ESTIMATION OF WEIBULL PARAMETERS USING SIMULATED ANNEALING AS APPLIED IN FINANCIAL DATA

ABUBAKAR HAMZA

UNIVERSITI SAINS MALAYSIA

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ESTIMATION OF WEIBULL PARAMETERS USING SIMULATED ANNEALING AS APPLIED IN FINANCIAL DATA

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ABUBAKAR HAMZA

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LIST OF ABBREVIATIONS

AIC	Alike information Criterion
ARR	Accounting Rate of Return
CDF	Cumulative density function
DE	Differential Evolution algorithm
DPB	Discounted Payback Period
EWD	Extended Weibull distribution
GA	Genetic algorithm
GOF	Goodness-of-Fit Tests
IRR	Internal rate of return
KS test	Kolmongrov-Smirov test
LSM	Least square method
MIRR	Modified Internal rate of return
MAE	Mean Absolute Error
MM	Method of Moments
MLE	Maximum Likelihood Estimation
MOA	Metaheuristics optimization algorithm
MPS	Malaysian property sector
NPV	Net Present Value
PB	Payback Period
PDF	Probability density function
PI	Profitability Index
R-square	Coefficient of Determination
RM	Regression Method
ROI	Return on Investment
ROA	Return on Asset
ROE	Return on Equity
RMSE	Root Mean Square Error
WD	Weibull Distribution
SA	Simulated Annealing Algorithm

LIST OF NOTATIONS

α	Scale parameters
β	Shape parameters
η	Location parameters
$E(X)$	Mean
$Var(X)$	Variance
ξ	Donate the shape and scale parameters
$\pi(\xi)$	Prior distribution
$\pi(\xi \mid x_1, x_2, ..., x_n)$	Posterior distribution
$f(x \mid \xi)$	Conditional probability density function of the model
$f(x_1, x_2, ..., x_n \mid \xi)$	Conditional joint probability density function of financial data

LIST OF APPENDICES

PENGANGGARAN PARAMETER *WEIBULL* MENGGUNAKAN PELUNAKAN SIMULASI BAGI APLIKASI DATA KEWANGAN

ABSTRAK

Analisis data kewangan yang tepat adalah penting untuk mewajarkan kemampanan untuk potensi pelaburan dalam syarikat. Taburan Weibull boleh digunakan untuk memeriksa gelagat pelaburan kerana fleksibilitinya untuk diubah menjadi jenis taburan lain. Walau bagaimanapun, pemilihan penganggar yang paling sesuai masih merupakan tugas yang mencabar. Kajian ini mencadangkan algoritma penyepuhlindapan simulasi (SA) dalam menganggar parameter taburan Weibull dengan aplikasi kepada data kadar pulangan dalaman yang diubah suai (MIRR). Objektifnya adalah untuk mengkaji potensi pelaburan syarikat patuh Syariah sektor hartanah Malaysia (MPS). MIRR dikira berdasarkan data yang diekstrak daripada laporan kewangan syarikat dari 2010 hingga 2018. Prestasi algoritma SA telah diterokai dari segi ketepatan dan ralat anggaran. Dapatan ini mendedahkan bahawa pengedaran Weibull sangat sesuai untuk menerangkan tingkah laku pelaburan MPS berdasarkan anggaran melalui algoritma SA. Oleh itu, pembelian saham dalam sektor ini sangat menarik untuk tempoh pelaburan jangka panjang, tetapi mungkin mempunyai risiko tinggi untuk melakukannya akibat turun naik dalam min dan varians anggaran. Selain itu, taburan Weibull dua parameter telah diperluaskan dengan memasukkan parameter tambahan untuk menangkap gelagat ketidakpastian dalam data kewangan. Dapatan menunjukkan bahawa model yang dicadangkan mengatasi model sedia ada dan lebih fleksibel daripada model sedia ada berdasarkan data simulasi yang digunakan. Tambahan pula, kajian lanjut dijalankan berdasarkan pendekatan Bayesian terhadap taburan Weibull pada andaian gamma sebelum seperti yang digunakan untuk data tuntutan insurans. Prestasi pendekatan Bayesian telah diterokai berbanding dengan algoritma SA dan Anggaran kebolehjadian maksimum (MLE). Dapatan ini mendedahkan bahawa anggaran melalui algoritma SA mengatasi penganggar lain untuk saiz sampel yang besar. Walau bagaimanapun, dalam kes saiz sampel yang kecil, pendekatan Bayesian dan MLE mendedahkan prestasi yang lebih baik dari segi ketepatan dan ralat anggaran. Ia disimpulkan dengan cadangan untuk menggabungkan dua penganggar untuk anggaran parameter optimum.

