. Gender differences may impact teachers’ perceptions of their students’ performance. as Legewie and Diprete (2012) discovered that certain instructors had gendered expectations of boys and girls.

Furthermore, teachers’ beliefs regarding gender differences may result in different interactions in the classroom (Spilt et al., 2012). Due to differing ideas, female and male students may not have the same possibilities in the classroom, which may hinder their future involvement in mathematics especially involving complex attributes in learning algebra.

As a result, understanding gender differences is crucial, especially in terms of how big of a difference exists, where the difference arises the approaches of teaching according to gender (Hasni Shamsuddin et al., 2020). Correspondingly to research conducted by Manandhar et al. (2022) regarding algebra on 360 students in grades 8 and 9 of public schools in Kathmandu Metropolitan City, there was only a significant difference in procedural knowledge, favouring male students over female students.

While this was going on, conceptual knowledge showed that gender had no impact on the responses of the students at this stage. Therefore, with identifying students' responses on gender, teacher deeply can identify the hiding obstacles on each item on the process of item development that may cause the differences between gender. Therefore, it will help researchers to gain deep knowledge about certain attributes to be highlighted more and go deeper with further research.

In algebra, students' score normally relies to the students' responses in conceptual understanding stages. The better students' result in conceptual understanding stage, the better students' score in procedural skill stage (Ardiansari et al., 2022 and Sarimah & Mohd Zaki, 2021). In fact, there were results of these two stages influenced the overall of students' score as mentioned by others researcher in their study (Ardiansari et al., 2022; Callou & Pereira, 2021; Zulfa et al., 2020).

However, there were contradict result with Chirove and Ogbonnaya (2021) whereby it showed no relationship between total score and these two stages. The relationship between HCI score towards students' responses on conceptual understanding and procedural skill stages is crucial to be considered due to confirm that students' score overall depends on students' responses on conceptual understanding and procedural skill stages.

Moreover, it as an enhancement to the CDA done in Malaysia such as Chin et al. (2020a); Hor (2015); Mohd Faizul Ridzuan and Lian (2019); Sia et al. (2018); Tan (2017) in order highlighting students' responses in conceptual understanding and procedural skill stages. In Malaysia, the researchers done CDA with focusing deeply on identifying students' responses analysis of error, strength and weaknesses and students' mastery level in attributes (Chin et al., 2020b; Hor, 2015; Sia et al., 2018).

As a result, the purpose of this study is not only to assess students' mastery levels in attributes, but also to determine the 'guess' parameter which students are unable to master the attributes but still respond correctly and 'slip' parameter indicates students are able to master the attributes but respond incorrectly, that affects students' performance in CDA.

Enlightening the parameter of 'guess' and 'slip' on each item will determine the qualities of an item. It is an additional method to validate the instrument by analysing the quality of each item (Rupp et al., 2010) precisely. The mastery and non-mastery level of students' algebraic skill in each item genuinely can be obtained precisely.

Hence, the DINA and model is chosen to outstand the function of both parameters (Wafa et al., 2020) . The reason is with identifying those parameters, students were unable to master the attributes but still performed correct answer and students able to master the attributes but failed to answer correctly can be detected so that it can influence the result of mastery and non-mastery students in each attribute.

Based on the issues raised above, it is clear why it is important to use CDA to access factorisation of algebraic fractions among form two students. If students can grasp this algebraic skill successfully, it has a significant influence on their progress in secondary school mathematics.

In addition, CDA is also can determine the quality of items based on the student's response and gender performance. To ensure the efficiency of the CDA towards diagnosing students' responses the process of development CDA itself, must be fit and valid.

1.4 Objective of study

The study converges on the development of cognitive diagnostic assessment (CDA) for measuring form two factorisation of algebraic fractions. The process of structuring attributes from two cognitive models (CM 1 and CM 2) and procedure to provide students' algebraic skill or attributes information are the focus on this research. The objectives of the study are based on two stages namely stage I: expert-based cognitive model and stage II: student-response based cognitive model. Hence the objectives study was divided into two stages as follows:

1.4.1 Stage I: Expert-based cognitive model

For stage I, the following objective study were used to develop the expert-based cognitive model:

1. To develop a valid form two cognitive diagnostic assessment (CDA) in factorisation of algebraic fractions.
2. To determine the attribute probabilities' arrangement following the complexity hierarchical arrangement from easy to difficult.
3. To determine the consistency of expert-based cognitive model and student-response based cognitive model of form two factorisation of algebraic fraction in CDA.

