BINARY ARTIFICIAL BEE COLONY OPTIMIZATION FOR WEIGHTED RANDOM 2 SATISFIABILITY IN DISCRETE HOPFIELD NEURAL NETWORK

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by

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LIST OF SYMBOLS

r	Weighted ratio of negative literals
Γ_{r2SAT}	Logical structure of Weighted Random 2 Satisfiability
и	Total number of 2SAT clauses
ν	Total number of first-order clauses
$Z_i^{(k)}$	The clauses for the different order of k
P _{RAN2SAT}	The Random 2 Satisfiability logical structure
${\pmb \eta}_i^{(k)}$	The number of negative literals exist in each clause
К	The total number of negative literals exist in the logical structure
λ	Number of literals
S_{i}	State of the <i>i</i> th neuron
$W_{(i,j)}$	Synaptic weight from unit <i>i</i> to <i>j</i>
$ ho_i$	Threshold constraints in DHNN
E _{P_{RAN 2SAT}}	Cost function of RAN2SAT
h_{i}	Local field
\wedge	Conjunction
\vee	Disjunction
-	Negation
$H_{P_{RAN2SAT}}$	Lyapunov energy function
$H_{P_{RAN2SAT}}^{\min}$	Minimum energy of DHNN for RAN2SAT
$H_{P_{RAN2SAT}}^{final}$	Final energy of DHNN for RAN2SAT
R	The rate of relaxation speed

Tol	Tolerance value
$A_x^{(k)}$	The arrangement of clauses with different order of k
P _{learn}	The decision output of the data set
$\max\left[n(S_i)\right]$	The highest frequency of the clauses
$P_{best}^{(k)}$	The best logic for different order of k
$P_{induced}^{(k)}$	The induced logic for different order of k
$x_{i,j}$ and $x_{k,j}$	Two best bees
$V_{i,j}$	New food source
ϕ	Control distance parameter
x_j^{\min}	Lower bound of bees
x_j^{\max}	Upper bound of bees
f_i	Fitness of the bees
p_i	Selection probability
SN	Number of clauses
$\overline{\vee}$	Not or
$\underline{\vee}$	Exclusive or
$\overline{}$	Not and
L_{j}	Candidate
f_{L_j}	Fitness of the candidate
$J_i^{(2)}$ and $J_i^{(1)}$	The fitness of the second and first-order clauses
N _{POP}	Number of population
N _{party}	Number of parties

P_{j}	Party
$v_i^{\ j}$	Voter
$f_{v_i^j}$	Fitness of the voter
N_{s_j}	The number of voters influenced by the candidate
$\sigma^{\scriptscriptstyle p}$	Positive advertisement rate
$\mathcal{O}_{v_i^j}$	The eligibility distance coefficient
$S_{v_i^j}$	The state flipping of each voter
$N_{v_i^*}$	The number of voters from another party influenced by the candidate
$\sigma^{^n}$	Negative advertisement rate
v_i^*	voter from another party
x_{iv}^{a} and y_{iv}^{b}	The entries corresponding with the attribute a and b
Γ_{train}	Train data
Γ_{test}	Test data
Γ^i_{best}	The best logic of <i>r</i> 2SAT
$\Gamma^{induce_i}_{j}$	The induced logic of r2SAT

LIST OF ABBREVIATIONS

ANN	Artificial Neural Network
HNN	Hopfield Neural Network
DHNN	Discrete Hopfield Neural Network
3SAT	3 Satisfiability
2SAT	2 Satisfiability
RAN2SAT	Random 2 Satisfiability
RAN3SAT	Random 3 Satisfiability
MAJ2SAT	Major 2 Satisfiability
Y-RAN2SAT	Y-Type Random 2 Satisfiability
G-RAN3SAT	G-Type Random 3 Satisfiability
r2SAT	Weighted Random 2 Satisfiability
CNF	Conjunctive Normal Form
GA	Genetic Algorithm
ABC	Artificial Bee Colony
MAE	Mean Absolute Error
TSP	Traveling Salesman Problem
SAT	Boolean Satisfiability
2SATRA	2 Satisfiability Reverse Analysis
3SATRA	3 Satisfiability Reverse Analysis
P2SATRA	Permutation 2 Satisfiability Reverse Analysis
E2SATRA	Energy Based 2 Satisfiability Reverse Analysis
S2SATRA	Supervised 2 Satisfiability Reverse Analysis
r2SATRA	Weighted Random 2 Satisfiability Reverse Analysis
Pr2SATRA	Permutation Weighted Random 2 Satisfiability Reverse Analysis

HTAF	Hyperbolic Tangent Activation Function
kSAT	k Satisfability
CAM	Content Addressable Memory
RAM	Random Access Memory
WA	Wan Abdullah
NAND	Not and
XOR	Exclusive or
NOR	Not or
ТР	True Positive
TN	True Negative
FP	False Positive
FN	False Negative

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PENGOPTIMUMAN BINARI KOLONI LEBAH BUATAN BAGI 2 SATISFIABILITI RAWAK BERWAJAR DALAM RANGKAIAN NEURAL HOPFIELD DISKRET

