MULTI-OBJECTIVE HYBRID ELECTION ALGORITHM FOR RANDOM *k* SATISFIABILITY IN DISCRETE HOPFIELD NEURAL NETWORK

SYED ANAYET KARIM

UNIVERSITI SAINS MALAYSIA

2023

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by

SYED ANAYET KARIM

Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

August 2023

ACKNOWLEDGEMENT

All mercy to the Almighty Allah who enlightened my intellect and enabled me to perform my thesis. I feel a great pleasure to express my recondite sense of gratitude to my esteemed and research supervisor Dr. Mohd. Shareduwan Mohd. Kasihmuddin, Senior Lecturer of Mathematics, Universiti Sains Malaysia for accosting me as his research student. I am truly obliged for his mentorship, providing constructive feedback, and always steering me in the right directions. I also pay honour and respect to my co-supervisors Professor Dr. Ahmad Izani Md Ismail and Associate Professor Dr. Saratha Sathasivam for their kind support. I would like to convey my best regards to AIRDG team members for their kind assistance during my research journey. I also acknowledge my gratefulness to the officers and staffs of the School of Mathematics, Institute of Post Graduate studies and Library management of USM for their kind positive feedback and guidelines.

I would like to express my sincere gratefulness and appreciation to Bangabandhu Sheikh Mujibur Rahman Science and Technology fellowship trust, Ministry of Science and Technology, Bangladesh for providing me with huge financial assistance during my PhD journey. This scholarship has allowed me to focus on my research and bring this thesis to fruition. I want to express my deep appreciation to all my respected teachers, colleagues, friends and loved ones who have supported me in various ways. I would like to extend my heartfelt thanks to Dr. Md. Nure Alam Abdullah, Professor of Physics, Jagannath University Bangladesh, Dr. Md Golam Kabir, Professor of Biochemistry, University of Chittagong, Bangladesh, Dr. ASM Moinuddin, Professor of Mathematics, University of Chittagong, Mr. Md Abu Yousuf, Mr. Mamun-or-Rashid, and Mr. Mosleh Arif for their vital inspiration to do my PhD in Malaysia. Also, my gratitude to Dr. Mohd Sohrab Hossain, Senior lecturer of UTP, Dr. Sayed Mohibul Hossain, Dr. Md Jamal Hossain, Mr. Shah Mahmud Yasin, Mr. Raihan Goni and Md Rabiul Amin for their kind guidance and help during my PhD journey.

I express my earnest gratitude to my parents and parents-in-law for their unconditional supports and blessings. Their trust in me and their willingness to provide assistance whenever needed has been instrumental in the completion of this thesis. Special thanks to my sister Dr. Syeda Nusrat Jahan, brother-in-law Dr. Md Joynal Abedin Muhuri and sister-in-law Jannatun Nur Piwly for their co-operation during my study period.

I gratefully acknowledge the contribution of my wife Engineer Jannatul Mouya Rima and my son Syed Azmain Iqtidar for their kind support which make me always strong and motivated. Their belief in my abilities, patience during moments of stress, and unwavering encouragement have been invaluable. I am deeply grateful for their love, understanding, and untiring presence throughout this journey. Their sacrifice reduces my family challenges and give me a huge mental support.

Finally, I am aware that no words can adequately express my gratitude to my wife, supervisor, family members and loved ones for their support throughout this thesis. Their presence and belief in me have made this accomplishment possible and for that, I am forever GRATEFUL.

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LIST OF SYMBOLS

$P_{RAN3SAT}^{k}$	Random *k* Satisfiability
$P_{RAN3SAT}^{1,2,3}$	Random *k* Satisfiability for *k*= 1,2,3
$P_{RANkSAT}^{2,3}$	Random *k* Satisfiability for *k*= 2,3
$P_{RANkSAT}^{1,3}$	Random *k* Satisfiability for *k*= 1,3
$P_{RANkSAT}^{HEA}$	Hybrid Election Algorithm for Random *k* Satisfiability
$P_{RANkSAT}^{EA}$	Election Algorithm for Random *k* Satisfiability
$P_{RANkSAT}^{ES}$	Exhaustive Search Algorithm for Random *k* Satisfiability
$P_{RANkSAT}^{GA}$	Genetic Algorithm for Random *k* Satisfiability
$P_{RANkSAT}^{ABC}$	Artificial Bee Colony Algorithm for Random *k* Satisfiability
P_{3SAT}	3 SAT clauses
P_{2SAT}	2 SAT clauses
$\eta_{\max}$	Maximum fitness of neuron states achieved in DHNN-RANkSAT
η_i	Current fitness of neuron states achieved in DHNN-RANkSAT
φ	Number of Learning
$J_i^{(k)}$	Clause combination for 3,2,1 literal.
f_{L_j}	Best fitness
S_i	State of the *i*-th neuron
W_{abc}	Synaptic weight from the unit *a* to *c*
σ	Advertisement rate
μ	Full Satisfied Strings
ρ	Ideal Solution Strings
h_i	Local field
CorD	Correlation Distance function
$\wedge$	Conjunction (AND)

$\vee$	Disjunction (OR)
$H_{P^k_{RAN3SAT}}$	Energy function Random 3 Satisfiability
$H^{min}_{P^k_{RAN3SAT}}$	Minimum Energy function for Random 3 Satisfiability
$f(F_{\max}, \gamma, S^{(i)}_{\max})$	Multi-objective functions that contain maximum fitness value with diversity ratio and ideal strings.
T_F	Tolerance value for fitness function
T_D	Tolerance value for diversity analysis
ν_A	Achieved a number of states for diversity
ν_T	The target number of states for diversity
β_C	Numerical calculation of storage capacity
ξ	Diversity of the logical rule in percentage
$G_{P^k_{RAN3SAT}}$	Number of Global Solution
$L_{P^k_{RAN3SAT}}$	Number of Local Solution