ESTIMATION OF WEIBULL PARAMETERS USING SIMULATED ANNEALING AS APPLIED IN FINANCIAL DATA

ABSTRACT

An accurate analysis of financial data is vital to justify sustainability for investment potential in a company. Weibull distributions can be used to examine investment behaviour due to their flexibility to be transformed into other types of distribution. However, the selection of the most suitable estimators is still a challenging task. The present study proposes a simulated annealing algorithm (SA) in estimating the parameters of Weibull distribution with application to modified internal rate of return data (MIRR).The objective is to examine the investment potential of the shari'ah compliance companies of the Malaysia property sector (MPS). The MIRR were computed based on the data extracted from the companies' financial reports from 2010 to 2018. The performance of the SA algorithm has been explored in terms of accuracies and estimation errors. The finding reveals that the Weibull distribution is well-suited to describing the investment behaviour of the MPS based on the estimates via the SA algorithm. Therefore, purchasing shares in this sector is very attractive for a long-term investment period, but may have a high risk of committing it as a result of fluctuations in the mean and variance of the estimate. Additionally, the two-parameter Weibull distribution has been extended by incorporating additional parameters to capture the uncertainty behaviour in the financial data. The findings reveal that the proposed model outperformed the existing model and is more flexible than the existing model based on the simulated data used. Furthermore, further study is conducted based on the Bayesian approach to Weibull distributions on the assumption of gamma prior as applied to insurance claims data. The performance of the Bayesian approach was explored in comparison with the SA algorithm and Maximum likelihood estimates (MLE). The finding reveals that the estimates via the SA algorithm outperformed the other estimators for a big sample size. However, in the case of small sample size, the Bayesian approach and MLE reveal better performance in terms of the accuracies and estimation errors. It is concluded with a recommendation of combining the two estimators for optimal parameter estimation.

CHAPTER 1.
INTRODUCTION

1.1 Background of the study

Owning company shares is a popular investment activity which is undoubtedly an important source of economic growth and stability. Investment analysis is the process of determining the operating and financial health of a firm from accounting and financial statements for a certain period (Bhunia et al., 2011). The goal of such analysis is to determine the efficiency and performance of the firm's management, as reflected in the financial records and reports. The analyst attempts to measure the sustainability of a firm and other indicators that the business is conducted profitably; ensuring enough returns to the shareholders to maintain at least its market value. Financial performance is the ability of a company to manage and control its resources. various metrics indicators have been used to evaluate and investigate investment behaviour, project profitability, and make investment decisions for a certain company (Fatihudin, 2018). Some of these metrics indicators include the Discounted Cash Flow (DFC) practices which take into account the value of money over time, such as Internal Rate of Return (IRR), and Net Present Value (NPV). Among the simplest are the Return on Equity (ROE), Return on Asset (ROA), Return On Investment (ROI), Earnings per share (EPS), Payback Period (PB), Accounting Rate of Return (ARR), Profitability Index (PI), the Discounted Payback Period (DPB), and Real Option (RO)(Leon et al., 2008).

Investment valuations based on the net present value (NPV) and the internal rate of return (IRR) represent the traditional techniques for stock valuation. These indicators are regarded as the most conceptually dependable tools utilized by financial analysts (Marchioni and Magni, 2018). However, despite the popularity of NPV and

IRR in examining investment performance, different shortcomings and limitations have been revealed (Xie and Chen, 2021). One of the main shortcomings of these models in the implementation of IRR in project evaluation is that a single investment project may have multiple values of IRR (Beaves, 1988; Osborne, 2010). To overcome the limitation of IRR, scholars first pioneered the modified internal rate of return (MIRR) during the 18th century and rediscovered it later in the 1950s. This model assumes reinvestment of cash inflows at the reinvestment rate and does account for the periodic free cash flows (Kierulff, 2008). Hence, MIRR solves the problem of multiple values of IRR. The MIRR here is the rate at which the NPV equals zero where the present value of the terminal value of the invested fund at the end equals the present value of the series of annual contributions over the investment period. Several studies indicate that MIRR is better than IRR models in which the DCF analysis is recalibrated to reveal more predictable values of NPV (Balyeat and Cagle, 2015; Mellichamp, 2017).

This thesis first and foremost investigates the primary objective of the modelling work by undertaking financial modelling for investment return in the Malaysian property sector (MPS) over its sustainability. The selection of potential investments has been carried out using modified internal rate of return (MIRR) techniques designed by (Sabri and Sarsour, 2019). The method has been used in determining the investment return on a project. Companies may employ different strategies for different tasks. For example, a company may use the PB approach for minor projects and the NPV can be used for large projects. Recently, sustainability has taken on increased significantly, with 'green buildings' becoming increasingly important to tenants, owners and property developers. The growing interest in investment in the property sector has triggered a need to examine the potentiality of

investment return in that sector (Low et al., 2020). Therefore, this study is intended to examine the investment behaviour of the MPS based on their financial data and to further describe long-term investment behaviour in the sector.