ABSTRAK

Salah satu alternatif untuk menambah baik pemodelan Rangkaian Neural Hopfield Diskret ialah dengan melaksanakan variasi peraturan logik yang berbeza. Dalam konteks ini, Satisfiabiliti sesuai sebagai peraturan logik dalam Rangkaian Neural Hopfield Diskret kerana kesederhanaan struktur, dan toleransi kesalahan. Oleh itu, tesis ini akan menggunakan 2 Satisfiabiliti Rawak Berjajar Tidak Bersistematik yang digabungkan dengan algoritma Binari Koloni Lebah Buatan dalam Rangkaian Neural Hopfield Diskret. Binari Koloni Lebah Buatan akan digunakan untuk mengoptimumkan struktur logik mengikut nisbah literal negatif dengan memanfaatkan ciri mekanisme penerokaan algoritma. Kemudian, algoritma Pilihan Raya akan digunakan untuk mendapatkan tafsiran yang berpuas hati tentang struktur logik yang betul dalam fasa latihan Rangkaian Neural Hopfield Diskret. Model yang dicadangkan ini akan digunakan dalam kaedah Analisis Songsang Yang Diperbaiki untuk mengekstrak hubungan antara pelbagai bidang set data kehidupan sebenar berdasarkan perwakilan logik. Tesis ini akan dibentangkan dengan melaksanakan set data simulasi dan penanda aras dengan pelbagai metrik penilaian prestasi. Berdasarkan penemuan, model yang dicadangkan mengatasi model lain.

BINARY ARTIFICIAL BEE COLONY OPTIMIZATION FOR WEIGHTED RANDOM 2 SATISFIABILITY IN DISCRETE HOPFIELD NEURAL NETWORK

ABSTRACT

One of the alternatives to improve the modeling of the Discrete Hopfield Neural Network is by implementing different variants of logical rules. In this context, Satisfiability is suitable as a logical rule in Discrete Hopfield Neural Network due to the simplicity of the structure, and fault tolerance. Hence, this thesis will utilize Non-Systematic Weighted Random 2 Satisfiability incorporating with Binary Artificial Bee Colony algorithm in Discrete Hopfield Neural Network. The Binary Artificial Bee Colony will be utilized to optimize the logical structure according to the ratio of negative literals by capitalizing the features of the exploration mechanism of the algorithm. Then, the Election algorithm will be utilized to obtain a satisfied interpretation of the correct logical structure in the training phase of the Discrete Hopfield Neural Network. This proposed model will be employed in the Improved Reverse Analysis method to extract the relationship between various fields of real-life data sets based on logical representation. This thesis will be presented by implementing simulated, and benchmark data sets with multiple performance evaluation metrics. Based on the findings, the proposed model outperforms other models.

CHAPTER 1

INTRODUCTION

1.1 Introduction to Neural Network

Artificial Neural Network or for short ANN is a mathematical model or computational model, that imitates the way the brain nervous system. ANN consists of large interconnected artificial neurons which are programmed to mimic the features of a biological brain. All ANN has a similar structure where different neurons are connected by weights that will develop the ANN by updating the connection weights (Yang *et al.*, 2021). ANN can be classified into two parts of working principles, (i) use the training samples to design and train the network to obtain prediction rules; (ii) predict the samples according to the obtained rules to verify its reliability with the accuracy of the test results (Liu *et al.*, 2021). Due to these principles, ANN has become a popular model for classification, pattern recognition, and prediction. Worth mentioning that, ANN consists of two structures of the network which are feedforward and feedback networks. One of the oldest feedback networks was proposed by Hopfield and Tank in 1985 namely Hopfield Neural Network (HNN).

Notably, the framework of HNN was inspired by a mathematical model of ferromagnetism in statistical mechanics namely the Ising model (Takabatake *et al.,* 2022). Subsequently, the energy function was introduced to study the stability of the network resulting in good memory association ability. The continuous HNN (CHNN) model that adopted an ordinary differential equation with constant coefficients is the first model proposed to solve the traveling salesman problem (TSP) which is a common optimization problem among researchers. In recent years, discrete HNN (DHNN) has caught the interest of several researchers because all neurons in the network can be updated asynchronously. The neuron in DHNN can be represented in

bipolar form {-1, 1} or binary form {0, 1} (Alway *et al.*, 2021). Generally, the Lyapunov energy function is utilized in DHNN to characterize the behavior of the networks and represents the problems to be solved (Cheng *et al.*, 2019). Additionally, DHNN provides a central memory feature or content addressable memory (CAM) to store a pattern in a matrix form (Alway *et al.*, 2020). Information is distributed in the content of biological memory, rather than a specific address.