LIST OF ABBREVIATIONS

ANN	Artificial Neural Network
HNN	Hopfield Neural Network
DHNN	Discrete Hopfield Neural Network
SAT	Boolean Satisfiability
3SAT	3 Satisfiability
2SAT	2 Satisfiability
RAN*k*SAT	Random *k* Satisfiability
DHNNRAN*k*SAT-HEA	Random *k* Satisfiability with Hybrid Election Algorithm in Discrete Hopfield Neural Network
DHNNRAN*k*SAT-EA	Random *k* Satisfiability with Election Algorithm in Discrete Hopfield Neural Network
DHNNRAN*k*SAT-ES	Random *k* Satisfiability with Exhaustive search in Discrete Hopfield Neural Network
DHNNRAN*k*SAT-GA	Random *k* Satisfiability with Genetic Algorithm in Discrete Hopfield Neural Network
DHNNRAN*k*SAT-ABC	Random *k* Satisfiability with Artificial Bee Colony algorithm in Discrete Hopfield Neural Network
RAN2SAT	Random 2 Satisfiability
RAN3SAT	Random 3 Satisfiability
HTAF	Hyperbolic Tangent Activation Function
RGS	Ratio of Global Solutions
ES	Exhaustive Search

LIST OF APPENDICES

ALGORITMA PEMILIHAN HIBRID OBJEKTIF UNTUK k SATISFIABILITI RAWAK DALAM RANGKAIAN DISKRIT NEURAL HOPFIELD

ABSTRAK

Dalam pembangunan penyelidikan Rangkaian Neural Buatan semasa, struktur logik simbolik memainkan peranan penting untuk menerangkan konsep kecerdasan. Rangkaian Neural Diskrit Hopfield yang sedia ada dengan struktur logik Satisfiabiliti yang sistematik gagal menghasilkan keadaan neuron akhir yang tidak berulang dan cenderung kepada penyelesaian minima tempatan. Dalam hal ini, tesis ini mencadangkan Satisfiabiliti k Rawak aras lebih tinggi untuk $k \leq 3$, yang mana k mempunyai maksimum tiga jenis gabungan logik (*k=1,3; k=2,3; k=1,2,3*) untuk melaporkan tingkah laku struktur logik peringkat tinggi. Untuk menganalisis gabungan logik Satisfiabiliti k Rawak, tesis ini akan menjalankan eksperimen dengan beberapa prestasi metrik. Analisis menunjukkan bahawa gabungan *k=2,3* bagi Satisfiabiliti k Rawak mempunyai tafsiran yang lebih konsisten dan penyelesaian global berbanding kombinasi lain. Selain itu, prestasi optimum logik Satisfiabiliti k Rawak boleh dicapai dengan menggunakan algoritma dalam fasa latihan Rangkaian Neural Hopfield Diskrit. Salah satu focus utama dalam pembinaan algoritma adalah untuk mencapai keseimbangan yang betul dalam strategi penerokaan dan eksploitasi. Dalam hal ini, tesis ini mencadangkan algoritma hibrid yang dinamakan Algoritma Pilihan Raya Hibrid yang boleh mengekalkan strategi penerokaan-eksploitasi dengan baik. Algoritma Pilihan Raya Hibrid yang dicadangkan ini memastikan nilai kecergasan dalam boleh dicapai optimum dalam satu lelaran. Dalam hal ini, kepentingan teras tesis ini adalah untuk memperkenalkan Algoritma Pilihan Raya

Hibrid dengan pelbagai objektif baharu yang boleh meningkatkan kapasiti simpanan Rangkaian Neural Hopfield Diskrit bersama-sama struktur logik yang pelbagai seperti perwakilan logik Satisfiabiliti k Rawak (untuk $k \leq 3$). Oleh itu, rangkaian hibrid yang dicadangkan telah dibandingkan dengan algoritma metaheuristik terkini dengan pelbagai prestasi metrik. Berdasarkan keputusan eksperimen dan statistik, model Algoritma Pilihan Raya Hibrid yang dicadangkan mengatasi semua model yang sedia ada. Tambahan, model yang dicadangkan telah melaksanakan teknik perlombongan logik untuk mengekstrak set data dunia sebenar. Model perlombongan logik yang dicadangkan menangani beberapa kelemahan utama dalam model perlombongan logik sedia ada seperti struktur logik yang tidak fleksibel, pemilihan atribut optimum dan pengiraan logik terbaik. Dalam model perlombongan logik yang diubah suai yang dicadangkan ini, fasa pra-pemprosesan juga telah disertakan untuk memilih atribut optimum set data. Prestasi model perlombongan logik yang dicadangkan akan dibandingkan dengan beberapa model perlombongan logik sedia ada dalam memproses pelbagai set data penanda aras daripada pelbagai bidang ilmu. Untuk memastikan kecekapan model yang dicadangkan boleh mentafsir tren set data kehidupan sebenar, kajian kes berkenaan dengan fobia matematik pelajar peringkat universiti Bangladesh telah digunakan untuk mengesahkan prestasi model perlombongan logik yang dicadangkan. Berdasarkan kajian kes, model perlombongan logik yang dicadangkan berjaya menerangkan faktor utama yang membawa pelajar ke arah fobia matematik.

MULTI-OBJECTIVE HYBRID ELECTION ALGORITHM FOR RANDOM k SATISFIABILITY IN DISCRETE HOPFIELD NEURAL NETWORK

ABSTRACT

In the current Artificial Neural Network research development, symbolic logical structure plays a vital role for describing the concept of intelligence. The existing Discrete Hopfield Neural Network with systematic Satisfiability logical structures failed to produce non-repetitive final neuron states which tends to local minima solutions. In this regard, this thesis proposed non-systematic Random k Satisfiability logic for $k \leq 3$, where k generates maximum three types of logical combinations (*k=1,3; k=2,3; k=1,2,3*) to report the behaviours of higher-order multiple logical structures. To analyse the logical combinations of Random k Satisfiability, this thesis will conduct experimentations with several performance metrics. The analysis revealed that the *k=2,3* combination of Random k Satisfiability has more consistent interpretation and global solutions compared to the other combinations. Moreover, the optimal performance of Random k Satisfiability logic can be achieved by applying an efficient algorithm during the training phase of Discrete Hopfield Neural Network. One of the major features of an efficient algorithm is to make a proper balance in the exploration and exploitation strategy. In this regard, this thesis proposed a hybridized algorithm named Hybrid Election Algorithm that can well maintain the exploration-exploitation strategy. This proposed Hybrid Election Algorithm ensures optimal fitness value of the neuron string within a single iteration. To improve the quality of the neuron strings during the training phase, this thesis will introduce a new multi-objective Hybrid Election Algorithm that can increase the

storage capacity of Discrete Hopfield Neural Network along with a diversified logical structure embedded with Random k Satisfiability (for $k \leq 3$) logical representation. Hence, the proposed hybrid network is compared with other state of the art metaheuristics algorithms through various performance metrics. Based on the experimental and statistical results, the proposed Hybrid Election Algorithm model outperformed all the existing models in the literature. Furthermore, the proposed model has implemented logic mining technique to extract real-world data sets. The proposed logic mining model is addressed some major limitations in the existing logic mining models such as non-flexible logical structure, optimal attribute selection, and computation of the best logics. Considering these views, this thesis proposed a newly Hybrid Election Algorithm Random k Satisfiability based Reverse Analysis model in the field of logic mining. In the proposed logic mining model, a pre-processing phase is included to select the optimal attributes of the data sets. The performance of the proposed logic mining model will be compared with several existing logic mining models in doing numerous benchmark data sets from various fields. To ensure the effectiveness of the proposed logic mining model that can interpret the behaviour of real-life data set, a case study regarding maths phobia of the university level students of Bangladesh has been conducted. Based on the case study, the proposed logic mining model explains the key factors which lead the students towards the maths phobia.