1.2 Stock Investment valuation

Investment is defined as the act of placing money either to begin or expand a project or to buy assets for which that money is utilized to generate or increase income over time (Balyeat and Cagle, 2015). It can be typically referred to as any procedure performed to generate income in the future. From a financial point of view, an investment may include the purchase of stocks, assets, or real estate property among numerous others (Bakar and Rossi, 2019). Modelling the behaviour of investment return is one of the critical and daunting financial objectives. Investors want to optimize their profits and regulators need predictive models. Investment valuation is the method of assessing and describing a company's performance in terms of rate of return (Li et al., 2018; Sabri and Sarsour, 2019). Jamei (2020) described stock investments as one of the key problems in global capital markets faced by financial management. The valuation of investment returns on stocks is an incredibly complex method that can usually be a mixture of art and science. Investors can be confused by the amount of information available which is used theoretically in valuing return on investment stocks. Choosing the most valid criterion for evaluating a company's performance and making an appropriate investment decision is one of the most attractive fields in finance, which is also one of the major concerns for all investors (Jamei, 2020). Forecasting and predicting the long-term investment return allows for the identification of investments with typically high returns. Such investment might otherwise not have been detected before it is too late to take any action to reduce the

risk of investment return. Furthermore, investigating an investment return allows for the estimation of the company's expected cost of handling returns (Marchioni and Magni, 2018).

Over the years, financial modelling has been a critical task in strategic business decisions across a variety of industries in many projects, and is widely regarded as a solid prerequisite for all types of investment evaluation. Financial analysis involving debt and equity funding structures (Rutkauskas et al., 2008). In finance, modelling is used to answer frequently difficult questions about future investment behaviour. This is accomplished by numerically representing a real-life situation and making smart financial decisions in all forms of capital projects. As a result, it has become a necessary activity for any project financial decision, as it is critical for any project evaluation which has an impact on the company's performance in the future. One of the major intentions is to build financial modelling, which is necessary for gaining a thorough grasp of the core concepts and elements that make up the model's main inputs and assumptions. The understanding of these important inputs before the actual financial modelling task in predicting the model's performance which has a significant impact on investment feasibility. A robust investment modelling and analysis of financing are primarily influenced by the modelling objectives, investment models, project ownership, and financing structures; and also the economic and financial key performance indicators required.

This research work proposes the simulated annealing algorithm in the Weibull distribution for stock investment valuation modelling. The study focuses on the shari'ah compliance companies of the MPS and publicly listed from 2010 to 2018. To determine the stock investment valuation modelling, it is prudent to consider the internal rate of return of business. Moreover, to find out the investment return of the

company, it is crucial to extract information such as dividend rate, stock issuance, bonus issues and share splits from a company financial report.

1.2.1 The stock share issuance

Issued shares are the subset of authorized shares that have been sold to and held by the shareholders of a company, regardless of whether they are insiders, institutional investors, or the general public (as shown in the company's annual report). Issued shares include the stock a company sells publicly to generate capital and the stock given to the existing shareholders as part of their compensation packages. Thus, authorized shares are the total amount a company can issue or sell, and issued shares are the portion of authorized shares that a company has sold or otherwise placed in the market, including shares they hold in their treasury (Rutkauskas et al., 2008).

A company issues a share only once; after that, investors may sell it to another investor on the secondary market. When companies buy back their shares, the shares remain listed as issued, even though they become classified as "treasury shares" because the company may resell them. For a small, closely-held corporation, the original owners may hold all of the issued shares (Gyimah, 2016). The number of issued shares is recorded on a company's balance sheet as capital stock, or owners' equity, while shares outstanding (issued shares minus any shares in the treasury) are listed on the company's quarterly filings with the Securities and Exchange Commission. The number of outstanding shares is also found in the capital section of a company's annual report (Tabibian et al., 2021).

1.2.2 Shares Issues, ownership, splits, bonus and treasury share

Corporations are owned by shareholders who invest money in the business by buying shares of stock. The portion of the corporation they own depends on the

percentage of stock they hold. For example, if a corporation has issued 100 shares of stock, and you own 20 shares, you own 20 percent of the company. Ownership of the company is determined by who owns the shares, and battles for ownership may take place when a person or entity acquires a sufficient number of shares to seek one or more seats on the company's board of director (Napp and Minshall, 2011). Ownership may also be measured by counting issued and outstanding shares, along with those that may become issued if all authorized stock options are exercised, which is known as the fully diluted calculation. In addition, ownership may be measured by using issued and authorized stock as a forecast of the position shareholders may be in at a future date. All board members must use the same calculation when making decisions or plans for the business (Michelon and Parbonetti, 2012; Schoenfeld, 2017).

The stock issuance process will incorporate data from the company's financial report, and the model also involves finding out the accumulation of share units. If a company fails to record any share issuance, then the share unit is assumed to be one share unit. The process of splitting the share unit from 1 to 2, means that the earned share unit will be multiplied by two. The stock price of a company will then be halved to improve and further enhance the liquidity of the company’s share capital that would be traded in a given market, which is why the share capital will end unchanged. Additionally, the issuance of a single bond by the ordinary share shows that the organization will award 50% of the share units to file up to 1.5 share units at the end of every year. If the evens are practised simultaneously, then the shareholders will then be issued three times more than the previous shares (Sidra and Attiya, 2013; Sabri and Sarsour, 2019). Some organizations are known for the practice of treasury stock that repurchases the shares of shareholders randomly for a given share price. During such instances, the main objectives of the organizations are to grow their capital with

the share unit. In most cases, the companies tend to ignore such practices by avoiding the offer of selling their shares back to the organization. Practically, when this happens, most of the dividends will then be liquidated to cash, which will be credited back to the account of the shareholders. Additionally, a couple of companies practice treasury shares by dividing all the dividends, which is commonly referred to as the treasury share profits. Under this condition, shareholders will get dividend in the form of cash. However, the share units will then be reduced to a given portion that will then be transferred to the annual report of the company. If a company wishes to hold the stock for a long period, the stock valuation will then be found by calculating the MIRR (Chen and Wang, 2012). The investment mechanism for holding such stock is by distributing the contribution of the investors at the start of the years for several years.