Note that, one of the drawbacks of DHNN is symbolic rule requires processing the input and representing the valuable information. Therefore, satisfiability (SAT) is used to represent the valuable information in DHNN. Additionally, Zamri et al. (2020) stated that the best way to observe the behavior of DHNN is by implementing a symbolic rule during the training and testing phase. Presently, there are few researchers suggested a new symbolic rule in DHNN such as Random 2 Satisfiability (RAN2SAT) (Sathasivam et al., 2020), Random 3 Satisfiability (RAN3SAT) (Karim et al., 2021), Major 2 Satisfiability (MAJ2SAT) (Alway et al., 2021), Y-Type Random 2 Satisfiability (YRAN2SAT) (Gao et al., 2022), G-Type Random 3 Satisfiability (GRAN3SAT) (Guo et al., 2022), and so on. Different variants of SAT are expected to perform differently in different areas of the real-life problem. Due to the explosion variants of SAT in DHNN, the benchmark training algorithm in DHNN is insufficient to find a consistent interpretation of the logical structure, especially the logic with the inclusion of first-order clauses. The inconsistent interpretation of the logic means that the logic is unsatisfied which will lead to wrong synaptic weight management. According to Mansor et al. (2017), the process of obtaining global solutions is always associated with the correct synaptic weight management. When the synaptic weight obtain is wrong, the final state obtained will be trapped in local solutions. According to Kasihmuddin et al. (2019), an optimal testing phase can be obtained by having an optimal training phase whereby optimal cost function is required to ensure that the synaptic weight obtained is correct. Zamri *et al.* (2020) stated that an optimal training phase can be obtained by employing a metaheuristic as the training algorithm in DHNN. Therefore, the incorporation of different metaheuristics in the training phase of DHNN is crucial to ensure an optimal learning environment for optimal DHNN. Besides that, the generalization ability is crucial in DHNN whereby the pattern extracted from the training data can give a good predictive in a new situation (Voyant *et al.*, 2017). Various methods can be used to improve the generalization ability such as cross-validation (Zhu *et al.*, 2019), and the training data must be a sufficiently large number (Castiglioni *et al.*, 2021). This method can ensure the model can classify a correct outcome for new data or situations that did not confront during the training. Hence, improving generalization ability can ensure the model can be used effectively in real-life applications by producing an accurate prediction of new situations.

1.2 Significance of Research and Problem Statement

An Artificial Bee Colony Algorithm (ABC) was proposed by Dervis Karaboga in 2008 which was inspired by the foraging behavior of honeybees to solve the continuous optimization. Overly the past decades, there is an increased interest in ABC to solve scientific and engineering problems which leads to the rapid development of ABC. Unfortunately, the original ABC can not be applied directly to discrete search space. Therefore, Jia et al. (2014) discretized the original framework of ABC by modifying the movement of the employed and onlooker based on the bitwise operation. The work replaced the real arithmetic operation used in the original ABC with the 'exclusive or' operator (XOR), 'and' operator (AND), and 'or' (OR). Although the work is able to show stellar performance in terms of convergence speed, final optimizing accuracy, and robustness, there is no effort to investigate the movement of employed and onlooker bees with the different bitwise operations. Note that, the binary ABC will explore the potential solution in the binary search space only. However, the work fails to justify the impact of the chosen bitwise operation in the binary search space. The current bitwise operation can perform worse with different types of problems. Hence, bitABC leaves open the possibility of an ideal bitwise operation. This thesis aims to address this problem by utilizing different bitwise operators in the food source equation of binary ABC. The significance of this study lies in the potential of different bitwise operators in binary ABC, the food source is crucial to control the exploration rate of the solution space. An efficient bitwise operator in binary ABC will speed up the convergence and diversify the solution space. Furthermore, the contribution of this study related to the further understanding of the behavior of different bitwise operators in binary ABC.

The symbolic rule via logical rule is one of the perspectives to improve the modeling of DHNN. Boolean Satisfiability or SAT is a logical rule that models the connections of the neurons in DHNN. Recently, there are variants of SAT proposed in DHNN such as 2 Satisfiability (Kasihmuddin et al., 2017), Random 2 Satisfiability (Sathasivam et al., 2020), Random 3 Satisfiability (Karim et al., 2021), Major 2 Satisfiability (Alway et al., 2021), Weighted Random 2 Satisfiability, and so on. Due to the rapid development of SAT in DHNN, the structure of SAT can be classified into systematic and non-systematic. As a side note, the systematic structure consists of a restrictive number of variables in each clause, meanwhile, the non-systematic structure consists of a non-restrictive number of variables in each clause. Sathasivam *et al.* (2020) were the core impetus of the non-systematic structure of SAT in DHNN. The

work proposed Random 2 Satisfiability in DHNN that comprises first and secondorder clauses in the logical rule. Worth mentioning that, Random 2 Satisfiability can promote the diversification of the logical rule during the training phase and manage to maximize the production of global solutions. Inspired by this work, Zamri et al. (2022) proposed a new approach to non-systematic SAT namely Weighted Random 2 Satisfiability with consideration of the different ratios of negative literals in the logical structure. The logic phase was proposed to ensure Weighted Random 2 Satisfiability has a correct number of negative literals according to the initiate ratio. A Genetic Algorithm was implemented in the logic phase to optimize the logical structure of Weighted Random 2 Satisfiability concerning the ratio of negative literals. Unfortunately, the Genetic Algorithm requires more than one iteration to obtain the correct number of negative literals in Weighted Random 2 Satisfiability. This is due to the mutation operator in the Genetic Algorithm tending to wrongly exchange the states of a chromosome which leads to a suboptimal solution. As a result, there is a high probability that the incorrect logical structure will be generated. Consequently, the logic phase will generate Random 2 Satisfiability which will be redundant to analyzing existing logical rules in DHNN. Therefore, this thesis aims to address the limitation by proposing different variant algorithms in the logic phase. The significance of this thesis lies in the potential of an improved algorithm to ensure that logical structure will obtain a correct number of negative literals with less error.

