DETECTION OF OUTLIERS AND STRUCTURAL BREAKS IN STRUCTURAL TIME SERIES MODEL USING INDICATOR SATURATION APPROACH

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by

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LIST OF SYMBOLS

$1[\bullet]$	indicator saturation function
α	level of significance
$\boldsymbol{b}$	the IIS vector of size (T/2×1)
$\hat{\beta}$	Unbiased least square estimator for IIS
b_1	First block/partitions
b_2	Second block/partitions
C_t	closed stock price at time t
c_a	critical value
c_t	cycle component of a longer period than seasonal component
d	number of diffused initial elements
ε_t	irregular component or error
$I_{j,t}(j)$	impulse indicator vector
H_1	Unknown location of structural break in the first block
k	exogenous variable
K_n	set of time indices in response to relevant indicators

λ	magnitude of outlier
L_1	unknown location in the first block/partition
m	number of elements in the state vector
N	Total number of x_a regressors
Q_t	m state disturbances with zero means and unknown variances.
p	number of falsely retained indicators
R_t	identity matrix of order of $m \times m$
$\tilde{r}_j$	retention rate
$\mathbf{S}_{\mathbf{b_1}}$	SIS vector consisting of step-indicators of δ_t
$\mathrm{r_t}$	return values
$\boldsymbol{\tau}_{\mathbf{L_1}}$	vector with outlier of unknown location
T	Number of observations
$\lvert t_j \rvert$	absolute value of t-statistics for the regressor
μ_t	trend (time component)
γ_t	periodic component of a fixed period (seasonal component)
$\hat{\delta}$	Least square estimator for SIS

x_a	regressors
y_t	univariate time series
Z_t	design vector of size of $m\times1$

LIST OF ABBREVIATIONS

AO	Additive Outlier
AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
BP	Bai And Perron’s
BSM	Basic Structural Model
CTSB	Consistent Timing Structural Break
DGP	Data Generating Process
DIS	Design Break Indicator Saturation
FTSE	Financial Times Stock Exchange
GARCH	Generalised Autoregressive Conditional Heterocedasticity
GDP	Growth Domestic Product
GETS	General-To-Specific
GUM	General Unresricted Model
ICA	Independent Component Analysis
IIS	Impulse Indicator Saturation
IO	Innovative Outlier
LLM	Local Level Model
LLTM	Local Level Trend Model
LS	Location Shift
MCO	Movement Control Order
MODWT	Maximal-Overlap Discrete Wavelet Transform

OLS	Ordinary Least Square
PICo	Problem, Interest, And Context
PWLAD	Penalised Weighted Least Absolute Deviation
QLR	Quandt Likelihood Ratio
RMSE	Root Mean Square Error
ROSES	The Reporting Standards For Systematic Evidence Syntheses
SIC	Schwartz Information Criterion
SIS	Step Indicator Saturation
SME	Small And Medium Enterprises
SSM	State Space Model
STSM	Structural Time Series Model
S&P	Standard and Poor’s
US	United States
USD	Us Dollar
WLAD	Weighted Least Absolute Deviation

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PENGESANAN TITIK TERPENCIL DAN PUTUSAN BERSTRUKTUR DI DALAM MODEL STRUKTUR SIRI MASA MENGGUNAKAN PENDEKATAN PETUNJUK KETEPUAN

ABSTRAK

Kehadiran nilai terpencil dan putusan berstruktur di dalam data siri masa berkemungkinan memberi kesan kepada penganggaran petunjuk ekonomi dan kewangan. Perubahan struktur yang disebabkan nilai terpencil dan putusan berstruktur juga mungkin akan mengurangkan ketepatan nilai penganggar dan menghasilkan nilai ramalan yang tidak tepat. Prosedur pengesanan nilai terpencil dan putusan berstruktur menjadi perhatian dalam kajian ini. Oleh itu, kajian ini mencadangkan teknik petunjuk ketepuan yang dikembangkan daripada model menyeluruh-kepada-khusus (GETS) untuk mengesan perubahan struktur di dalam data siri masa. Kajian ini bertujuan untuk menyiasat prestasi petunjuk ketepuan nilai terpencil dan putusan berstruktur dalam kerangka keadaan-ruang. Teknik yang dicadangkan mampu mengesan lokasi, tempoh, saiz dan bilangan perubhaan struktur di dalam data siri masa. Setakat yang diketahui, semua kajian lepas menggunakan *Autometrics* yang berfungsi di dalam *OxMetrics* untuk mengoperasikan pendekatan petunjuk ketepuan unutk proses penjanaan data (DGP) statik. Oleh itu, kajian ini cuba untuk mengisi kekosongan jurang dengan menggunakan pakej *gets* di dalam R untuk menyiasat prestasi petunjuk ketepuan dalam model dinamik iaitu model struktur siri masa. Prestasi petunjuk ketepuan diukur melalui simulasi Monte Carlo menggunakan konsep potensi dan tolok. Keputusan dari simulasi Monte Carlo mendedahkan pemilihan bersusun mengalahkan prestasi pemilihan tidak bersusun di dalam pemilihan model automatic GETS. Selain itu, nilai

optimum tahap keertian yang dicadangkan adalah pada $\alpha = 1/T$. Petunjuk-petunjuk yang kekal menunjukkan sepadan dengan krisis kewangan global 2008-2009. Secara keseluruhan, teknik ini menawarkan pendekatan yang efektif untuk mengesan lokasi, saiz, dan tanda putusan berstruktur di dalam kerangka struktur siri masa. Hala tuju penyelidikan pada masa hadapan mempertimbangkan pelbagai jenis petunjuk ketepuan termasuk penglibatan pemboleh ubah penjelas di dalam model struktur siri masa.