On the other hand, if the company announces their profit every year, then the dividends will be ploughing back into the initial investment and then deposited with a level contribution to increase the share units. At the end of the anticipated years, the company will then allow all their shareholders to get a certain number of share capital, which indicates the profit of the investors for some periods (e.g t years). If the share capital happens to be lower than the total contribution, it is then expected that MIRR will turn out to be in a negative form (Sabri and Sarsour, 2019). When an investor is faced with varieties of options and choices while considering investment decisions, he needs to decide on the number of asset selections, as well as the amount of investment.

Therefore, making a proper decision on the amount of money that should be invested in each asset is of great importance (Raei and Bahrani, 2012). The major role the investment decision plays is to invest in stocks so that the investor faces a risk-return trade-off while making his decision on his investment (Rahmani et al., 2019).

An investment decision is equivalent to the investor investing the optimal return from a set of possible investments by considering the return, risk, and many other factors. With this, investment decisions can be solved by using different approaches such as Linear programming problems, the Bayesian approach, Multi-Purpose Planning, Randomized Control Models; and recently, the application of artificial neural networks, and metaheuristics algorithms which have contributed tremendously in optimizing, forecasting and modelling of investment behaviour.

1.2.3 Modelling of Investment Return

The mathematical model used in the investment returns valuation will enable investors to systematically make optimization decisions. Making investments require a large amount of capital invested in the initial year, mostly in fixed assets such as property, machinery, or equipment. Due to the significant amount of cash outflows required, companies perform a capital investment analysis to evaluate the profitability of an investment and determine whether it is worth it. This is especially important when a business is presented with multiple potential opportunities and needs to make an investment decision based on the investment returns available. To assess the profitability of capital investment, companies can build a capital investment strategy to model the key valuation metrics including the DCF, NPV, IRR, PB and so on.

Therefore, this study is interested in the complexities of the methods of generating cash flow, which is achieved by using financial data. The study examines the MIRR which is crucial in examining the investment process. The use of mathematical models allows different criteria, and circumstances to be taken into consideration and variants of return-on-investment project scenarios to be produced. These criteria define the efficiency of a project in producing cash flows, which is optimal in proving project viability as compared to many other measuring criteria. The

efficiencies of mathematical models in forecasting investment behaviour for planning purposes have been investigated (Burgelman and Vanhoucke, 2018). In this respect, it appears that mathematical and statistical models, rather than traditional accounting measurements, are required. The significance of mathematical models stems from their potential to be utilized as a criterion to address the most recent limits of accounting measurements and hurdles to performance evaluation accuracy. It is critical to highlight that choosing a suitable criterion for prioritizing and evaluating shareholder performance is critical. This eventually becomes the basis for establishing managers' rewards, and it assists in making better investment decisions as well as achieving their benefits and goals.

Investments are made in generating value from their savings, but because of market volatility and uncertainty in financial conditions, they face several difficulties when making decisions. The complexity associated with financial data makes it difficult to be comparable, and non-adjustable to uncertainty (Filiasi et al., 2014). There is a need to develop a model framework based on a statistical distribution such as the Weibull distribution to address these challenges. This research work proposes the stock investment valuation model by using the Weibull distribution. The stock investment valuation modelling focuses on the Shari'ah compliance companies of the Malaysian property sector (MPS) and publicly listed organizations from 2010 to 2018 To determine the stock investment valuation model, it is prudent to consider the internal rate of return of business. And to find out the internal rate of a company's return, it is crucial to derive the company's financial information, such as the dividend rate, and the stock issuance share split.

1.2.4 Weibull distribution

The Weibull distribution is a continuous probability distribution that can fit an extensive range of distribution shapes. It is named after Swedish mathematician Waloddi Weibull, who described it in detail in (Weibull, 1951), although it was first applied by Rosin (1933) to describe a particle size distribution. It is commonly called "lifetime distributions". Lifetime distribution data analysis is frequently referred to as "Weibull analysis." It has been applied in a wide variety of applications in different areas such as reliability engineering, physical sciences, medical sciences, social sciences and other engineering filed such as quality control, maintenance and replacement, inventory control, astronomy has been listed in (Akgül et al., 2016; Ishaq & Abiodun, 2020; Jiang et al., 2017).