Weighted Random 2 Satisfiability managed to promote diversification of the logical combination because of the different order of clauses and different dynamics of negative literals. However, the inclusion of first order and a high number of negative literals in the logical structure made the benchmark training algorithm in DHNN inefficient in finding a consistent interpretation of the logical rule embedded. Note

that, Exhaustive Search is a benchmark training algorithm in DHNN whereby this algorithm utilizes a trial-and-error mechanism to obtain an approximate solution. According to Zamri et al. (2022), the possible satisfied interpretation for the first-order clause is 50% and the inclusion of different ratios of negative literals in the logical structure made the logical structure difficult to be satisfied. As a result, an Exhaustive Search consumed more computation time to complete the whole search process which leads to an increase in error. This gets worse when the number of neurons increases, there is a high probability that an Exhaustive Search fails to obtain a consistent interpretation of the logical rule. This will lead suboptimal cost function that will impact the process of generating synaptic weight by using the Wan Abdullah method where the synaptic weight obtained will be randomized. As a result, the global solutions of the logic in DHNN will reduce dramatically and HNN is unable to display the behavior of the Weighted Random 2 Satisfiability. Therefore, the current training phase requires an efficient algorithm that can optimize the fitness of the neuron state iteratively. By employing an efficient training algorithm, the correct synaptic weight can be obtained which leads to high global solutions obtained.

Logic mining is a subset of data mining where the information from the data sets were extracted in the form of a logical rule. Rather than becoming a black box model that delivers the final result, logic mining improves pattern representation by translating the final neuron state into the logical rule. Last few years, there is an increased interest in logic mining in Discrete Hopfield Neural Networks. One of the recent works of logic mining was proposed by Kasihmuddin *et al.* (2022) whereby the work suggested a modification in pre-processing stage of logic mining. The statistical method was utilized to find the relationship between the attributes in the data sets to obtain optimal attributes that will be presenting the data sets in Discrete Hopfield Neural Network. Although the work managed to provide a significant improvement, the proposed logic mining is prone to several weaknesses. First, the work is limited to systematic Satisfiability, and only one logical rule is considered to represent the data sets. In this case, the logical rule that represents the data sets is not varying. By this standard, the model will fail to produce the diversified induced logic. Secondly, the Reverse Analysis (RA) that is used to find the pattern of the train data sets is biased toward positive outcomes. The logic that represents the data set was considered a high frequency of clauses from the entry with the positive decision. Consequently, the model unable to maximize the correct classification of negative outcomes. This thesis aims to address the limitations of the existing logic mining model by proposing a new approach of Reverse Analysis method with non-systematic Satisfiability which is Weighted Random 2 Satisfiability. The significance of this study lies in the potential to improve the generalization ability of the logic mining model. Moreover, the study will contribute to a better understanding of the development of the Reverse Analysis method with different variants of symbolic rule.

1.3 Research Questions

In order to ensure the consistency between the research problems and research objectives, this thesis proposes four important research questions. Note that, these questions are the guides for this thesis to address all the problems stated in Section 1.

 How different bitwise operations in the binary Artificial Bee Colony algorithm will affect the search space that has the capability to find a solution with lesser error?

- 2. How binary Artificial Bee Colony algorithm with the optimal bitwise operation will be implemented in the logic phase to generate a correct number of negative literals in the logical structure with regard to the initiate ratio?
- 3. What is the alternative training algorithm in DHNN that has the capability to obtain satisfied interpretation of non-systematic Weighted Random 2 Satisfiability especially when the number of neurons increases?
- 4. What are the crucial developments that must be implemented in the current logic mining to ensure the information that was extracted from the data set has the ability to predict and classify?

1.4 Research Objectives

This thesis is centered on modeling a Discrete Hopfield Neural Network using a non-systematic logical rule, namely Weighted Random 2 Satisfiability, that will be generated by the logic phase. The proposed model will be optimized by using a natureinspired algorithm that has the ability to distribute the negative literal correctly. Subsequently, the logical rule will be trained by socio political based algorithm that has the capability minimizing the cost function of the model. In this context, the proposed model must be computationally stable and has the capability to minimize learning iteration. Ultimately, this thesis will utilize the proposed model to extract the pattern from the real-life data set which has the capability to classify and predicts. Therefore, the objectives of this thesis is as follows:

 To propose several variants of binary Artificial Bee Colony Algorithm inspired by bitABC. The proposed models will utilize different bitwise operations in the update rule equation. By using these models, the search space of the solution with the different bitwise operations will be evaluated.