1.4.2 Stage II: Student-response based cognitive model

Meanwhile, to develop the student-response based cognitive model, the following objectives study were used as a basis development:

4. To identify students' responses on correctly response/incorrectly response, mastery/ non-mastery level for items and attribute and according to conceptual understanding and procedural skill in CDA.
5. To gauge any significance differences of mastery level between male and female students for each item and attribute in CDA.
6. To gauge the relationship between hierarchical consistency index (HC)I score, conceptual understanding, and procedural skill.
7. To gauge parameter of 'guess' and 'slip' of students' responses in CDA.

Hence, as consequences the research questions are also constructed based on two stages as aforementioned.

1.5 Research questions

The research questions (RQ) highlighted as follows based on the objectives stated and formatted in two stages as follows:

1.5.1 Development research questions in stage I: Expert-based cognitive model

The three research questions (RQ) involved in Stage I are as follows, aiming to develop the expert-based cognitive model. These research questions are:

1. a) How valid is the designed cognitive diagnostic assessment (CDA)?

b) How reliable is the designed cognitive diagnostic assessment (CDA)?

2. Do the attributes probabilities follow the difficulties level in CDA?
3. Are the expert-based cognitive models consistent with the student-response based cognitive models of form two factorisation of algebraic fractions in CDA?

Next, move to RQ in stage II involving the development of student-response based cognitive model.

1.5.2 Development research questions in stage II: Student-response based cognitive model

For Stage II, four research questions (RQ) were formulated to develop a student-response-based cognitive model. These RQ are:

4. What are the students' responses to the items and attributes in factorisation of algebraic fractions in CDA?
   a) Which items and attributes might students correctly respond to?
   b) Which items and attributes might students incorrectly respond to?
   c) Which attributes are able to be mastered by students?
   d) Which attributes are unable to be mastered by students?
   e) Does any significance differences of students' responses between male and female students for each attribute in CDA?
5. What are the students' responses to algebraic skills in CDA according to conceptual and procedural skill stages?
   a) Which items and attributes might students correctly respond to and able to master according to conceptual understanding and procedural skill stages?

b) Which items and attributes might students incorrectly respond to and unable to master according to conceptual understanding and procedural skill stages?

c) Does any significance differences of students' responses between male and female students in CDA according to conceptual understanding and procedural skill stages?

6. Is there a relationship of students' responses on conceptual understanding and procedural skill stages towards HCI score for both cognitive model CM 1 and CM 2?

7. What are the parameters of 'guess' and 'slip' of students' responses in CDA?

a) What are the parameters 'guess' and 'slip' of students' responses on each item and attribute?

b) What are the parameters 'guess' and 'slip' of students' responses on each item and attribute for both cognitive models (CM 1 and CM 2)?

Thereby, due to few research questions being constructed to identify the differences according to gender and the regressions between the variables, then the null hypothesis is constructed to prove the earlier assumption is true.

1.6 Null Hypotheses

All hypotheses were developed based on prior research regarding gender differences in students' responses (Farhana Aida et al., 2020; Hasni Shamsuddin et al., 2020; Ying et al., 2020) and the relationship between variables (conceptual understanding and procedural skill stages) that influence students' responses (Ardiansari et al., 2022 and Manandhar et al., 2022).

Since the TIMSS report (2019) frequently brought up these issues, it was ideal to explore gender performance in learning algebra, including factorisation of algebraic fractions. By identifying the performance of these groups, male and female students were able to understand the gaps in their understanding of algebraic learning from the perspective of gender differences and may impact teachers' perceptions of their students' performance (Legewie and Diprete, 2012).

Furthermore, teachers' beliefs regarding gender differences may result in different interactions in the classroom (Spilt et al., 2012). Due to differing ideas, female and male students may not have the same possibilities in the classroom, which may hinder their future involvement in mathematics. As a result, understanding gender differences is crucial, especially in terms of how big of a difference exists, where the difference arises the approaches of teaching according to gender (Hasni Shamsuddin et al., 2020).

However, the identification of gender responses between genders stated the instability results. In studies done by Hasni Shamsuddin et al.(2020) and Ying et al. (2020), there was a significant gender difference in the assessment. In contrast, there were no significant differences in the studies done by Farhana Aida et al. (2020) and Manandhar et al. (2022).