DETECTION OF OUTLIERS AND STRUCTURAL BREAKS IN STRUCTURAL TIME SERIES MODEL USING INDICATOR SATURATION APPROACH

ABSTRACT

The presence of structural changes, specifically outliers and structural breaks, adversely affects the estimation of economic and financial indicators in terms of the model accuracy and forecasting performance. Focusing on the detection of outliers and structural breaks, which has recently gained growing research interest, this study aimed to examine the performance of indicator saturation, as an extension of the general-to-specific (GETS) modelling, in detecting these structural changes in structural time series model framework. The proposed technique is capable to detect the location, duration, magnitude and number of structural changes in time series data. To date, prior studies only considered using *Autometrics* embodied in *OxMetrics* to apply this approach in static data generating process (DGP). Addressing this gap, this study used the *gets* package in R to examine the performance of indicator saturation in dynamic model viz state space model. Through Monte Carlo simulations, the performance of indicator saturation was evaluated in terms of potency and gauge. Based on the simulation results, the sequential selection algorithm outperformed the non-sequential selection approach in the automatic GETS model selection procedure. The results also suggested $\alpha = 1/T$ as the optimum level of significance level. The results of actual data applications further revealed that the retained indicators in Shariah-compliant stock indices matched the global financial crisis (2008–2009). Conclusively, the proposed indicator saturation approach serves as an effective approach to detect structural changes with unknown magnitude and structural break

signs at unknown locations in a structural times series framework. Future research direction is to consider other types of indicator saturation, including the addition of explanatory variables in the structural time series model.

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Time series data involves a chronological order of data points with equal time intervals. These time intervals, which can be on hourly, weekly, monthly, or annual basis, are known as time series frequency. Assuming that the gathered historical data points in time series data have constant variance (homoscedasticity), time series analysis is instrumental in trend determination and modelling and forecasting of certain values. However, any disturbances or structural changes in time series data result in inconsistent variances, which affect the reliability and validity of modelling and forecasting. Apart from poor forecasting performance, structural changes adversely affect the estimation of economic signals and seasonally adjusted series. Moreover, the presence of structural changes yields systematic forecasting bias, misspecification of estimated models, and potential distortion of parameter estimation and inference. Fox (1972) and Peña (1990) similarly highlighted how the presence of outliers adversely affects parameter estimation.

Common cases of structural changes for time series data are outliers and structural breaks. An outlier refers to an unusual data point that significantly differs from other data points. The structure of the data influences how an outlier is defined. Johnson and Wichern (1992) defined an outlier as a case of inconsistent data point, while Barnett and Lewis (1994) defined an outlier as a case of deviation of a data point from other data points. In a more recent study, Carreño et al. (2020) defined an outlier as an anomaly, conflicting observation, or contaminant. According to Chang et al. (1988), there are two common types of outliers in time series data, which are additive outlier (AO) and innovative outlier (IO). An AO affects only one data point in the time series data, but an IO affects all subsequence data points. For example, in the case of modelling a time series using autoregressive-moving average (ARMA) model that involved autocorrelation (ACF) and partial autocorrelation (PACF), Ledolter (1989) found that the presence of outliers in the time series distorted the model's parameter estimation, resulting in inaccurate forecasting performance.

On the other hand, structural break or location shift (LS) refers to an unforeseen change at any data point, which alters the mean or variance of the time series data. For example, structural breaks occur when there are unexpected changes in the relationship between macroeconomic variables, such as the relationship between inflation and unemployment or the relationship between oil prices and exchange rate. Other examples of unprecedented events that cause structural breaks include World War I, World War II, Asian Financial Crisis 1997, and global financial crisis in 2008–2009. From the economic perspective, an abrupt and unexpected change shifts the relationship between related variables. Apart from major financial events, abrupt technological, social, policy, and legislation changes trigger the shift of related parameters. The extent of the influence of structural break on an empirical model depends on the location, magnitude, and type of the structural break. However, it is not possible to acquire the details on the occurrence of structural breaks in real world data. Similar to outliers, poorly handled structural breaks in the time series data produce distorted parameter estimation and inaccurate forecasting (Castle & Hendry, 2019).