- 2. To implement a binary Artificial Bee Colony algorithm in the logic phase to optimize the logical structure according to the ratio of negative literals. The binary Artificial Bee Colony algorithm with the optimal bitwise operation will be evaluated by comparing with the benchmark metaheuristic in Discrete Hopfield Neural Network.
- 3. To implement Election Algorithm as a training algorithm in Discrete Hopfield Neural Network. An Election Algorithm will improve the fitness function of the training phase iteratively to ensure that satisfied interpretation of Weighted Random 2 Satisfiability is obtained.
- 4. To introduce the new approach of the Reverse Analysis method that employs non-systematic Weighted Random 2 Satisfiability in doing various real-life data sets from the field of health to business. The proposed logic mining will extract the pattern in the form of induced logic from the data sets using a Discrete Hopfield Neural Network. This induced logic has the capability to classify and predict.

1.5 Methodology

In recent years, there are various types of binary Artificial Bee Colony algorithms proposed. In this thesis, five variants of the binary Artificial Bee Colony algorithm were proposed. The new variants of the binary Artificial Bee Colony algorithm were inspired by the work of Jia *et al.* (2014) whereby the arithmetic operation in the update rule of the original Artificial Bee Colony algorithm was replaced with a bitwise operation. The binary Artificial Bee Colony algorithm consists of three important components such as employed bees, onlooker bees, and scout bees. Note that, the bitwise operation will control the movement of the employed and onlooker bees in the search space of the solution. In this context, five variants of binary Artificial Bee Colony algorithms will utilize five different bitwise operations which are not and (NAND), exclusive or (XOR), and (AND), or (OR), and not or (NOR). Note that, these models will utilize the same number of scout bees. In order to ensure the model obtained the optimal solution rapidly the rate of scout bees increased. These models will be implemented in the logic phase to optimize the logical structure according to the ratio of negative literals. Then, the impact of different bitwise operations in the update rule of the binary Artificial Bee Colony algorithm will be evaluated based on the mean absolute error and the convergence analysis. While this methodology can be an optimal approach to investigate the behavior of different bitwise operations in the search space, there are a few limitations to ensure the reproducibility of the objective in this thesis. First, the operators in the Artificial Bee Colony algorithm employ more exploitation mechanisms than exploration. However, the exploitation mechanism occurs in the early stages (employed) managed to avoid the solution being trapped in the suboptimal solutions (Su et al., (2022). Second, this thesis only considered five bitwise operations to improvise the solution space in the logic phase. Although combining with different bitwise operations in the update rule equation of the Artificial Bee Colony algorithm, there is only five unique search spaces can be obtained.

The logic phase that acts as a logic optimizer requires effective and efficient metaheuristics to reassure a correct logical structure according to the ratio of negative literals generated in the Discrete Hopfield Neural Network. In this thesis, five variants of binary Artificial Bee Colony algorithms with different bitwise operations will be implemented in the logic phase. Note that, the bee represents the logical structure, and the food source carried by the bee reflects the number of negative literals that the

logical structure has. The first stage of the binary Artificial Bee Colony algorithm in the logic phase is to initiate the random logical structure. Then, the employed bee will explore the new food source around the hive (search space for the solution). In this stage, the search space of five variants binary Artificial Bee Colony algorithm will be different due to the movement of the bees being controlled with the different bitwise operations. Subsequently, the onlooker bee that waits in the hive will gain information about the new food source from the employed bees. During this stage, only the most profitable food source will be selected by onlooker bees. The onlooker bees will exploit the information obtained by using the same mechanism as the employed bees. The bee will be evaluated based on the amount of the food source carried. If the food source obtained is insufficient which indicates an incorrect number of negative literals in the logical structure obtained, then, the scout bee will be spawned. The correct number of negative literals depends on the ratio of the negative literals. In this thesis, the range of the ratio of the negative literals is $r \in \{0.1, ..., 0.9\}$ with a step size $\Delta r = 0.1$. In terms of the performance of the logic phase, the proposed model will be evaluated based on the error analysis and the convergence analysis. After obtaining an optimal bitwise operation for the logic phase, the performance of the model will be compared with three benchmark metaheuristics in Discrete Hopfield Neural Network: Genetic Algorithm, Particle Swarm Optimization, and Exhaustive Search. While this methodology can be an optimal approach to creating a Discrete Hopfield Neural Network with a logic optimizer that generates logical structure according to the predetermined ratio of negative literals, there are a few limitations to ensure the reproducibility of the objective in this thesis. First, the value of $r \in \{0,1\}$ is neglected which indicates all literals are positive and negative, respectively. This is due to ensure the accurate synaptic weight obtained which avoids the same neuron state trained (Zamri *et al.*, 2022). Second, the operators for the employed and onlooker bees in the Artificial Bee Colony algorithm are based on the work of Jia *et al.* (2014) instead of the original work by Karaboga (2008). This is because the logic phase search space only considers the binary representation solutions.