Analysts use the Weibull distribution frequently because it is so adaptable to varying conditions. The nature of the distribution changes significantly based on the values of the parameters. In fact, with certain parameter values, the Weibull distribution is equivalent to other probability distributions. The ability to analyze profitability trends and provide profitability forecasts in financial data sets (McCool, 2012; Rinne, 2008). It has been extensively used in the modelling and analysis of real-life data in emerging areas such as banking, risk management, financial and actuarial sciences, and economics, among others. The quality of statistical procedures mainly depends on the considered probability model of the studied phenomenon which may be associated with uncertainty in the data (Alshenawy et al., 2020). The popularity of the Weibull distribution is attributable to the fact that it is flexible in providing a useful fitting for different kinds of data, especially in emerging areas such as financial and investment data sets (stock prices movement and claims frequency in actuarial data) in addition to its traditional survival and engineering applications (Zhao et al., 2021).

In summary, the Weibull distribution is versatile in data fitting with growing, constant, and decreasing failure rate functions (Xie et al., 2004).

Several methods have been developed in the field of mathematical statistics to estimate the parameters of Weibull distribution based on simulated and real-life data sets. A more common technique includes probability-weighted Moment (Ribereau et al., 2016), Maximum likelihood method (Glas, 2017), Least square method (Hansen, 2008), Regression method (Datsiou and Overend, 2018) and Metaheuristics optimization algorithm (Okafor et al., 2018). The Bayesian approach can also be used in which the estimate returns the posterior distribution of the parameters given the data. One important advantage of the Bayesian approach over the maximum likelihood method is the possibility to inject prior knowledge into the parameters (Garnier-Villarreal and Jorgensen, 2020). Interval priors can be used to restrict one parameter within some given bounds, for example, a parameter that can only be positive. Likewise, if a distribution can be positively or negatively skewed, but the data are known to always be positively skewed, the parameter(s) that determine the skew could be constrained by the use of an interval prior. The appropriate method of analysis varies according to the type of data generated, and in some cases, based on the probability distribution employed. Among them are the MLE, which under regulatory conditions is graded as asymptotically unbiased with minimal variance (Glas, 2017).

The simulated annealing algorithm (SA) is one of the first stochastic metaheuristics algorithms inspired by the thermodynamic process. This process which has been applied in metallurgical engineering by Kirkpatrick et al. (1983) in describing the solidification process involves changing a material's condition while lowering its energy state to the lowest possible level, after the metal is detached from the heat source and the molten material is manually cleaned, the temperatures are gradually

reduced according to a pre-determined rate. The energy has achieved its minimal level as heat is transferred to the surrounding environment, causing the metal to crystallize into a single massive crystalline lattice structure and solidify (Optimization). Because optimal results need a very slow reduction of the temperature with control from iteration to iteration, the SA algorithm can be delayed in obtaining the optimal solution (well-organized and perfect structure). If crystallization occurs too quickly, the resulting lattice structure is unlikely to be optimum which is an imperfect structure (Bertsimas and Tsitsiklis, 1993).

In this study, the performance of the SA algorithm has been explored in estimating the parameters of Weibull distribution with particular application to financial data. The MIRR data will be utilized in this study to model the behaviour of investment returns. The purpose is to investigate the potential of investment in the Shari'ah compliance companies of the MPS. This has been under taken by examining the investment behaviour using the Weibull distribution. The SA algorithm has been successfully used in a variety of optimization applications such as (Abbasi et al., 2006; Yang and Cho, 2019). It has shown outstanding performance in parameter estimation for linear and non-linear functions. To the best knowledge of the researcher, the parameters of Weibull distribution have not been estimated using the SA algorithm based on the financial data set (i.e. MIRR data from the MPS). The MIRR data from the MPS has been used for the analysis of long-term investment return. A comprehensive review regarding Weibull distribution, Extended Weibull distribution, investment return, and metaheuristics algorithm have been presented under the literature review in Chapter 2.

1.3 Problem Statement

Scholars and practitioners in the field of finance and investment have underlined the need of continuing to understand what drives investment decisions among existing and prospective investors. Despite the need for accurate investment models for examining and forecasting investment growth, only a few studies were conducted on investment modelling (Bakar and Rosbi, 2019; Jamei, 2020; Rutkauskas et al., 2008). Most of these studies have been conducted in a short time investment period and only focused on the survey, academic or industrial literature and stock price movement. The existing studies on equity investors have all paid attention to the broad categories of economic and behavioural factors that influence investor decisions (Anderson, 1990; Asquith and Mullins Jr, 1986; Sidra and Attiya, 2013). Some of the existing studies focused on the investors' behaviour. None of these studies focused on using the Weibull distribution in modelling investment behaviour based on financial data. Therefore this research adopts the Weibull distribution in modelling investment behaviour for long-term investment returns. The proposed study sought to examine investment behaviour using Weibull distribution and its extended version. The MPS is one of the most important sectors of the economic indicators used in the computation of the cost of living index in the country (Choon et al., 2011; Thim et al., 2012). Moreover, the MPS has contributed tremendously to economic growth in Malaysia (Nguyen, 2011; Yop, 2021). However, to the knowledge of the researcher, recent research on the MPS is still limited; thus, there are several opportunities to explore investment in MPS. Nevertheless, no evident study has been undertaken on the modelling of the long-term investment return of the shari'ah compliance companies of the MPS based on the assumption of Weibull distribution. The behaviour of the investment return could be used in determining the 'profitability' of investment in that

sector for both short-time and long-term periods. This research is not only observing the investment return of the MPS but also analysing the viability of the investment as a whole. Specifically, the behaviour of investment return will reflect how a better-informed investor could make a sustainable decision making to invest in that sector.