CHAPTER 1

INTRODUCTION

The development of Artificial Intelligence (AI) through Artificial Neural Network (ANN) is one of the foremost scientific and industrial revolution breakthroughs. It is gaining popularity as a method for addressing a wide variety of significant challenges. This chapter will address the role of Artificial Intelligence and Artificial Neural Network in the context of this thesis. This discussion continues with problem statements, research objectives, research motivations, and the significance of research in terms of its objectives, scope, and thesis creation. A graphic synopsis of this research method study is included in order to provide a clear picture of this thesis subject matter.

1.1 A Primer of Artificial Intelligence and Artificial Neural Network

In the 19^{th} century, the fundamental research movement get one step forward with the concept of 'Building technology through Thinking'. This concept arose since learning, making judgements, and forming decisions were kept challenges for scientific development. To tackle these challenges in the same stride, human intelligence is sometimes legged off due to its limited range. To overcome the lack of human intelligence, researchers have developed a mechanical concept known as AI, which artificially mimics human intelligence. Therefore, the concept of AI keeps advancing by replacing human intelligence. The main goal of AI is to know intelligent entities, and how intelligence works that reproduce intelligent human behaviour. This intelligent behaviour can be understood and explained through a computational process. Moreover, this intelligent computerisation system produced outstanding results in speech recognition, image processing, natural language processing, and intelligent robotics

(Zhang and Lu, 2021). Also, one of the most popular areas of AI is neural symbolism, which combines the benefits of using symbols to present information with a clearer picture of how intelligence works.

McCulloch and Pitts (1943) developed a historical transition to show how the human nervous system can be used to explain ‘intelligence’ through neural symbolism. The authors artificially imitate brain synapses rather than simulating the human mind which is later named ANN. One of the solid features of ANN is mimic ability on how the brain works such as it can build a model or system that can execute a collective decision system. As a result, AI practitioners highly rely on the feature of ANN which results in a lack of in-depth understanding of the operations conducted by ANN models. A large number of interconnected neurons make up this ANN idea, and each one of them executes a mathematical action which can illustrate the intelligence manner more clearly. Since ANN is a highly non-linear dynamical system, implementing an appropriate logic will yield the best result. From this paradigm, mathematicians discovered a wealth of opportunities for having a research direction. "Neo-connectionism" raises a close relationship between computer scientists and mathematicians in the study of ANN, which has the potential to circumvent the shortcomings of AI in a different way.

In the year of 1949, a distinguished psychologist named Donald Hebb expressed his idea of how neurons communicated and stored knowledge in the brain structure (Hebb, 1949). The Hebbian Learning Rule was born out of this theory, which led to the invention of learning rules for the McCulloch-Pitts brain models and ANN. Starting from that time to till now, this ANN is open research platform such as a data processing model that can acquire, retain, and use experimental knowledge in an attempt to mimic the intelligent brain that is human brains capabilities. To achieve great success in a wide range of endeavours, researchers are now using appropriate ANN application mechanisms. Figure 1.1 shows a general connection of AI, ANN and Mathematics where these can be correlated each other that make the bridge to depict the concept of intelligence more clearly.

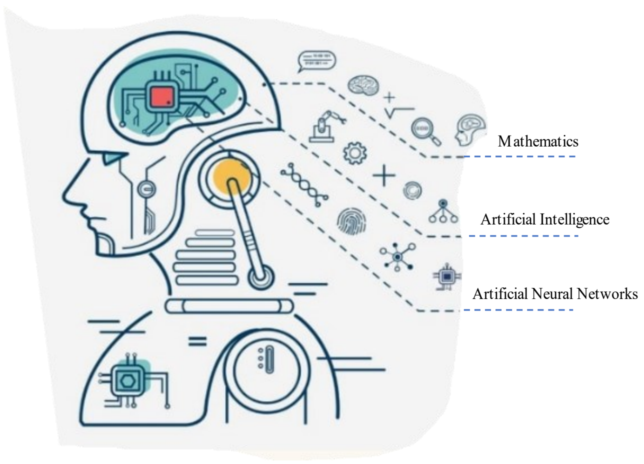

Figure 1.1 The connectionist of ANN, AI and Mathematics.

In the 21st century, ANN keeps its own footstep in every discipline of human civilization. By deploying several mechanisms in ANN, it is successfully embedded in wastewater treatment (Jawad et al., 2021), bio-diesel-based fuel energy (Tuan Hoang et al., 2021), medical science (Bukhari et al., 2021), food process modelling (Bhagya Raj & Dash, 2020), tourism industry (Talwar et al., 2022) and many more sectors. Moreover, ANN is playing a vital role in the study of data mining also (Baashar et al., 2022). From the vast applications of ANN, the importance and acceptance from the other ANN researchers can easily be understood. More importantly, it is very much necessary to focus on the structure and foundational elements of ANN since the dynamic behaviour of ANN strongly depends on its fundamental structure.

Meanwhile, ANN is generally classified into two major parts according to its connectivity and structure. The first one is feed-forward networks such as radial basis function neural networks and the second one is feedback networks such as the Hopfield Neural Network. Since this thesis is about modelling structures of ANN that can solve optimization or constraint satisfaction problems, Hopfield Neural Network (feedback network) is more useful from this point of view. Noteworthy, Hopfield Neural Network considers a simple summation function and threshold activation function to navigate the units of the network so that every neuron in the network is connected to each other except itself. The inclusion and exclusion of layers with the synaptic connection of the neurons and architectural design in the Hopfield network can be impactful for solving several types of discrete and continuous optimization problems. In this concern, this thesis will focus on this Hopfield Neural Network that has simple architecture by recurrent networks with symmetric synaptic connections.