Meanwhile, many previous studies claimed that the overall student score mostly relies on students' conceptual understanding stages rather than procedural skill stages (Ardiansari et al., 2022; Callou & Pereira, 2021; Zulfa et al., 2020). In fact, students' responses were quite good in the conceptual understanding stage, and they mastered the skills excellently (Ardiansari et al., 2022). Nevertheless, there were a few studies that showed inconclusive results where there was no relationship between conceptual understanding and students' scores (Chirove & Ogbonnaya, 2021).

Therefore, the instability of results, the null hypotheses were utilised in this research to investigate the differences of gender response and the relationship between stages and scoring (students' responses). The following null hypotheses were developed as follows and is shown in Table 1.1 according to RQ's:

Table 1.1

Null hypotheses in CDA

Research question	Null hypothesis
RQ 4(e)	H_o1: There is no significant difference of the students' responses between male and female students for each attribute in form two CDA of Factorisation of Algebraic Fractions.
RQ 5(c)	H_o2: There is no significant difference of the students' responses between male and female students for each attribute in form two CDA of Factorisation of Algebraic Fractions.
RQ 6	H_o3: There is no relationship between students' responses on HCI scores with the conceptual understanding and procedural skill stage.

1.7 Significance of studies

The cognitive diagnostic assessment (CDA) was developed and assessed in this study to determine the competence level of each associated CDA attributes for the topic factorisation of algebraic fractions among form two students. Numerous stakeholders benefited from the study's conclusion as it served as a resource for student assessments.

1.7.1 Teachers

Through this study, teachers had a valuable resource to develop assessments that could measure students' cognitive processes for specific topics taught in the classroom. The use of CDA as a diagnostic assessment in this study enabled teachers to provide quality education by thoroughly analysing students' responses, considering students' answers in relation to items and attributes, mastery levels in each attribute overall, as well as according to conceptual understanding and procedural skill stages.

Additionally, this assessment helped in identifying crucial factors such as 'guess' and 'slip' parameters that needed to be investigated by teachers to ensure students' competency levels in factorization of algebraic fractions. Furthermore, by implementing the CDA to diagnose students' competency in Form Two factorization of algebraic fractions, teachers were able to predict students' competency by modelling their cognitive thinking and evaluating the consistency between the expert-based model and the student-response model.

Through this process, teachers gained a deeper understanding of students' cognitive processes in learning the factorization of algebraic fractions. They could identify students' development of cognitive thinking from one attribute to another, which was crucial for assessing students' proficiency in this topic. Most importantly, examining the connections between attributes was essential because it showcased students' proficiency in this area. If the connections between attributes diminished, it could indicate issues in students' cognitive processes. Hence, teachers could precisely identify students' errors in cognitive thinking and address them specifically.

Teachers were able to analyse students' responses deductively, categorizing them as correct or incorrect for each item and attribute. Subsequently, these responses were further analysed based on the mastery level for each attribute, according to the cutoff criteria established in this study. The identification of mastery levels was essential, as it allowed teachers to precisely track students' progress in learning factorization of algebraic fractions based on their responses to items representing specific attributes.

Students were considered to have mastered an attribute if they could correctly answer at least two-thirds of the items related to that attribute. Hence, analysing students' progress in this manner provided teachers with deep insights into their mastery levels, facilitating a comprehensive understanding of students' learning achievements.

1.7.2 Students

The CDA developed for Form Two factorisation of algebraic fractions in this study was an assessment tool designed for students. This CDA could diagnose students' mastery at an early stage as it was conducted formatively in the classroom, depending on the teacher's planning. Consequently, students could perform self-evaluation based on the CDA results and work towards self-improvement in preparation for final examinations.

By analysing the CDA results, students could identify the attributes they had mastered well and those they had not. Therefore, students could take a strategic approach by narrowing down their focus of study. In other words, they could concentrate more on the attributes they had not yet mastered, effectively addressing the challenges they faced.

1.7.3 Ministry of Education (MOE)

The measurement and evaluation of students' mastery level in the factorisation of algebraic fractions in Form Two is a significant agenda for the Ministry of Education in Malaysia. In this context, the use of cognitive diagnostic assessment (CDA) can be employed as a valuable tool for assessing and measuring students' proficiency in this area.