Maximum likelihood estimation (MLE) has been the most widely used method for estimating the parameters of the Weibull distribution. Over the years, the Bayesian estimation approach has been receiving great attention from most researchers such as Calabria et al. (1994), who study the objective of Bayesian analysis for Weibull distribution with application to the random censorship model. Singh et al. (2013) proposed a Bayesian estimation of parameters of inverse Weibull distribution is presented. Classical and Bayesian estimation of Weibull distribution in presence of outliers by (Gupta and Singh, 2017). An engineering approach to Bayes estimation for the Weibull distribution is considered by (Ajmal et al., 2022).

Owing to the rapid growth of the number of claims in most developing countries and the obvious economic and social ramifications lead to large claims data generated by insurance companies with relatively heavy tails (Bolancé et al., 2010; Vatamidou et al., 2014). Several standard probability distributions have been used over the years for modelling real-life datasets. However, research has shown that most of these distributions do not adequately model some of these heavily skewed datasets which creates a problem in statistical theory and applications. Therefore, distributions with high flexibility and accuracy is used (Saxena and Rao, 2015). Furthermore, The capacity to estimate claims frequency is dependent on knowledge and interpretation of loss distribution, which is critical in calculating premiums, expected profits, and reserves when making insurance decisions (Embrechts et al., 2013). Therefore, the modelling of claim data is an important technique for actuaries to observe the

behaviour of claims data for sustainable decision-making. An analysis based on Bayesian approach is quite useful as it allows one to learn about the whole distribution of a chosen quantity, rather than just a single statistic. Additionally, the assignment of probability distributions to unknown quantities allows for the reflection of uncertainty. This uncertainty is represented as the prior distribution, which is then updated with data. Then, once suitable models for the parameters are obtained, one can study the distribution of the variables of interest, which is the end goal of Bayesian analysis. To the best knowledge of the author, the Bayesian approach has not been employed based on gamma prior with particular application to financial data. In this study, the Baysian approach will be employed based on the insurance claims data to simplify the computations, conjugate priors (priors for which the related posterior belongs to the same family of distributions). This also makes obtaining a closed-form expression for the estimator easier. Before the advent of modern processing power, the only approach for employing Bayesian estimates in a mathematically tractable manner was the use of conjugate priors.

On the parameter estimation methods, the simulated annealing algorithm (SA) has been used by scholars as a probabilistic-based technique introduced by Kirkpatrick (1983). Specifically, the SA algorithm is one of the most widely used metaheuristic algorithms employed purposely for global optimization in a large search space for various optimization problems. One of the motivations of our work is the work of Abbasi et al. (2006) in which the SA algorithm was applied in estimating the three-parameter Weibull distribution using simulated data. However, no study extended the work of Abbasi et al. (2006) to cater for real-life financial data. This study bridge the gap by utilising the SA algorithm in estimating the parameters of Weibull distribution based on the financial data (i.e. MIRR data from the MPS). The study on the stock

investment modelling using the Weibull distribution-based financial data from the shari'ah compliance companies of the MPS is a brand-new model. Henceforth, the financial data is expected to fit the Weibull distribution optimally. Hence, the performance of the SA algorithm is projected to outperform the existing parameter estimation methods as applied to the financial data. The expected outcome should be in good agreement with performance evaluation metrics.

1.4 Research Objectives

The main objective of this research is to explore the performance of the SA algorithm in estimating the parameters of Weibull distribution with particular applications to financial data. The objectives of this study can further be divided into the following specific objectives:

1. To present an investment model framework based on the MIRR data of the shari'ah compliance companies of the MPS for long-term investment periods;
2. To propose a new extended version of the Weibull distribution by imposing growth rate parameters on the existing Weibull distribution as applied to financial data;
3. To propose a Bayesian approach to estimate the parameter of the Weibull distribution on the assumption of gamma prior as applied to insurance claims data;
4. To explore the effectiveness of the SA algorithm in estimating the parameters of Weibull distribution based on simulated and real-life financial data sets;
5. To compare the performance of the SA algorithm with five others parameter estimation methods based on the real and simulated data set.