Subsequently, a correct structure of Weighted Random 2 Satisfiability with the desired number of negative literals generated by the logic phase will be trained in the training phase of the Discrete Hopfield Neural Network. Inspired by the work of Sathasivam et al. (2020b), Election Algorithm will be utilized as a training algorithm in Discrete Hopfield Neural Network in order to ensure a satisfied interpretation of the logic. In the Election Algorithm, an individual will represent the solution string whereby the individual can be either voter or a candidate based on fitness. In order to ensure the model effectively find the solution in all defined spaces, a systematic solution space partition was employed in the Election Algorithm. Note that, there are three iterative operators in the Election Algorithm: positive advertisement, negative advertisement, and coalition. The positive advertisement will exploit the known region of the search space to find an approximate solution. Then, the negative advertisement will explore the unknown search space to improve the fitness of the solutions. Finally, the coalition stage will be executed to evaluate the best candidate (maximum fitness) found. The best candidate indicates a consistently satisfied interpretation of the logic. As a result, the optimal cost function can be obtained, and a correct synaptic weight will produce. Therefore, Discrete Hopfield Neural Network has the assurance to obtain global solutions. To measure the performance of the Election Algorithm in the training phase of the Discrete Hopfield Neural Network, a comprehensive comparison will be conducted between the Election Algorithm and an Exhaustive Search in terms of error iterations. While this methodology can be an optimal approach to reveal the new

benchmark metaheuristic in Discrete Hopfield Neural Network, there are a few limitations to ensure the reproducibility of the objective in this thesis. First, the computation of the similarity of belief between candidates and voters is according to Sathasivam et al. (2020b) instead of the original work by Emami and Derakhshan (2015). This is because Satisfiability only deals with binary or bipolar representation. Second, this thesis does not compare the performance of the Election Algorithm with another state-of-the-art metaheuristic in Discrete Hopfield Neural Networks such as the Genetic Algorithm. The main reason is the work by Sathavisam et al. (2020b) shows that Election Algorithm is capable to outperform Genetic Algorithm as a training algorithm in Discrete Hopfield Neural Network. One of the main weaknesses of existing logic mining is the systematic structure of the logic that represents the information from the data sets. Hence, the search space for the correct classification of the logical outcomes will be restricted to the same induced logic obtained. In this thesis, an improved RA namely the Weighted Random 2 Satisfiability-based Reverse Analysis method with permutation was proposed to address all the major weaknesses of the existing RA. There are three domain layers in logic mining whereby preprocessing layer, the logic phase, and the DHNN layer. In the first layer, the data set will be split into two where 60% of the data will be classified as a train data set meanwhile, whereas the remaining 40% will be classified as a test data set (Jha & Saha, 2021). Then, a new perspective of choosing optimal attributes will be proposed whereby Jaccard Index will be utilized to evaluate the relationship between input attributes and decision attributes. Note that, the proposed model will consider the permutation operator that exchanges the placement of the attribute to represent the data sets. In the second layer, the non-systematic Weighted Random 2 Satisfiability will extract the information from the data sets. Then, the proposed model will generate

more than one logical rule to represent the data sets based on the highest total number of correct classifications of the logical outcomes. The logic will be generated by using Objective 2 whereby the binary Artificial Bee Colony algorithm will be implemented in the logic phase to control the distribution of negative literals in the logical structure. In this thesis, Discrete Hopfield Neural Network will be utilized instead of Continuous Hopfield Neural Network because the purposes of both types are different. According to Yu et al. (2020), Continuous Hopfield Neural Network is used for optimization problems with continuous variables, whereas Discrete Hopfield Neural Network is used for pattern recognition that is stored in associative memory. Therefore, in the third layer, Discrete Hopfield Neural Network will train these logical rules by using the training algorithm in Objective 3. The satisfying interpretation of the logic will lead to a correct synaptic weight obtained by using the Wan Abdullah method (Abdullah, 1992). Subsequently, the induced logic in Discrete Hopfield Neural Network can be obtained by converting the final neuron state into the logical rule. The induced logic will be used to compare with the test data set. The induced logic with the best performance will be considered the best-induced logic that represents the data set. Note that, the proposed model will be tested with 10 real-life data sets with various field areas. The performance of the proposed model will be evaluated during the testing phase in terms of the confusion metric. The proposed model will be compared with the existing logic mining model. While this methodology can be an optimal approach to extracting logic from real-life data sets, there are a few limitations to ensure the reproducibility of the objective in this thesis. First, the data set used in this thesis can be obtained from UC Machine Learning Repository where the data does not require any pilot study. Using the benchmark data sets, the proposed model can be compared with another existing method in the field of feature selection. Second, the proposed logic mining utilizes Hyperbolic Tangent Activation Function to retrieve the final neuron state. This is due Hyperbolic Tangent Activation Function providing relaxation property as the Discrete Hopfield Neural Network evolves to a stable state (Mannsor and Sathasivam, 2016).



Figure 1.1 The Overall Thesis Flowchart

1.6 Organization of Thesis

The remaining of this thesis is as follows. Chapter 2 will briefly review the main domain of this thesis. There are five domains: Discrete Hopfield Neural Network, Boolean Satisfiability, Artificial Bee Colony algorithm, Socio-Political mechanism-based algorithms, and Data Mining in Neural Networks. In the first domain, the development of the Discrete Hopfield Neural Network will be further discussed. The second domain will review the different variants of Boolean Satisfiability. From this section, there are two categories of Boolean Satisfiability in Discrete Hopfield Neural Networks such as systematic and non-systematic structure. The third domain will review the discretized Artificial Bee Colony algorithm. This thesis will utilize the same framework as the work of Jia *et al.* (2014) whereby the bitwise operation will replace the previous method that controls the movement of the bees. The fourth domain will

review different variants of socio-political mechanism-based algorithms. This domain will focus on the discretized Election Algorithm where the distance formulation will compute the similar belief of the candidate and voters. The last domain will review logic mining which is the subset of data mining. At the end of Chapter 2, the strength and weaknesses of the existing work can be obtained.