1.2 Hopfield Neural Network

In 1982, Dr John Hopfield rebuilt ANN by incorporating a dynamic input in the energy analysis of the feedback neural network system, which is called the Hopfield Neural Network (HNN) (Hopfield J.,1982). In this work, the author explained that AI cannot purely mimic the human brain and also introduced a modified structure of ANN that can solve different optimization problems. Structurally, the sufficient condition for the stability of HNN is that the weighted coefficient matrix is symmetric and has zero diagonal elements. By taking advantage of the coefficient matrix of HNN, the behaviour of solution memory mimics actual human intelligence. Noteworthy, HNN is considered the most studied attractor-memory model due to the feature of associative memory system which is known as Content Addressable Memory (CAM), that is very beneficial for optimizing a model (Sani and Shermeh, 2022). Mainly, the synaptic weights of the neurons are stored in this system block CAM. In this CAM, the unit of memories is stored by its content instead of its address locations. Moreover, in a feedback network with symmetric weight, this HNN demonstrated stable equilibrium states that update asynchronously. One of the characteristics of the Hopfield network is that there exists a relatively straightforward method for setting up the connections between nodes, in such a way that any desired set of patterns may be transformed into ‘stable firing patterns’.

Moreover, HNN can explain a 2D complex visual system as well as the minimized energy. This network has dynamic behavior in which each state has its own attraction basin. Furthermore, the strengths of this HNN are that the neurons are highly interconnected since it has no hidden layer, which allows it to evolve freely and lowers the energy of the network. Although there are no self-loops in neurons, the neuron arrays are fully linked to one another. The ability of neurons in HNN has made a significant contribution to solving numerous optimization issues such as the travelling salesman problem (TSP), the linear programming problem, and computation in model neural circuits (Yadav et al., 2022).

Though HNN is very effective, it has a major drawback with its storage capacity. With N nodes of HNN, the number of stored patterns is limited to $0.14N$. In this regard, HNN needs to compromise with the low quality of CAM which retrieved the sub-optimal final neuron states. Although Zhao et al., (2022) attempted to increase the storage capacity of HNN but it does not create major development in getting more storage capacity. From this circumstance, it is immediately necessary to address this drawback since the neuron interpretation is based on the initial neuron state that converges to local or global minimum solutions. The general neuron node architecture of HNN is shown in Figure 1.2. The signals of neurons in a closed feedback loop back and forth arrangement can be seen where $x_{1,...}, x_n$ are input values with synaptic weights (W_{ij}) through N_n nodes and $y_1,...y_n$ are output values.

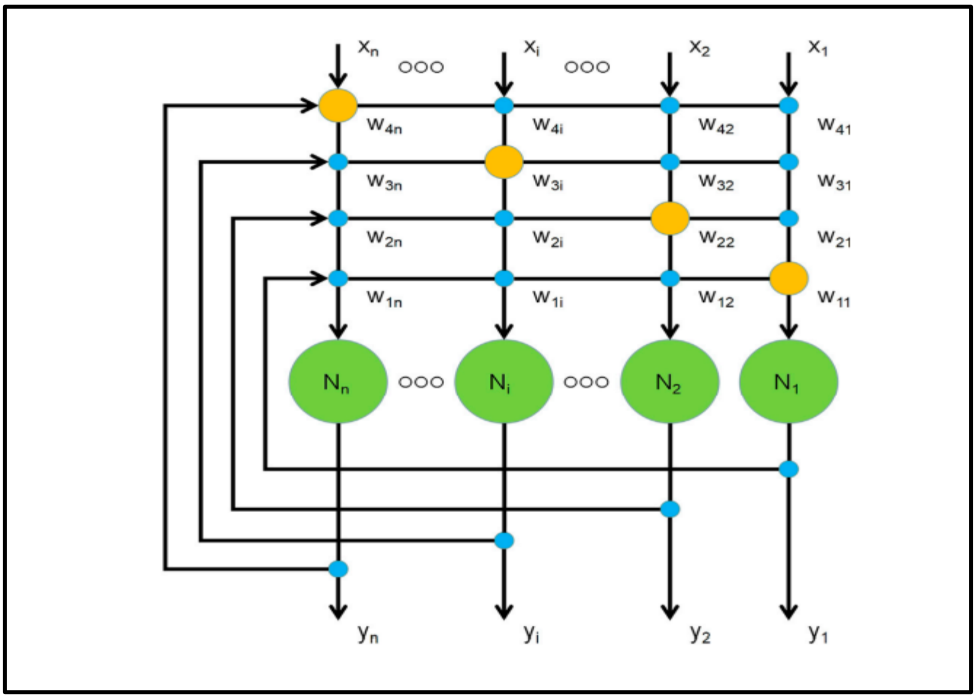

Figure 1.2 Neuron node Structure of Hopfield Neural Network (Yu et al., 2022)

Another weakness of traditional HNN is lower neuron interpretability, incorporating the symbolic logic can be a better idea which may be easily embedded in Discrete HNN as a single intelligence. The input of symbolic logic in Discrete HNN can be capitalized as the effective logical rule being trained and retrieved with the aim to generate the solutions with global minimum energy.

1.3 Symbolic Logical Representation in Discrete Hopfield Neural Network

It is known to all that the DHNN mechanism is built in a black box model for which the internal operation managements are unknown. This creates another fact that the convergence of neuron state from the initial state to the final state is unknown. In this regard, it creates uncertainty for the researchers whether they optimize their problem in proper way or have no idea how to optimize a problem. This uncertainty creates a new interest for the researchers to introduce symbolic logic in DHNN. This can be clearer for the researchers to trace the neuron arrangement compared to the traditional black box model.

The crucial step in revising and correcting symbolic knowledge using neural networks is to create the three-link chain shown in Figure 1.3. In order to enable a rule-refinement system, the method uses neural networks as the empirical learning algorithm. Symbolic knowledge, which can take any logical form, is the first link in the three-link chain. By switching the representation of the rules from symbolic to neurally based, this step improved the conventional neural training techniques by using the rules. Then utilizing any heuristic or metaheuristic form of training method, the DHNN is trained in the second step of the chain. The last step is to take trained DHNN and extract rules from it.