1.5 Purpose of the Study

The main purpose of this study is to explore the performance of the SA algorithm in estimating the parameters of Weibull distribution with particular applications to

financial data. In this thesis, three situations related to investment modelling and parameter estimations will be been explored and evaluated using the Weibull distribution. Firstly, to fit the financial data on the Weibull distribution and eventually use it to examine the investment behaviour of the shari'ah compliance companies of the Malaysia property sector. The analysis involves the estimation of the Weibull distribution parameters with application to financial data. Secondly, to propose a three-parameter Weibull distribution by including a growth rate parameter on the existing Weibull distribution. Finally, to present a Bayesian approach to Weibull distribution with parameters following a gamma distribution with application to insurance claims data. The current study will assist investment decision-makers in making a proper long-term investment decision with low risk and higher investment returns. The proposed method can be used in the computational finance, and financial mathematics communities in providing an alternative method of carrying out computations and to the insurance company in forecasting the behaviour of insurance claim data which enables the insurance fund managers to have a better understanding of claims data. Knowledge of insurance claims distribution can be used in advising insurance companies to consider reinsurance. Understanding how insurance claims data are distributed can also aid in recommending insurance companies to seek reinsurance.

1.6 Significance of the Study

The study put earnestly endeavours to contribute knowledge to the few numbers of literature on investment modelling techniques based on financial data for appropriate decisions among investors in Malaysia. Expected findings may aid in the formulation of investment decisions among investors, which, if implemented, may significantly improve investment decisions. The shari'ah compliance companies of the

MPS used in this case may be able to advice and have knowledge about their clients. The study highlighted the essence of understanding probability and loss distribution that is used by financial analysts and actuarial scientists in general insurance. This will enable the insurance company to make decisions such as estimating premiums, expected profits and reserves.

1.7 Scope of the Study

This study focuses on estimating the parameters of Weibull distribution with particular application to financial data. The extended version of the Weibull distribution; the simulated annealing algorithm (SA) and the Bayesian approach (BA) based on gamma prior. The study has been conducted by using MIRR data of 62 shari'ah compliance companies of the MPS from 2010 to 2018 and simulated and real-life insurance claims data.

1.8 Limitations of the Study

Studies in financial data fitting based on statistical distribution are limited. The related studies on investment modelling-based financial data such as MIRR and insurance claims data are also scarce, which in turn created limitations when referring to existing literature. The research could have considered many companies when extracting information on company financial reports, which is also difficult and time-consuming. The research could have considered insurance claims data beyond two companies but due to time constraints, the study was limited to two insurance claims data. It is difficult to get access to real-life data from insurance companies.

1.9 The Organization of the Thesis

The work is organized into seven main chapters that are chapters one to seven. Chapter One covers the introduction, which is made up of the background of the study include; covers the Weibull distribution and its modifications to improve financial data fitting, a statement of the problem, objectives of the study, limitations of the study, and chapter organization. The work has been organized as an intensive review of investment modelling frameworks and various statistical distributions are given in Chapter two.

This literature review includes the financial models, Weibull distribution, metaheuristics optimization algorithms (MOA) and their modifications that have been proposed in recent years. The first part reviews the internal rate of return modelling, reviews statistical distribution for investment return analysis and continuous modifications of the Weibull distribution. The second part reviews the extended Weibull distribution model, parameter estimation methods have been presented. The third part reviews MOA as parameter estimation methods of various statistical distribution models. The remaining parts review various mathematics and statistical models, actuarial modelling process and Bayesian methodology.

Chapter three covers the research materials and methods which include the Weibull distribution, the MIRR model framework, the methods of parameters estimation which cover; the traditional numerical method, maximum likelihood estimation (MLE), least square method (LSM), and methods of moments (MM) and the concept of MOA covering; the simulated annealing algorithm (SA), differential evolution algorithm (DE), and genetic algorithm (GA).

Chapter four presented the SA algorithm in estimating the parameters of Weibull distribution with particular application to the modified internal rate of return

(MIRR) data. The performance of the traditional numerical method and MOA in estimating the parameters of the Weibull distribution which has been discussed in chapter three has been measured using the goodness-of-fit tests for the SA algorithm in the Weibull distribution. The Means and Variance of transformed MIRR data on Weibull distribution have been calculated.

Chapter five presented the simulation study on the extended Weibull distribution. The two-parameter Weibull distribution has been extended by incorporating the growth rate parameter into the existing model. The mathematical properties of the proposed model have been derived. The SA algorithm has been used in estimating the parameters of the proposed model in this chapter.

Chapter six presents the Weibull distribution on the assumption of gamma prior. The parameters of the proposed Weibull distribution were estimated based on the Bayesian approach with particular application to insurance claims data. A comparison of the Bayesian approach was presented with the SA algorithm with different sample sizes. Chapter seven concludes this thesis by summarizing the main objectives, findings, and suggestions for further study.

CHAPTER 2.
LITERATURE REVIEW

2.1 Introduction

Capital expenditures in various areas and spheres of the economy, including entrepreneurship, are referred to as investments. The investment environment is revived when the economy is stable. This situation enables not only long-term investment returns but also the application of sustainable management strategies in the investment long-term investment process, to improve investment attractiveness and subsequently profitability in business performance. Investment is unquestionably a source of economic growth, prosperity, and stability. Various models have been developed in an attempt to effectively assess investment activity in various sectors of the economy. This study is built based on a mathematical models framework in providing empirical evidence on the theoretical viability of the investment behaviour in shari'ah compliance companies of the Malaysian property sector (MPS). This chapter review some related studies available on investment decision models, modified internal rate of return (MIRR) modelling, the Weibull distribution, extended Weibull distribution, and parameters estimation methods.