Chapter 3 imparts an explanation of the existing work allied to the model proposed in this thesis. The first section will briefly describe the formulation of Random 2 Satisfiability proposed by Sathasivam *et al.* (2020a). Note that, the logical rule utilized in this thesis was inspired by Random 2 Satisfiability but with consideration of the ratio of negative literals. The next section will formulate the implementation of Random 2 Satisfiability in a Discrete Hopfield Neural Network. Subsequently, the state-of-the-art metaheuristics in such as Genetic Algorithm, Particle Swarm Optimization, and Exhaustive Search will be further described. These metaheuristics will be implemented in the logic phase to validate the performance of the model proposed in this thesis. The last section will briefly describe the existing logic mining models exist which will be utilized to validate the performance of new approaches of logic mining proposed in this thesis. This Chapter mainly discusses the existing work that can be implemented in the proposed model of the thesis.

Chapter 4 will formulate the concept of the proposed model in this thesis. The first section will describe the logical rule utilized in this thesis namely non-systematic Weighted Random 2 Satisfiability. Since this logical rule has unique features whereby the distribution of the negative literals can be controlled in the logic phase. Therefore, five different variants of the binary Artificial Bee Colony algorithm with different bitwise operations will be implemented in the logic phase to optimize the logical structure. Then, Election Algorithm will be implemented as a training algorithm in

Discrete Hopfield Neural Network to ensure a consistent interpretation of the logical rule. In the last part of Chapter 4, the new approach of the Reverse Analysis method in logic mining will be described. At the end of Chapter 4, a further understanding of the methodology for each objective in this thesis is obtained.

Chapter 5 will highlight the capability of the proposed model in doing simulated data. The experimental setup of the binary Artificial Bee Colony algorithm and Election Algorithm can be obtained in this Chapter. The performance of five variants of the binary Artificial Bee Colony algorithm will be compared in terms of Mean Absolute Error and convergence analysis. As a result, an optimal bitwise operation in optimizing the logic phase is obtained. Then, the optimal binary Artificial Bee Colony algorithm will be evaluated with state-of-the-art algorithms in Discrete Hopfield Neural Network. Therefore, the logic phase has the assurance to achieve the correct logical structure. In the last part of Chapter 5, the performance of the Election Algorithm as a training algorithm in the Discrete Hopfield Neural Network will be compared with an Exhaustive Search. Note that, an efficient training algorithm is crucial in Discrete Hopfield Neural Network to ensure that the logical rule achieved satisfied interpretation which leads to optimal synaptic weight management. At the end of Chapter 5, research questions 1 until 3 can be answered.

In Chapter 6, the proposed model will be extended to the logic mining perspective which real-life data sets will utilize to test the proposed model. The performance of the proposed model will compare with the existing logic mining such as Supervised 2 Satisfiability Reverse Analysis, 2 Satisfiability Reverse Analysis with Permutation, 2 Satisfiability Reverse Analysis, 3 Satisfiability Reverse Analysis with Permutation, and the standard model of Weighted Random 2 Satisfiability Reverse Analysis. The capability of the logic mining models will be evaluated based on the confusion metrics: Accuracy, Precision, Sensitivity, Specificity, and Matthew Correlation Analysis. At the end of Chapter 6, the credibility of the new approach of logic mining with non-systematic Satisfiability to represent the real-life data sets can be obtained.

Chapter 7 will give a summary of this thesis as well as discusses the directions for future work. Finally, Appendix A contains an illustrative example of a binary Artificial Bee Colony algorithm in the logic phase. Appendix B illustrates the process of the Election Algorithm as a training algorithm in Discrete Hopfield Neural Network. Appendix C will demonstrate the new approach to logic mining. Appendix D will represent the source code of the proposed model.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, five important domain that contributes to the proposed model will be reviewed in each section. The first section will review the development of a Discrete Hopfield Neural Network in presenting various optimization tasks. Note that, this thesis will utilize Discrete Hopfield Neural Network to process the behavior of the data set. Hence, several features can be considered. Next, the second section will briefly review the boolean satisfiability which is the symbolic rule in Discrete Hopfield Neural Network. By reviewing the existing logic in the Discrete Hopfield Neural Network, a better understanding of the weakness and strengths of the systematic structure and non-systematic structure. Therefore, a clear direction with different perspectives on the satisfiability of the Discrete Hopfield Neural Network is attained. Then, the third section will review the development of the Artificial Bee Colony algorithm as a popular swarm intelligence. This review shows the evolution of the Artificial Bee Colony algorithm from a continuous to a discrete framework. A discrete Artificial Bee Colony algorithm is essential to this thesis because Discrete Hopfield Neural Network is only compatible with binary/bipolar representation of neurons. The next section will review the concept of a socio-political-based algorithm. In this section, Election Algorithm is commonly utilized as a training algorithm in Discrete Hopfield Neural Networks due to the effective operators. The last section of this chapter will review the evolution of data mining in neural networks. Worth mentioning that this thesis will utilize one of the subsects of data mining which is logic mining. In this last section, the problems and research gaps of the existing logic mining models can be addressed.