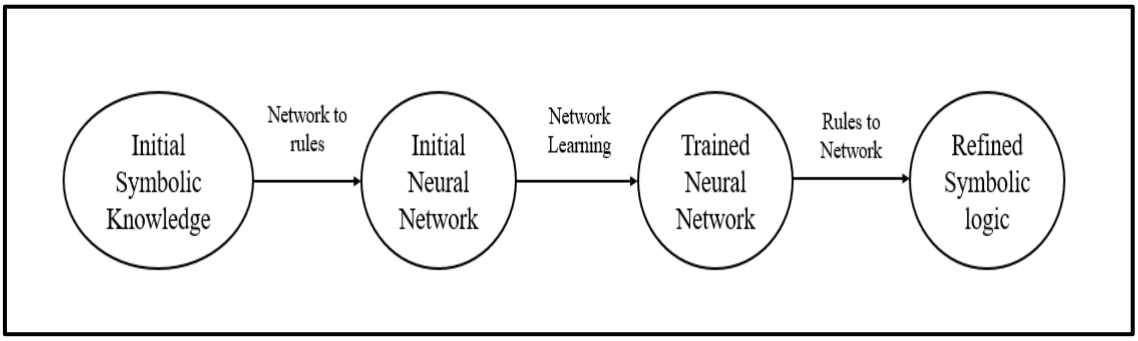

Figure 1.3 Symbolic representation of DHNN.

Meanwhile, Wan Abdullah, (1992) attempted to introduce a fundamental symbolic representation by implementing logic satisfiability in DHNN by computing the synaptic weights. The key intention of this work was to check the logical inference through the minimization of logical inconsistencies on a symmetric neural network.

Hence, the proposed method by Wan Abdullah is called the core method where it represents the logic in DHNN through symbolic forms. Later, Kasihmuddin et al., (2018), Mansor et al., (2017a), and Sathasivam et al., (2020a) introduced systematic and non-systematic logic, which created more ways to understand the importance of studying symbolic logic in DHNN. Notably, these research studies provide a new insight into the idea of intelligence through symbolic logic. Another important point is that the logic properties are always fixed. For this prospect, the connection of the neuron will be more transparent compared to the traditional DHNN model. This perspective is completely different from other solution-based paradigm where ANN was utilized as a tool to find the interpretation that maximizes the number of clauses. Noteworthy, the DHNN model will be better optimized if the appropriate logic applied in this DHNN model.

In other words, optimal logic development in DHNN is still worth exploring. By including the logic, DHNN may transcend to multiple applications, including data mining and classification. The previous discussions have made it evident that symbolic logical rules can be used to describe DHNN more effectively, and it make one step closer to understand an alternate form of intelligence. In this thesis, the optimization of the classic DHNN model is based on multiple perspectives about the implementation of higher-order symbolic logical rules, the optimization of the training phase by incorporating multi-objective metaheuristics, and logic mining. Concerning such prospects, this thesis will also highlight the flaw regarding the storage limitation of DHNN and develop a specific framework for solving and exploring the modified DHNN. As a result, the research objectives that follow the problem statements of the proposed research study are addressed in detail in the next following discussions.

1.4 Problem Statements

The discussions of the previous sections highlight several research gaps that this thesis considers as problem statements. The aim of the problems statements is to address the current drawback of the symbolic logic and the training mechanism of DHNN. The key points of these problem statements of this thesis are given below:

(a) Existing satisfiability is made up of a series of clauses having a systematic logical structure and specified literals. The nonflexible logical framework of traditional systematic satisfiability logic leads to a lack of interpretation. Meanwhile, systematic logic only focus on single combination and synaptic weights are always keep equal magnitude. For this reason, it may produce repetition of the final neuron states which promotes to make overfitting global minima solutions. On the other hand, the advantage of incorporating non-systematic satisfiability logic is crucial to improve the flexibility of the logical structure in order to prevent the overfitting solutions. The non-systematic logic can utilize more different combinations which can create different final neuron states and that lead to global minimum energy. Sathasivam et al., (2020a) first time introduced a non-systematic logical structure named Random k Satisfiability for $k \leq 2$. Though this Satisfiability structure is more flexible in terms of systematic logical combinations but this current study only focused on first and second order logical structure and there was no performance analysis between the Random 2 Satisfiability and systematic Satisfiability. Moreover, Random 2 Satisfiability is composed of first order clauses can create more logical inconsistencies so that the Random 2 Satisfiability structure caters low dimensional decision system, as well as created lower probability to minimize cost function and comparatively higher Lyapunov energy function which resulting suboptimal testing phase. Notably, introducing higher order logic (such as third order) can create different variants of non-systematic logic which generate higher probability to minimize cost function that leads to achieve optimal testing phase. Moreover, a higher-order non-systematic logic also can provide more logical combinations, and DHNN may effectively 'behave' according to the non-systematic logical rules.

(b) The efficiency of a metaheuristic algorithm depends on how well its operators maintain the exploration and exploitation strategy. Researchers generally focus on such type of metaheuristics which are able to make effective partitioning of solution spaces that reduces the time complexity and create simple independent computations. In this regard, a research work by Emami, (2015) introduced a swarm-evolutionary based algorithm named Election Algorithm (EA) which have several influential operators that can tackle the critical numerical optimization problems. However, this algorithm has few major limitations: it stacked at local optima or failed to generate global minimum value for higher number of simulations due to the inability of the operators in searching solution space and this work utilized Euclidean distance function for calculating fitness value of the population that decrease the speed of the algorithm. Additionally, this work only directed for solving continuous optimization problems. Later, Sathasivam et al., (2020b) also utilized this population-based algorithm to check the capacity of this algorithm with non-systematic logic in DHNN. Though this experimentation provided a new direction to do discrete constraint satisfaction problems through non-systematic logic but this algorithm was only implemented as the learning algorithm, without direct intervention in the testing phase. Furthermore, it did not provide any unique way or mechanism in the structure of this algorithm that can adopt logical diversity and highest fitness in the same stride. More importantly, the current structure of Election Algorithm cannot work well with the exploration and exploitation strategy because the local search operator (Positive advertisement operator) does not enough efficient to improve the quality of local solutions (Emami, 2019). Hence, it is necessary to implement additional mechanism in the current structure of EA with higher order non-systematic logic such that it can ensure to improve the local solution and maintain exploration-exploitation strategy that can address the flaws of the mentioned studies.