Therefore, it is pivotal to explore the presence of outliers and structural breaks in time series data since valuable details like the duration, location, magnitude, and number of structural changes are not known. Fox (1972) highlighted the implications of AO and IO, which led to numerous studies on the development of appropriate detection techniques based on a specific model or type of structural changes. For instance, Box and Tiao (1975) examined the influence of AO and IO in time series data using the step and pulse functions. Tsay (1988) performed a conceptually similar study to examine the presence of LS in the ARMA model, which inspired Chang et al. (1988) to develop the iterative detection procedure that can detect the presence of outliers in the ARMA model. Following that, Harvey and Koopman (1992) applied the outlier detection procedure in Gaussian state-space model. After Harvey and Koopman (1992), McCulloch and Tsay (1993), Shephard (1994), De Jong and Penzer (1998), Chow et al., (2009) and You et al., (2020) finally formalised the development of the outlier detection procedure for the detection of AO and IO in state-space model. With that, the current study aimed to examine the application of this detection procedure in a model-based framework, specifically the state-space model framework.

1.2 Problem Statement

An accurate forecasting based on historical data points is particularly important when it comes to making a decision or formulating a policy. The process of making good decisions requires adequate understanding on the predictability of an event based on several considerations, such as the amount of data available, the extent of one's understanding on the potential factors that influence forecasting, and the influence of forecasting on the event of interest (Hyndman & Athanasopuolos, 2018). It is rather straightforward to forecast certain events. For example, since daily temperature values in Malaysia do not fluctuate much, the forecasting of daily temperature can be highly accurate since the weather conditions are fairly consistent. Malaysia is located in the tropical climate region, near the equator, and surrounded by seas despite the occurrence of monsoon seasons, the daily temperature values do not vary substantially. With adequate historical weather data on daily temperature, humidity, and rainfall, as well as a reliable statistical model, the overall forecasting performance can be highly accurate.

However, it is highly challenging to forecast stock prices since there are limited information and understanding on the potential factors that influence the movement of stock markets despite the extensive amount of historical data on stock prices. It can be an internal factor or external factor, or even both for examples, macroeconomic variables, political turmoil, general election, disease outbreak, trade war, or policy changes. In other words, stock prices are highly unpredictable, and it is highly crucial that forecasters, investors, or analysts aware of the uncertainties before they make any decision.

Therefore, this has led to the next question: how to perform an accurate forecasting given the uncertainties and unexpected changes? A Greek philosopher, Heraclitus, described that change is the only constant in life. The world continuously changes over time these changes may be gradual or abrupt. The finite and constant values of mean and variance over time makes a time series stationary, but all social, political, and economic data continuously change over time, resulting in changing values of mean and variance for the time series. Structural changes alter the values of mean and variance, which negatively influence the empirical model performance, resulting in poor forecasting. There are two main sources that are unpredictable, which are evolutionary process and sudden shocks (Castle & Hendry, 2019). Events like the

World War I in 1914, Asian Financial Crisis 1997, and global financial crisis in 2008–2009 clearly demonstrated the non-stationary nature of data. However, the majority of economic theories highlighted the equilibrium state in analysis despite the constant changes and uncertainties due to the influence of various factors in the actual settings. These factors may be significant elements that cause changes in the data. Therefore, a good forecasting model is expected to capture the actual pattern of these structural changes (Hyndman & Athanasopuolos, 2018).

In this unprecedented situation, a good forecasting model can significantly benefit the government (Hyndman & Athanasopuolos, 2018), especially in the implementation and improvement of policies and precautionary measures. Unfavourable implications can be avoided if major shifts can be properly captured and handled. Unpredicted changes or shifts distort parameter estimation in an empirical model and lead to inaccurate forecasting. In practice, the duration, magnitude, number, and type of changes or shifts are unknown (Castle & Hendry, 2019).

A structural time series model captures the stochastic trend, seasonal, and irregular components for forecasting. However, it is not capable to identify the presence of any anomalies in the time series. Therefore, a detection procedure that can capture structural changes is instrumental because these changes substantially cause major shifts in distribution, alter the relationships between variables, and produce inaccurate forecasting. With that, the current study evaluated the performance of indicator saturation approach in identifying unknown outliers and structural breaks in a model-based framework, specifically the structural time series model.

1.3 Research Objectives

In general, this study aimed to address the gaps in the detection of outliers and structural breaks in the time series model using the indicator saturation approach and to evaluate the application of the indicator saturation approach using actual data through different perspectives of analysis. The specific objectives of this study are as follows:

1) To propose a model-based detection procedure using the indicator saturation approach to detect outliers and structural breaks in state-space model through general-to-specific (GETS) modelling
2) To develop an algorithm for the indicator saturation approach using the *gets* package in R for the detection of outliers and structural breaks in structural time series model
3) To apply the indicator saturation approach in structural time series model framework for the detection of outliers and/or structural breaks in Shariah-compliant stock indices

1.4 Scope of Study

The current study aimed to demonstrate the performance of indicator saturation approach in detecting outliers and structural breaks in state-space model framework. Therefore, this study exclusively focused on local level model (LLM), local level trend model (LLTM), and basic structural model (BSM). Furthermore, among the different types of indicator saturation, this study selected impulse indicator saturation (IIS) and step indicator saturation (SIS). However, considering the focus of the current study to detect the presence of outliers and structural breaks, this study did not explore how the selected approach can influence GETS modelling, model treatment, and forecasting.