2.2 Review on Investment Modelling

Investment decision-making is viewed as a complicated process involving uncertainty, risk and unanticipated investment return. Investment techniques are the subject of several studies. The modern portfolio theory (MPT) developed by Markowitz (1952) is regarded as one of the most widely used methodologies for analysing investment efficiency. The idea demonstrated how investors would optimally set up a company with two investment tools accessible and make investment

decisions. This theory provides a foundation for building and selecting portfolios based on expected investment returns and the investor's level of risk tolerance (Fabozzi, Gupta, & Markowitz, 2002). Over the years, research orientations have emerged; taking on a different perspective focusing predominantly on the fundamental directions of how investment decisions are made and how preferences are shaped to the investment decision.

Investment modelling has shown to be an important task in making essential strategic business decisions across a variety of markets on different ventures. It is widely regarded as a solid prerequisite across all forms of investment valuations and analysis which include debt and equity financing structures (Merikas et al., 2010). In all aspects of investment research, a model is planned and recognized for addressing sometimes difficult potential problems by representing a real-life scenario to allow realistic and rational financial decisions. As a result, financial modelling has become one of the basic tools for any equity investment, as it is essential in investment evaluation (Almeida-filho et al., 2020). Investment decisions are common in the day-to-day business of activities. A study conducted by Zopounidis et al. (2015) pointed out that the use of advanced methods to facilitate investment decision-making has increased in the literature, owing to other factors and the complexities of the financial markets. Despite developments in these methodologies, such decisions have grown increasingly complex over time and continue to face several challenges. The globalized world has seen to fit the amount of data and the number of choices available, as well as the points of view that a decision-maker is believed to be a greater one. Many companies are often faced with financial decisions that usually boil down to making an investment or discontinuing an asset or a project. These decisions can usually make or break a company and are hence significantly important. The decisions

involve estimating appropriate cash flow, investment value, and the discount rate applicable based on the risk and investment return. The discount rate is the required return or the cost of capital for an asset or investment with a specified amount of risk. Estimating the discount rate correctly is most important as small changes in value can change the decision. The financial decision models are the most common models used in financial analysis to make investment decisions. However, because this market is more competitive, a bad judgment might result in financial/economic loss that is often irreversible (Xidonas et al., 2021). Given the complexities associated with financial data and the various targets that financial decisions should cover, a multi-criteria decision-making model should be viewed as an alternative that aids decision-making. The strategic goals of financial decisions include maximizing profitability, liquidity, financial appreciation, and social benefit while minimizing risk; costs and environmental impact from investments among other things. It should also be noted that in certain cases, the goals may be conflicting (Xidonas et al., 2021). The typical case of target conflict is the study of the trade-off between maximizing the return on an investment portfolio and minimizing the risk involved with it, which is often calculated in terms of variance of investment return. This conflict is addressed in MPT by Markowitz (1952) which has been thoroughly researched in the literature, and this involves a large range of variants and developments.

The relevance of this study prompted literature reviews related to investment decisions in a comprehensive way. Several studies have been conducted by different researchers in an attempt to model investment and examine the potential of investment in a particular project. A study was carryout in address the problem associated with the underlying uncertainty in investment by Sherif (2001). The model has been developed based on interval mathematics and possibility distribution. The finding of

the study revealed that the model could be used to measure and rate investment behaviour. Chan (2002) adopted the modelling approach proposed by Tiao et al. (1981) based on the multiple time series, in constructing a stochastic investment model for price movement, inflation rate, share dividends, and long-term interest rates. The method has the benefit of being straightforward and transparent. In univariate time-series analysis, the sequential and iterative phases of preliminary representation, estimate, and diagnostic testing are similar to those of the well-known Box-Jenkins process. In contrast to some other stochastic asset models in the literature, no prior correlation analysis is required. Rutkauskas et al. (2008) proposed a modelling approach to investment decisions based on the sustainable development concept associated with the financial market. The study included a FOREX experiment as well as some mature and emerging capital markets. The study presented the suitability of the model for investment decisions and reliability evaluation of the portfolio. Analysis has been made as a primary tool for establishing a plan for long-term investment returns. The study provided examples of putting appropriate investment strategies based on a creative and efficient financial tool for investors and the stock treasury. The model performance was analyzed based on real data from financial markets. A model of investment decisions making in stock market investment based on a hybrid discrete option system was developed by (Robin, 2012). The study aimed at modelling the decision to purchase or sell stocks using a binary logic model with latent classes, which characterizes risk perception. The model was developed to address the problem of complexity underlying the judgment mechanism in computation using data from a Swiss bank, comprising 25989 transactional. The model's parameters are interpretable and measure interesting behavioural processes associated with investment decisions. A hybrid algorithm was developed based on the autoregressive moving average