(c) One of the major challenges of DHNN mechanism is the capability of the neurons to converge towards optimal solutions. Then the selection of training method is very important to ensure the optimal synaptic weight. Recently, Guo et al., (2022) incorporated Exhaustive search algorithm in the training phase of DHNN with a special type of logic to obtain correct synaptic weight that led to highest fitness value. Unfortunately, this work implemented Exhaustive Search algorithm in the training phase of DHNN which has no capability to improve the fitness neurons in every iteration and when number of neurons increases the computational time is higher which created a major computational complexity. Moreover, as the restricted learning was applied, the satisfiability of the logic embedded into DHNN may not be properly verified. It needs to address that poor training mechanisms generate suboptimal synaptic weights and suboptimal final neuron state which prevent to get maximum fitness. Notably, lower fitness value does not create any significant impact in solving optimization problems. Meanwhile, another study by Sathasivam et al., (2020d) introduced different metaheuristics such as Genetic Algorithm, Artificial Bee Colony algorithm in the training phase of DHNN with systematic logic to check the efficiency of each metaheuristic algorithm and analyse which one can get the highest fitness value through restricted learning environment. Nevertheless, this work failed to get the correct interpretation that make the logic fully satisfiable and also no guarantee to get highest fitness value as the number of neurons increases. Therefore, it is necessary to improve current training phase of DHNN by including an efficient algorithm that can guarantee the highest fitness value for higher number of neurons within a few iterations.

(d) To analyse the efficiency of a metaheuristic algorithm, several researchers employed different strategy to achieve their individual desired goals. In the point of symbolic logical rule, ANN researchers have common trend to neglect the effect of positive and negative literals in the clause arrangements since the only target is to achieve fully satisfiable logic for which diversity of the symbolic logic failed to explore. One of the latest works by Kho et al., (2022) introduced a metaheuristic named ant colony optimization algorithm in the training phase of DHNN. This experimentation result revealed that there is only one objective of this work was to converge global solutions. Regrettably, there was no effort to introduce diversity in the logical rule which can improve the structural issue of the existing logical rule that effects to encompass the bigger picture of intelligence in DHNN. Another major flaw of the current structure of DHNN is that it has limited storage capacity. Though several researchers attempted to increase the storage capacity of DHNN but the mechanism to increase the storage capacity is worth investigating. It needs to mention that there is no research work that accommodate multiple constraints such as achieving maximum fitness as well as keeping logical diversity and a dynamic mechanism to improve the storage capacity of DHNN. Considering above constraints, this thesis will propose a multi-objective concept with effective metaheuristic algorithm in the training phase of DHNN that can ensure to achieve highest fitness value with the variation of logic and increase the storage capacity of DHNN.

(e) Logic mining is a variant of data extraction process by leveraging the Boolean logic and DHNN. It is important to point out that how a model can be applied in the logic mining study to illustrate a clear view of a data set. Current studies of the logic mining analysis focus on obtaining optimal induced logic. The research work by Kho et al., (2020) made a fruitful improvement to do analysis the behaviour of the data sets through logic mining. However, this study has some major drawbacks. Firstly, this author created the logic mining model by disregarding the process for attribute selection. Secondly, there are no filtering mechanism before the data converted to the logical rule. In this regard, there is a strong possibility to select less important attribute which effect in interpreting the behaviour of the data sets and generate suboptimal induced logic. Meanwhile, Alway et al., (2020) introduced another dimension in the study of logic mining in terms of systematic satisfiability representation to ensure optimal induced logic. Unfortunately, this work also has some limitations. Firstly, this study did not make any step in the logic mining process that can increase the search space and there were no efforts for having flexible logical structure. Secondly, a limited performance metrics were involved to justify their model performances. In these consequences, the model cannot explore the search space and create poor justification which did not provide effective value to deliver the behaviour of the data sets for real life problems analysis. By considering the above-mentioned drawbacks, it needs to employ a new impactful logic mining model which is able to overcome all the mentioned problems.

1.5 Research Questions

According to the problem statements there are several questions arise. These questions need to get research elaborately. Based on the comprehensive discussion of previous sections, the research questions of this study are mentioned as follows:

(a) What are the potential formulations and structures for higher order non-systematic logical combinations that are highly comparable to the current systematic and non-systematic logics which represent high dimensional decision system in Discrete Hopfield Neural Network?

(b) In the context of dynamic metaheuristics, how does the architecture of existing metaheuristic algorithm can be formed that can optimize the local and global search operators in the training phase of Discrete Hopfield Neural Network?

(c) How can the existing training mechanism of Discrete Hopfield Neural Network be more efficient that ensure to achieve only a single objective in the context of higher-order logical rule?

(d) In the framework of non-systematic logic, how multiple objective functions can be incorporated in a metaheuristic that can achieve maximum fitness, create diversity for the logical rule and increase the storage capacity of Discrete Hopfield Neural Network?

(e) In order to interpret the appropriate behaviour of a data set through logic mining, which crucial improvement should be implemented in the current logic mining models to find best induced logic for extracting the behaviour of the benchmark and Case study data sets?

1.6 Research Objectives

This thesis is focused on utilizing higher-order non-systematic logical rule. The proposed model will be incorporated with swam-evolutionary inspired methods that has the ability to include multi-objective nature. In this context, the proposed model must be capable in generating less computational effort and make the simulation within fewer iterations. Moreover, this thesis will form a new logic mining model that can extract the pattern from the data sets and case study. From the research questions, the objectives of this study are portrayed as follows:

(a) To formulate different new combinations of higher-order non-systematic logic named Random *k*- Satisfiability (for $k \leq 3$) that can create three-dimensional decision system where each clause can contain with a combination of 3 literals or 2 literals or 1 literal.

(b) To propose a new operator named Caretaker Party in the advertisement campaign of Election algorithm which is able to expand the solution search space as well as improve the quality of the local solutions which is later introduced as Hybrid Election Algorithm.

(c) To introduce a metaheuristic algorithm for higher order non-systematic logic that can ensure highest fitness value with less computational effort. In this context, the proposed Hybrid Election Algorithm will be used to optimise the training phase of the Discrete Hopfield Neural Network by assuring that the proposed Hybrid Election Algorithm can achieve highest fitness value with the fewest number of iterations.

(d) To formulate the multi-objective function associated with the higher order non-systematic Random k Satisfiability logic that consist of maximum fitness, promote variety in the logical rule and develops the storage capacity through Hybrid Election Algorithm in Discrete Hopfield Neural Network.