This study only compared the existing algorithm in literature using the *gets* package in R programming language, instead of PcGive module embodied in OxMetrics.

Adding to that, this study evaluated the performance of indicator saturation approach in gets package from two viewpoints. Firstly, ISS was applied to identify the presence of single AO and multiple AOs separately using Monte Carlo simulations, while SIS was applied to identify the presence of single level shift (LS) and multiple shifts. The Monte Carlo simulation settings involved varying durations, locations, magnitudes, and numbers of AO and LS. The performance of indicator saturation in this study was determined based on the potency and gauge. Secondly, as for the use of actual data, the current study obtained stock market indices from Shariah counterparts to evaluate the performance of indicator saturation. The obtained real data were modelled by reference models based on the null hypothesis (of no outliers and structural breaks). The main purpose of using actual data was to examine the detection of outliers and structural breaks with respect to the occurrence of global financial crises, COVID-19 pandemic, and/or other major events. However, this study did not compare the performance of indices using any advanced statistical analysis. Furthermore, instead of using the original stock prices, the return values (r_t) of stock prices were considered. This study compared the detection of these changes using the gets package in R to the resultant outcomes of *Autometrics*.

Besides that, this study limited the number of observations in actual data. Only related data sets from 2007 were gathered considering that the majority of Islamic stock indices were initiated in that year. All historical data on stock prices from December 2005, including conventional stock indices, were excluded from this study. Therefore, this study did not consider the influence of major events prior to 2007, such as the Asian Financial Crisis 1997.

Furthermore, all results of stock prices in this study were discussed in a standardised unit of currency, specifically US dollar, in order to avoid potential comparison bias. Therefore, it should be noted that the use of any other local currency may not reflect the actual overview of stock markets in a particular country.

1.5 Significance of Study

A structural time series model can capture the behaviour of the series through trend, seasonal, and irregular components. Through the presence of auxiliary residuals in the structural time series, outliers or structural changes can be identified. High residual values in the structural time series model reflect the presence of outliers or structural changes. However, Harvey and Koopman (1992) elaborated that these residuals can be serially correlated even in a correctly specified model with known parameters. The study then suggested incorporating these auxiliary residuals into Bowman-Shenton test to develop a more formal technique that can detect unusually large residuals. Addressing that, the current study presented a reliable alternative that can concurrently detect outliers and structural breaks, specifically through the application of indicator saturation approach.

The application of the indicator saturation approach for economic data provides more accurate parameter estimation, model specifications, and forecasting performance. The current study's findings on the detection of outliers and structural breaks for accurate forecasting can substantially benefit the society. Considering the country's aim of becoming an Islamic financial hub in the region, accurate estimation and forecasting of financial instruments in consideration of various influencing factors are pivotal. The current study exclusively focused on selected Shariah-compliant stock indices around the world to develop new techniques that can reliably detect outliers and structural breaks in high-frequency financial time series. This study provided significant insights on how to detect outliers and structural breaks in Islamic stock indices, which can help in the formulation of a more strategic economic policy for higher GDP growth.

Accordingly, there is an organisation with a global network of expert Shariah scholars, which performs independent screening on the Shariah compliance of relevant firms. The establishment of Shariah-compliant stock indices promotes the active involvement of Islamic fund managers in the capital markets. Based on Shariah principles, Islamic investment should exclude any practices with the elements of usury (*riba*), gambling (maisir), and ambiguity (gharar). Islamic investors are prohibited to consider financial instruments with fixed income, such as fixed rate of return on investment and grant no voting rights (Walkshäusl & Lobe, 2012). Besides that, Shariah principles strictly prohibit producing or delivering services that against Islamic teaching, such as alcohol, pork, prostitution, and pornography (Securities Commission, 2002).

Furthermore, the current study focused on Shariah-compliant stock indices in Malaysia due to the small number of constituents in Shariah-compliant indices, as compared to conventional indices. From a theoretical viewpoint, this reduces the amount of net market capital fund in the index, which subsequently leads to lower profit yields from this index. This study also presented an additional toolkit to measure the performance of Shariah-compliant and conventional indices.

This study presented significant theoretical findings on the detection of outliers and structural breaks in structural time series model. This study offered valuable insights on the behaviour of each component in the structural time series model and detection of unanticipated shocks. The improved alternative algorithm introduced in this study was expected to help improve parameter estimation and forecast accuracy, which can significantly benefit researchers, policymakers, and analysts in capturing unexpected events and making good decisions based on the historical data.

1.6 Thesis Organisation

Overall, the thesis consists of six chapters. **Chapter 1** presented the background of study and problem statement. The objectives of this study were also listed. Besides that, the scope and significance of this study were described in this first chapter.

Chapter 2 presents the systematic review of literature on various methods for the detection of outliers and structural breaks. This chapter also reviews the detection of outliers and structural breaks using the indicator saturation approach.

Chapter 3 presents the methodology used in this study. This third chapter describes the application of indicator saturation approach through GETS modelling to detect outliers and structural breaks in the structural time series model. The characteristics of the structural series model in this study are also described. Besides that, this chapter discusses how the performance of indicator saturation approach in detecting the structural changes.