The development of this tool was undertaken with the aim of fulfilling the objectives of classroom-based assessment (CBA), which entails the ongoing evaluation of students' progress and understanding within the educational setting. Hence, the utilisation of CDA can serve as a comprehensive framework for the Ministry of Education (MOE) to offer a valuable and practical diagnostic assessment tool across various subject areas in all schools in Malaysia.

The primary purpose of CDA is not to engage in a comparative analysis of students' outcomes but rather to assist educators in assessing students' cognitive development and enhancing their instructional approaches as necessary. Furthermore, the utilisation of CDA as a national diagnostic assessment in Malaysia can serve to identify the levels of mastery as well as the strengths and weaknesses exhibited by students.

Consequently, the outcomes are enhanced in terms of validity and reliability, as all Malaysian students utilise a standardised diagnostic assessment tool that consists of similar items and attributes.

1.7.4 Other Researchers

Presently, a significant number of researchers in the field of psychometric education are actively investigating strategies for effectively integrating Cognitive

CDA into the Malaysian education system. Researchers interested in studying the cognitive processes of students when learning specific topics across various subjects may find the CDA as one of the tools to be a valuable resource.

The CDA can serve as a foundation for analysing students' cognitive thinking in numerous ways, aligning with the objectives of the study. There is a discrepancy in the body of research that examines students' cognitive reasoning and their ability to articulate their thought processes using statistical data. This study can also serve as an exemplar for future research endeavors aiming to employ CDA as a diagnostic tool in exploring various subject areas of interest.

1.8 Limitation of study

There are few limitations of this study that are highlighted associated with the current study that may affect generality of result.

1.8.1 Sample constraint

Selection of schools as the sample is the core constraint. Random sampling was not feasible, considering the administrative constraints. The sample is then chosen convenience sampling as the students are selected by the administrators of the school after multistage proportionate stratified random sampling was utilised to select each school according to district in Kedah and Penang state (Dillon & Obrusnikova, 2011).

The target sample includes students from classes that consist of student performance levels that are medium, moderate and high skills. The administrators of school were suggested to select students from multiple stage of abilities, yielding the range of data results is wider.

1.8.2 Time constraint

Time constraint during the assessment of the instrument was one aspect that needed to be considered. As usual, before the instrument was tested on students, students were briefed on the objective of this assessment. Based on the instrument of diagnostic assessment, the objective was one hundred percent to diagnose students' performance rather than total score. Students were given one hour to settle down for the assessment.

However, the limitation exists due to time constraint because of students' factors such as anxiety, the apparatus of examination tools broken such pencils or pens among participants. Therefore, based on these conditions, students who encounter these problems may be given extra time to complete it for around 10 to 15 minutes permission from school administrators.

Students are given the courtesy to prevent a negative effect on data collected for the purpose of study. Other than that, because of limitation of time, the sample is shielded by Kedah and Penang.

1.8.3 Research design

The research design in the study was fully quantitative approaches due to limitation of time to get thorough data via interviewing students' difficulties of algebraic skills experienced and think a loud session.

To address these issues, a preliminary test (pencil and paper test) was done to gather data on students' cognitive processes, particularly their errors in algebraic skills, as a guide to the researcher in developing the CDA's instrument.

1.9 Definition of term and operational definitions

Definition of terms and operational definitions was tried to highlight the definition of related variables accordance to the purpose of this study. The definitions of the variables stated were consistent all the way until the end of this thesis. Hence, the definition and operational definitions of the variables stated as follows:

1.9.1 Cognitive diagnostic assessment (CDA)

CDA is one of assessment based on psychometric model that conducted formatively to get information about students' performance which attribute mastered by students and vice versa (Wen et al., 2020). In order to get useful information of students' responses on algebraic skill, CDA developed based on cognitive models (CMs) and Q- Matrix is created to choose related attributes.

In this study, CDA was developed to highlight students' mastery level on each attribute tested in factorisation of algebraic fractions. In this study, the purpose of CDA was to determine attribute probabilities that indicated students' mastery level on each of algebraic skill as known as attribute in factorisation of algebraic fractions.

Hence, the researcher identified which attributes were correctly/incorrectly respond to and able/unable to be mastered by students, and according to conceptual and procedural skill stages.

1.9.2 Algebraic attribute(s)

Algebraic attribute can be described as a skill containing method of learning in which rules, procedures, algorithms, sense making, practical reading, and the development of algebraic expressions are all thoroughly incorporated (Friedlander & Arcavi, 2012).