(e) To propose a new logic mining model which introduces log-linear technique with permutation operator in the pre-processing phase along with Random k Satisfiability logic and Hybrid Election Algorithm will be imposed in improving the training phase of Discrete Hopfield Neural Network.

1.7 Motivations

This thesis mainly focuses on remodelling of DHNN by using higher-order non-systematic symbolic logic with a hybrid metaheuristic algorithm. In this consequence, the problem statements and research questions with research objectives of this thesis open a bigger picture that motivates to do research in such areas to overcome the flaws of the existing models. Here, the key motivations of this thesis are described in details.

(a) Flexible logical structure is a critical issue that needs to address in the study of Satisfiability representation in order to investigate logical viability and interpretability. One of the prime options for the logical structure investigation might be Random k Satisfiability because of its non-systematic structure. By understanding this prospect, Sathasivam et al., (2020a) explored this non-systematic structure and achieved optimal final neuron states that minimizes the cost function and it retrieved almost 90% global minimum solutions. Despite the fact that this study limited the investigation of logical interpretation by attempting only Random 2 Satisfiability $(k \leq 2)$ that proposed the structure exclusively for a 2-dimensional decision system. This thesis focuses on finding the optimum combinations that introduce the higher order Random k Satisfiability in order to address this significant drawback. This Random k Satisfiability contains a number of combinations for first, second, and third order logics (for $k \leq 3$). Hence, these three different combinations of Random k Satisfiability can overcome all of systematic satisfiability and Random 2 Satisfiability flaws.

(b) Generally, Election Algorithm is a combination of swarm and evolutionary algorithms, so that its potentiality is highly acceptable due to its efficient operators which was pointed by Emami, (2015). Having inspiration by the prospects of this algorithm, Sathasivam et al., (2020b) applied this Election Algorithm with non-systematic logic in Discrete Hopfield Neural Network that provide a direction for discrete constraint satisfaction/optimization problems. This study showed that the operators of this algorithm remarkably facilated the training process through Random 2 Satisfiability representation by achieving highest global minima solutions. Though the operators of this Election Algorithm are effective but it has imbalance to maintain global and local search operators. Importantly, a metaheuristic algorithm must keep its intensification and diversification strategy. This strategy can be kept by balancing the local and global search operators. Nevertheless, in the aforementioned studies have some flaws but having such features and experimental results motivates to improve the architecture of this traditional Election Algorithm which may ensure to keep the intensification-diversification strategy. Point to be noted that current structure of Election Algorithm showed the inability of the advertisement campaign in improving the quality of local solutions through 2^{nd} order non-systematic logical representation. To overcome such major problem, this thesis will introduce an additional operator named ‘caretaker party’ is included in this Election Algorithm which is later named as Hybrid Election Algorithm. With the inclusion of this ‘caretaker party’ operator and higher order non-systematic logic, it will enhance the quality of local solutions and as well as balance the global and local search operators.

(c) Recently, a lot of research has been done on how to solve real-world optimization problems efficiently with metaheuristic algorithms. One of the flaws of DHNN is that it results suboptimal solutions due to its inefficient training phase. In this regard, the single objective and major challenge for DHNN is to avoid suboptimal final neuron states and achieve maximum fitness value. In this context, applying different types of metaheuristics in the training phase of DHNN is also a fruitful idea to check whether the model can converge to global solutions with less computational time or not. Meanwhile, several researchers did research to keep the maximum fitness value by employing different types of metaheuristics in the training phase of DHNN. Following this, Kasihmuddin et al., (2016) proposed Genetic Algorithm in DHNN training phase and used systematic logic to maximise fitness value by getting more global minima solutions. This work provided an inspiration to utilize algorithm in the training phase of DHNN that can ensure maximum fitness value but this algorithm failed to keep its highest fitness value or ratio of global solutions for higher number of neurons. From this, it is highly important to select such type of metaheuristics which is able to stable its highest fitness value as the number of neurons increases. Understanding this perspective, Sathasivam et al., (2020b) introduced a population-based algorithm named Election Algorithm with non-systematic logic in the training phase of DHNN. In this work, the authors found 90% global minimum solution for higher number of neurons, which makes it interesting to do more research. However, the authors only looked the non-systematic logic up to the second order, which made the DHNN model less effective because it may take more iterations to get the desired cost function. Motivated by the efficiency of this Election Algorithm and research scope from the non-systematic logic, a higher order non-systematic logic that can build three-dimensional decision system with hybrid Election Algorithm can be employed to overcome logical and metaheuristics flaw of the mentioned studies. The proposed algorithm will entirely focus only the single objective that is keeping highest fitness value for higher number of neurons and make single iteration to achieve desired cost function.

(d) Multi-objective concept initialization in metaheuristics can bridge the gap between Discrete Hopfield Neural Network and symbolic logical rule which create a new dimension in ANN research. Latest work by Gao et al., (2022), Zamri et al., (2022a) primarily focused on a single objective that is achieving maximum fitness value. In their works, there is no intension to accommodate multi-objective functions to tackle critical real world optimization problems, whereas now a days almost all the optimization problems have several independent objectives. In the context of ANN research, multi-objective function can be composed by the inclusion of diversity in the logical rule which can create another footstep ahead in understanding the concept of intelligence through symbolic logic. As a result, motivation for several objectives including maximal fitness and diversity emerges. Furthermore, achieving maximum fitness value or a single objective cannot clearly reveal the metaheuristics performances. According to Li et al., (2022), the optimal solution of a multi-objective function is a better trade-off solution than the single objective function. Importantly, multi-objective functions create multiple options for optimal solutions can be good in terms of improving the search capabilities of the algorithm since the computational effort is diversified to explore different regions of the search space. Hence, multi-objective functions for an optimization problem improves the knowledge of the problem domain, much more than single-objective function. As a result of these findings, this thesis proposed a novel metaheuristic called the Hybrid Election Algorithm, which combines multi-objective functions including highest fitness value and creating variety in the logical rule. The proposed Hybrid Election Algorithm leverages the multi-objective paradigm, in which both fitness and diversity are represented in Discrete Hopfield Neural Network by higher order Random *k* Satisfiability combinations. Notably, the proposed multi-objective Hybrid Election Algorithm maximises the fitness and employs *k*-ideal solution strings with a diverse logical rule, which also increases the storage capacity of Discrete Hopfield Neural Network.