Chapter 4 discusses the results of this study on the performance of IIS (in detecting outliers) and SIS (in detecting structural breaks) using the Monte Carlo simulations. This study specifically structured Monte Carlo simulations based on five different cases: (1) no outlier or structural break in the generated time series; (2) detection of single AO; (3) detection of multiple AOs; (4) detection of single LS; (5) detection of multiple structural breaks.

Chapter 5 discusses the results of this study on the empirical applications of IIS and SIS using actual data. This study focused on matching outliers and/or structural breaks to the economic events during the specified timeframe. The forecasting performance outcomes with and without step-indicators are also discussed in this chapter.

Chapter 6 concludes the overall study's results on the detection of outliers and structural breaks in structural time series model using the indicator saturation approach. Recommendations for future research are presented in this final chapter.

CHAPTER 2

SYSTEMATIC LITERATURE REVIEW

2.1 Introduction

In machine learning literature, it is important to perform data cleaning prior to the modelling of data using an algorithm. The process of data cleaning includes detecting outliers and structural breaks. Both outliers and structural breaks adversely affect the model estimation and forecasting performance (Castle et al., 2015a). Therefore, the detection of outliers and structural breaks in time series data is pivotal. The conventional way of detecting outliers and structural breaks in time series data is through visual inspection (e.g., Box and Whisker plot or time series plot), which is not effective since the time series data consist of trend and seasonal components.

2.2 Outlier Detection Methods

There are several outlier detection methods available, which are summarised in Table 2.1. However, the use of these methods may involve certain constraints. Firstly, masking effect may occur when there are multiple outliers (Chen & Liu, 1993). As a result, certain outliers may be overlooked. Secondly, smearing effect may occur Justel et al., (2001). In this case, “good data” may be misidentified as outliers. Thirdly, data may be incorrectly distributed, resulting in misclassification of outliers. Lastly, the magnitude of the identified outliers cannot be determined.

Table 2.1: Summary of outlier detection methods

Method	Description
Likelihood ratio test	Fox (1972) introduced the likelihood ratio test method, which inspired Chang et al. (1988) and Tsay (1988) to introduce the iterative approach in detecting outliers in ARMA and ARIMA models. This method relies on the maximum likelihood ratio statistics and standardised estimated errors in time series data.
Observation influence	Abraham and Chuang (1993) employed influence function to measure the small change effect of data distribution. Small changes, which include missing data, are omitted during the estimation process.
Gibbs sampling	The Gibbs sampling method was first proposed for the detection of outliers and estimation of the AR parameter McCulloch and Tsay (1993). Each parameter requires conditional posterior distribution, including the magnitude of outliers. The probability is carried out based on the prior and posterior probabilities. Justel et al. (2001) conducted an extended study using the form of patches.
Independent component analysis (ICA)	Baragona and Battaglia (2007) used the ICA method to detect outliers in multivariate time series. The method demonstrated the best performance in single outlier detection, even at the end of time series. However, this method does not consider time dynamics of observations.
Weighted least absolute deviation (WLAD)	When it comes to the LAD regression framework, Giloni et al. (2006) highlighted the estimator's lack of robustness in the presence of outliers. The study then introduced the WLAD method to reduce the influence of outliers and subsequently, improve the robustness (Giloni et al., 2006). However, the presence of a large number of outliers in time series adversely affect the robustness of the method.
Penalised weighted least absolute deviation (PWLAD)	Gao and Fang (2016) proposed the PWLAD method to improve the robustness of WLAD and detection of outliers. After all, it is widely known that the PWLAD method can improve the robustness of an estimator. However, the application of the method in AO detection is deemed not practical.
Maximal-overlap discrete wavelet transform (MODWT)	Rashedi et al. (2020) combined the Tukey method and MODWT to detect outliers in Saudi Arabia's closed price stock market. The Tukey method detects outliers in the original series, and MODWT transforms (and sustains) the series to remove the detected outliers. However, this method may not yield its full potential when there are insufficient number of observations to reach the level of wavelet transform. An iterative process is necessary for the detection of outliers.
Penalised weighted least absolute deviation (PWLAD-LASSO)	Addressing the limitation of PWLAD algorithm, Jiang et al. (2021) proposed PWLAD-LASSO that can concurrently attain robust variable selection in linear regression model and detect outliers. However, this method requires one to select a proper weight for the initial estimator.

2.3 Structural Break Detection Methods

There are several structural break detection methods available, which are summarised in Table 2.2. However, the use of these methods may involve certain constraints. For instance, the Chow test method is less powerful in detecting multiple structural breaks in time series data. On the other hand, the Quandt likelihood ratio (QLR) test method is computationally costly and time-consuming. Berg et al. (2008) highlighted that Bai and Perron’s (2003) sequential algorithm may inaccurately estimate in the presence of more than two cases of structural breaks. Likewise, the consistent timing structural break (CTSB) test method can only work in the presence of single structural break Olmo et al., (2011).