(e) In the growth of logic mining research, this multi-objective Hybrid Election Algorithm can assign precisely. Meanwhile, attribute selection is an important factor in the logic mining research. Point to be noted, lack of appropriate attribute selection renders the induced logic uninterpretable and results into poor induced logic. Several researchers (Alzaemi et al., 2020; Kho et al.,2020) paid attention to find only optimal induced logic through systematic logic mining model. Even though the mentioned logic mining models by the authors created induced logic which deemed optimal but the major problem was that non-significant attributes were incorporated into the induced logic which lead to suboptimal knowledge extraction. Understanding the importance of this prospect, Jamaludin et al., (2021b) introduced another new strategy to select proper attributes through a statistical method named which is known as log-linear method. In this work, the data modelled according to the appropriate attributes but it did not operate any intelligence system to store the pattern of the data set. Motivated from the prospects of this log-linear method, this thesis will employ log-linear technique in the pre-processing phase before entering the training phase of the Discrete Hopfield Neural Network which has a significant impact on the selection of appropriate attributes. Thus, the implementation of log-linear method into logic mining will create optimal induced logic that has the capability to predict and classify. Another significant feature that can play important role in logic mining study is the inclusion of permutation operator because it has the feature to widen the search space by rearranging ordered sequence of the attributes of a data set. Motivated from this feature, Jamaludin et al., (2022b) analysed the effect of permutation operator and improved the performances of the logic mining model as the number of logical combinations has increased. Based on this significance, this thesis will incorporate the permutation operator, the pre-processing phase with non-systematic logic, as well as an effective training mechanism such as the Hybrid Election Algorithm that can address the shortcomings of previous studies and build a dynamic logic mining model that is highly capable for prediction and classifications.

1.8 Methodology and Limitations

The hybrid network is created by embedding the higher order non-systematic logical rule in DHNN. During the training phase of the DHNN, a new and improved metaheuristic Hybrid Election Algorithm (HEA) will be employed. Different existing models will be compared with this proposed method. The newly updated Random *k* Satisfiability Reverse Analysis approach will be used in conjunction with this Hybrid Election Algorithm model in order to do logic mining study. When compared to other models, the ability of the induced logic in the Random *k* Satisfiability Reverse Analysis approach will be demonstrated and discussed. The methodologies with limitations are explained as follows:

Methodology for Objective 1. In the current research progress, ANN researchers focus on implementation of symbolic logic in DHNN to do various constraint satisfaction or optimization problems. In this regard, a new type of non-systematic higher order logical structure named Random *k* Satisfiability (for $k \leq 3$) is proposed. This proposed logic is formed in 3-dimensional decision system and will be applied with different unique combinations. Having formation of the proposed logic, each literal of the clauses is represented as neurons. The synaptic weight of the neurons will be computed through Wan Abdullah method. After getting the zero-cost function, the correct synaptic weights will be stored as Content Addressable Memory. Meanwhile, the different types of Random *k* Satisfiability combinations enforced with DHNN will be analysed with benchmark Exhaustive Search algorithm. To check the efficiency of each Random *k* Satisfiability combinations, several performance metrics will be applied. During the testing phase, the global minima solutions will be utilised to assess the effectiveness of each Random *k* Satisfiability combinations.

Limitation of Methodology 1. In this methodology, there are certain restrictions to ensure reproducibility for future research. Firstly, the clauses are arranged only in conjunctive normal form and the synaptic weights must be calculated by using Wan Abdullah method since this Wan Abdullah method need less computation effort and easy to implement in DHNN. Secondly, there should not be any redundant literals in the logical rule. Another notable limitation is that it fully follows to the Random k Satisfiability propositional logic which is encoded to the DHNN system. It does not compare other logic options such as fuzzy or quantum logic. Furthermore, this paradigm only applies in three-dimensional decision systems with bipolar states.

Methodology of Objective 2. Hybridization of a metaheuristics is a real challenge because there is a strong possibility to degrade the performance of that hybrid metaheuristics. In this thesis, metaheuristic hybridization concept is utilized in the tradition Election Algorithm. According to Emami, (2019), existing Election Algorithm needs to optimize the operators of the advertisement campaign (key part of Election Algorithm) since it stuck to local optima. In this context, existing operators cannot fulfil the required expectations for improving quality of local solutions. It needs to mention that hybridized algorithm can maintain the exploration and exploitation strategy (Biabani et al., 2022). In these circumstances, the architecture of the Election Algorithm will be improved or hybridized by including another additional operator in the advertisement campaign which is named as Hybrid Election Algorithm. 'Caretaker party' is the additional operator which is placed after positive advertisement, negative advertisement and coalition part**.** The key strength of this caretaker party is that it only keeps such strings which have highest fitness value. Notably, the advertisement campaign of this HEA model also parted into two phases. The three operators (Positive, Negative and Coalition) are in the maximal fitness phase and the newly formed

Caretaker party will be dealt in the diversity phase. This proposed Hybrid Election Algorithm will be embedded with DHNN to make effective and efficient training phase specially as the number of neurons increased.

Limitation of Methodology 2: While this methodology can be an optimal approach to reveal the needs of metaheuristic algorithm in DHNN. At first, it does not fully follow the foundational work by Emami, (2015) since it was applied for continuous optimization problems. In the diversity phase, only caretaker party operator involves whether the influence of positive advertisement, negative advertisement and coalition operators are disregarded. Additionally, if the candidate of the population achieved highest fitness in the positive advertisement, it cannot directly put into the caretaker party. The simulation continues until completing all operations of the operators have been done. This can create more computational problems for HEA.

Methodology of Objective 3. The proposed Random *k* Satisfiability logic with DHNN requires such type of effective and efficient training phase so that the model remain stable as the number of neurons increases. To address this point, this thesis proposed an improved metaheuristics named Hybrid Election Algorithm which is composed of evolutionary and swarm intelligence concept. The single objective of the proposed metaheuristic is to gain highest fitness value and this objective is calculated by following Wan Abdullah method. Through this method, the number of unsatisfied clauses is directly proportional to the cost function of the proposed logic in Objective 1. Meanwhile, the fitness function will be higher when the number of satisfied clauses in the Random *k* Satisfiability logic increased. In the proposed Hybrid Election Algorithm, the fitness function will be improved after utilizing Objective 2, where several operators- positive advertisement, negative advertisement, coalition and finally