Table 2.2: Summary of structural break detection methods

Method	Description
Chow test	This method was introduced by Chow (1960) shows that a location of structural break is needed to be known exogenously to test whether it is a structural break or not. The Chow test is used to assess the stability of regression coefficients in the model. Any coefficient instability means a structural change in the regression model. It is useful to identify a single structural break. Detection of break date based on F-statistics. (Mohamed & Nageye, 2021), (Dao, 2022) and (Ghouse et al., 2023)
Quandt likelihood ratio (QLR) test	QLR is an extension from Chow test proposed by Quandt (1960). This test differs substantially because it avoided location of structural breaks exogenously. Hence, it able to detect unknown break date. Quandt statistics known as the most significant Chow test statistics from all break dates selected. Recent applications of QLR test can be found in (Lu et al., 2022a; Martinho, 2022) and (Lu et al., 2022b)
Bai and Perron’s (BP) test	Bai & Perron, 1998 (2003) proposed this test to identify multiple structural breaks in time series data and testing their statistical significance. This test works in two separate parts. First, it identifies multiple structural breaks regardless their statistical significance based on sequence of sup F-statistics. Second, it used a series of test statistics to test their statistical significance. Besides, (Bai & Perron, 2003) demonstrate their ideas in testing the multiple structural breaks performed well in large samples. The latest development of by (Perron et al., 2020) allow estimation of a trend function and testing for structural changes regardless of whether the stochastic component is stationary or contains an autoregressive unit root.

Method	Description
	Then, (Esteve & Prats, 2022) used the test statistics to test jointly for structural changes in mean and variance proposed by Perron et al. (2020). Recent study by Prados de la Escosura & Rodríguez-Caballero (2022) employed the Bai and Perron's procedure to capture the structural breaks in real house prices with annual data for the case of Australia for the period 1870–2020. (Ghouse et al., 2023) investigates the spillover effects of the waves of Covid-19 that affected the performance Pakistan's stock exchange.
Cumulative sum (CUSUM)-based approach	The CUSUM test is based on the analysis of the scaled recursive residuals and has a significant advantage of not requiring prior knowledge of the point at which the hypothesized structural break takes place (Dao, 2021). Olmo et al. (2011) introduced the CTSB test to maintain the integrity in financial system by capturing potential insider trading prior to the release of market sensitive information. Recent application of CUSUM-based approach can be found in (Dao, 2021), (Dao, 2022), and (Ali et al., 2022).
Wild Binary Segmentation (WBS)	The algorithm was originally introduced by (Fryzlewicz, 2014) for consistently estimating the number and locations of multiple change points in data. There are several studies that extend the idea of (Fryzlewicz, 2014) by combining the WBS algorithm with CUSUM-based algorithms, such as (Cho & Fryzlewicz, 2015), (Barigozzi et al., 2018), and (Li et al., 2022). However, the limitations of these algorithms are that they only detect breaks in the first-order moment structure and affect the estimation accuracy of the break number and location.

Overall, it is fundamental to detect both outliers and structural breaks in order to examine the relationship between short-term and long-term effects in financial time series data. However, there have been limited studies on the detection of these structural changes using financial time series data. This propelled the current study to develop a robust method that can concurrently detect outliers and structural breaks using financial time series data.

2.4 Improving Detection of Structural Changes Using Indicator Saturation Approach

The indicator saturation approach in GETS model selection is the most recent development in literature on the detection of outliers and structural breaks. The applications of IIS and SIS in structural time series model framework have continued to gain growing research interest. For instance, Marczak and Proietti (2016) applied IIS and SIS in basic structural time series (BSM) framework to detect the presence of outliers in the European industrial production time series data. Prior studies also proved the capacity of IIS and SIS in detecting outliers and structural breaks when the duration, location, magnitude, and number of structural changes are unknown.

Hendry (1999) first introduced the indicator saturation approach using the US food expenditure data from 1931 to 1989. Hendry and Krolzig (2004), Hendry and Santos (2005), Santos (2008), Santos et al. (2008), and Johansen and Nielsen (2016) then extended the proposed approach through the development of IIS. Using the US ex-post real interest rate sample data, Castle et al. (2012) revealed that the performance of IIS exceeded the performance of Bai and Perron's test. Following that, Doornik et al. (2013) developed SIS for the detection and modelling of structural breaks using step-indicators. In a more recent study on SIS, Castle et al. (2015a) compared the performance of SIS with other structural break detection methods, such as LASSO and least angle regressions.

To date, there has been no systematic review on the detection of outliers and structural breaks using indicator saturation approach, especially in the case of Shariah-compliant stock indices. This study aimed to address this particular gap through a systematic review on how to detect outliers and structural breaks and model the financial time series data. The current study performed a systematic literature review on the detection of outliers and structural breaks in financial time series model, which significantly contributed to the existing body of knowledge. This led to the formulation of more specific research questions, which are discussed in the subsequent section.

2.5 Data Abstraction and Synthesis

An integrative review was considered in this study, which included both qualitative and quantitative methods. Whittemore and Knafl (2005) identified integrative review as the most ideal approach to analyse data because it allows researchers to compare the extracted data iteratively. This study performed data abstraction with respect to the formulated research questions. All 36 articles were analysed to extract relevant data with respect to the research questions listed in Table 2.7. Following that, thematic analysis was performed to identify any underlying patterns and trends on the detection of outliers and structural breaks using the indicator saturation approach. Relevant themes were generated, in which any similar or related ideas were grouped together (Mohamed Shaffril et al., 2021). As a result, four themes emerged: (1) automatic model selection; (2) indicator saturation approach; (3) algorithm development; (4) empirical applications. The usefulness and accuracy of these themes were reviewed through a series of discussion with the expert reviewers. The points of arguments and inconsistencies were evaluated and discussed to the point of agreement.

Table 2.3: Themes of thematic analysis

No.	**Studies**	**Location Shifts**		**GETS**	**Indicator Saturation**			**Algorithms**				**Empirical Applications**
		Outliers	**Breaks**		**IIS**	**SIS**	**DIS**	***PcGets***	***Autometrics***	**R**	**HP**	
1	Hendry (1999)		/	/	/							US Food Expenditure
2	Hoover & Perez (1999)			/							/	Lovell’s (1983) data
3	Krolzig & Hendry (2001)			/				/				Lovell’s (1983) data
4	Krolzig (2003)			/				/				UK consumers’ expenditure
5	Hoover & Perezà (2004)			/								Cross-growth country
6	Hendry & Krolzig (2004)			/				/				Data generating process (DGP)
7	Hendry & Santos (2005)			/	/			/				DGP
8	Santos (2008)		/	/	/			/				DGP
9	Johansen & Nielsen (2008)	/	/		/				/			
10	Santos et al. (2008)	/		/	/				/			DGP
11	Castle & Hendry (2009)	/			/				/			Euro GDP Growth
12	Santos & Oliveira (2010)		/	/	/			/				France Inflation rate
13	Castle & Hendry (2010)		/	/	/				/			DGP

No.	Studies	Location Shifts		GETS	Indicator Saturation			Algorithms				Empirical Applications
		Outliers	**Breaks**		**IIS**	**SIS**	**DIS**	***PcGets***	***Autometrics***	**R**	**HP**	
14	Castle et al. (2011)		/	/	/				/			US real interest rate
15	Ericsson (2017)											UK and US economic variables
16	Sucarrat & Escribano (2012)			/						/		Exchange rate
17	Castle et al. (2014)		/	/	/				/			US and UK Philip curve
18	Castle et al. (2012)		/	/	/				/			US real interest rate
19	Bergamelli & Urga (2014)		/	/	/				/			US interest rate
20	Castle et al. (2015a)		/	/		/			/			DGP
21	Doornik & Hendry (2015)	/		/	/				/			DGP
22	Panday (2015)		/	/	/				/			Nepal exchange rate
23	Aloui et al. (2015)		/									Islamic stock
24	Hendry & Mizon (2016)		/	/	/				/			DGP
25	Johansen & Nielsen (2016)	/		/	/				/			Fish data
26	Marczak & Proietti (2016)	/	/	/	/	/			/			Euro IPP
27	Pretis et al. (2016a)		/	/			/		/			Volcanic eruptions

No.	Studies	Location Shifts		GETS	Indicator Saturation			Algorithms				Empirical Applications
		Outliers	**Breaks**		**IIS**	**SIS**	**DIS**	***PcGets***	***Autometrics***	**R**	**HP**	
28	Stillwagon (2016)		/	/	/	/			/			Exchange rate
29	Majdoub et al. (2016)		/									Islamic and conventional stocks
30	Ericsson (2017)		/	/	/				/			US federal debt
31	Pretis et al. (2018)	/	/	/						/		UK sulphur dioxide emission
32	You et al. (2020)	/										Ecological momentary assessment data
33	Qiao et al. (2020)	/										Wireless sensor networks
34	Phoong et al. (2020)		/									Crude oil prices
35	Telli & Chen (2020)		/									Cryto-currency markets
36	Rashedi et al. (2020)	/										Saudi Arabia’s closed price stock market

2.6 Findings of Thematic Analysis

2.6.1 Automatic Model Selection

The development of equation in simple-to-general modelling was initially based on economic theory. However, there are several limitations to the simple-to-general modelling, which were highlighted by Hendry and Krolzig (2001): (1) no clear stopping point in model selection procedure; (2) too many models due to divergent and dependence in searching path strategy; (3) potentially pernicious to proceed with alternative hypothesis if the model does not pass the diagnostic tests. Following that, Hendry and Krolzig (1999) reviewed the application of the GETS approach based on Lovell's (1983) study of data mining. Hendry and Krolzig's (1999) pioneering study prompted more studies, such as studies by Krolzig and Hendry (2001), to explore algorithm development for multipath search procedure in *PcGets*. In particular, Krolzig and Hendry (2001) found that automated GETS modelling outperformed the manual model selection due to its capacities to deal with more variables than observations and explore infinite searching paths. Apart from these two key strengths, Hendry and Krolzig (2004) added that the automatic model selection also outperformed manual selection in terms computational speed and capability of handling the complexity in detecting multiple breaks. Hendry and Krolzig (2005) provided similar views on the capability of handling more variables than observations as the key strength of the GETS approach embodied in *PcGets.* All reviewed articles in the current study also demonstrated the preference to use the GETS approach in automatic model selection procedure.

2.6.2 Indicator Saturation Approach

There are three types of indicator saturation in literature, namely IIS, SIS, and design break indicator saturation (DIS). Hendry (1999) pioneered the application of indicator saturation to detect unknown number of breaks at unknown location, magnitude, and time. This pioneering work has led to the development of numerous indicators in static regressions model (Hendry & Santos, 2005). Santos et al. (2008) demonstrated that the indicator saturation approach did not produce distorted model estimation and selection, but the dummies coefficient estimated by ordinary least squares were not consistent. Meanwhile, Castle et al. (2009), Castle et al. (2011), Castle et al. (2015b), and Marczak and Proietti (2016) used IIS in automatic model selection procedure to detect multiple breaks. Doornik et al. (2013) then introduced step-indicators as an extension of IIS to detect structural breaks in time series data. However, studies have proved that SIS is more effective than IIS in the detection of multiple breaks (Castle et al., 2015a; 2015b; Stillwagon, 2016; Marczak & Proietti, 2016). Several prior studies also proved that step-indicators function in both static and dynamic models (Doornik et al., 2013; Pretis et al., 2016a; Stillwagon, 2016). In a more recent study, Marczak and Proietti (2016) examined the performance of IIS and SIS in basic structural model (BSM) framework using Monte Carlo experiments (replicated 1,000 times).

2.6.3 Algorithm Development

Referring to Lovell's (1983) works in data mining experiment using limited *MATLAB* code (HP1999), Hoover and Perez (1999) started the development of data mining algorithm in GETS modelling. Essentially, the development of HP1999 algorithm involves the formulation of general unrestricted model (GUM), multiple-path searching strategies, encompassing test, diagnostic tests, and info criterion as a tiebreaker. Accordingly, Hendry and Krolzig (2001) improved this algorithm in *PcGets*, an *Ox* package. At this point, *PcGets* extended HP1999 with additional pre-search, iterative multiple-path searching strategies, and theoretical aspects in model selection. Following that, Doornik (2009) presented substantial improvement to the GETS algorithm via *Autometrics* embodied in *OxMetrics*—the *Autometrics* algorithm was found to systematically improve multiple-path searching strategies) through the tree-search method and to raise computational speed that can prevent same model estimation and delayed diagnostic testing (Doornik, 2009).

Sucarrat and Escribano (2012) recently introduced GETS algorithm in R package, specifically known as *AutoSearch*. Following that, Pretis et al. (2018) introduced the *gets* package as a successor of *AutoSearch*. To date, it is the only free and open-source software available to provide GETS modelling of the mean of a regression, GETS modelling of a conditional variance regression, and indicator saturation using *isat* function. Moreover, the *isat* function enables IIS, SIS and trend-indicators to detect and estimate outliers and structural breaks in time series data. Pretis et al. (2018) recently proved that the *gets* package can substantially increase the computational speed with *turbo = TRUE* and *max.paths = NULL* arguments in *isat* function.

As of January 2022, there are several software available for GETS and indicator saturation, which are presented in Table 2.8. In particular, the *gets* package in R (version 0.20, as of September 2019) serves as the first software within and beyond the R universe, with a full set of features for user-specified GETS and ISAT, namely user-specified model, user-specified diagnostics, and user-specified goodness-of-fit criteria (Sucarrat, 2020).

Table 2.4: List of software available for GETS modelling and indicator saturation

	HP1999 (MATLAB)	***Autometrics* (OxMetrics)**	***Grocer* (Scilab)**	***genspec* (STATA)**	**EViews**	***gets* (R)**
GETS of linear regression	Yes	Yes	Yes	Yes	Yes	Yes
GETS of variance model						Yes
GETS of probit model		Yes	Yes			
ISAT of linear regressions		Yes	Yes		Yes	Yes
User-specified GETS			Yes			Yes
User-specified ISAT						Yes
User-specified diagnostic			Yes			Yes
User-specified of goodness-of-fit						Yes
Menu-based GUI		Yes			Yes	
Free and open source	Yes*		Yes	Yes*		Yes

Note: * denotes that, despite having free and open source module, HP1999 (MATLAB software) and *genspec* (STATA software) are embodied in non-free and closed source software environment.

2.6.4 Empirical Applications

The majority of prior studies presented empirical applications in GETS modelling based on UK and US economic data (Hendry, 1999; Hoover & Perez, 1999; Krolzig, 2003; Ericsson & Reisman, 2012; Bergamelli & Urga, 2014; Castle et al., 2015b; Ericsson, 2017). However, there have been limited studies on the detection of outliers and structural breaks in Shariah-compliant stock indices. Prior studies focused on the detection of either outliers or structural breaks in Shariah-compliant stock indices, as shown in Table 2.9. Although the current study focused on a limited timeframe up to 2020, the detection of outliers and structural breaks has continued to gain research interest given the growing number of articles published after 2020 in Scopus and Web of Science. For instance, Pellini (2021) used *Autometrics* to