2.2 Discrete Hopfield Neural Network

Hopfield and Tank (1985) proposed Hopfield Neural Network (HNN) to solve combinatorial problems. The initial task for the proposed model is to solve the Travelling Salesman problem with less computational effort. This research shed new light on HNN future applications. Recently, Li et al. (2022) proposed HNN to solve the capacitated location routing problems (CLRP) by using the improved framework of HNN that solves traveling salesman problems. HNN incorporates a multi-start mechanism in order to achieve optimal performance. The model is capable to produce superior performance and satisfactory efficiency for practical use. Thus, this work shows that the original HNN can be improved. Symbolic integration is another interesting application of HNN. In 1992, Wan Abdullah proposed logic programming in HNN by capitalizing on the energy minimization of the Lyapunov energy function. The work is the first attempt to obtain synaptic weight in HNN by comparing the cost function with the Lyapunov energy function. The significance of having synaptic weight in HNN is to minimize the final neuron state into a global minima solution. This work was improved by Sathasivam (2010) that proposed a new relaxation method to accelerate the convergence property of HNN. This study investigates the relaxation rate that controls the energy relaxation process. The HNN model with a relaxation rate has a better performance compared to those without using a relaxation rate. The best relaxation rate is $2 \le R \le 4$. Kasihmuddin *et al.* (2019) proposed Mutation HNN (MHNN) that consolidates with the global search capability of the estimation of distribution algorithms (EDA). MHNN managed to produce diverse final neuron states with optimal energy. However, the work stated that an optimal training phase is required to ensure an optimal testing phase which leads to convergence of DHNN. Therefore, Zamri et al. (2020) proposed DHNN with Clonal Selection Algorithm

(CSA) to improve the manual transactions system of granting employees resources access in the field of data science. CSA was utilized to enhance the learning phase of DHNN. This finding shows that the modified HNN can be used to solve unstimulated data. Worth mentioning that, the common ground of all the mentioned studies is a modification of HNN managed to improve the original model. However, there is no effort in proposing an optimizer layer of the logical rule in DHNN.

Author(s)	Detail of studies	Summary and Findings
Hopfield and Tank (1985)	The work proposed HNN	The proposed model
	to solve combinatorial	managed to solve the
	problems.	Travelling Salesman
		problem with less
		computational effort.
Li et al. (2022)	HNN incorporates a	The improved HNN is
	multi-start mechanism to	capable to solve CLRP
	solve CLRP.	with superior
		performance.
Wan Abdullah (1992)	Logic programming in	The work is the first
	HNN by capitalizing on	attempt to obtain synaptic
	energy minimization.	weight in HNN by
		comparing the cost
		function with the
		Lyapunov energy
		function.
Sathasivam (2010)	The work investigates the	The Sathasivam
	compatibility of the new	Relaxation method was
	relaxation method in	proposed with the best
	HNN.	relaxation rate is
		$2 \le R \le 4$.
Kasihmuddin <i>et al.</i> (2019)	Mutation HNN (MHNN)	MHNN managed to
	consolidates with the	produce diverse final
	global search capability of	neuron states with optimal
	the estimation of	energy.
	distribution algorithms	
	(EDA).	~~
Zamri <i>et al</i> . 2020	DHNN with Clonal	CSA in the training phase
	Selection Algorithm	helps the model to obtain
	(CSA) to improve the	the optimal training phase.
	manual transactions	
	system of granting	
	employees resources	
	access.	

Table 2.1 Summary of Discrete Hopfield Neural Netowork

2.3 Boolean Satisfiability

Boolean Satisfiability (SAT, for short) is the decision mechanism that determines whether the formula is satisfiable (Hidouri *et al.*, 2021). SAT is crucial in DHNN because it can store valuable information which leads to possible outputs either True or False (Mansor *et al.*, 2020). Recently, researchers suggested different variants of SAT in DHNN. Consequently, SAT in DHNN was split into two classes which are systematic SAT and non-systematic SAT (Guo *et al.*, 2022). In this section, both classes will be explored to gain more understanding of the behavior of different SAT in DHNN.

2.3.1 Systematic Satisfiability

Systematic SAT is a logical rule that contains a strict number of literals in each independent clause (Karim *et al.*, 2021). Sathasivam (2010) was the first person to implement SAT in DHNN. The study proposed Horn Satisfiability or for short HornSAT in DHNN that constituted clauses with at most one positive literal and the literal can be repeated. Although the work is successfully implementing a symbolic rule in DHNN, the work did not investigate different variants of SAT in DHNN. The work by Kasihmuddin *et al.* (2017) proposed a hybrid approach of DHNN and Genetic Algorithm (GA) in doing 2 Satisfiability (2SAT). Note that, 2SAT is the decomposition of two literals per clause. The proposed hybrid model is able to reduce the computational complexity and produced more global solutions with smaller errors. The performance of 2SAT in DHNN is able to achieve 90% production of global minimum solutions which refers to the credibility of 2SAT in representing the way of the neurons behave in DHNN. In another development, Mansor *et al.* (2017) suggested a higher order of SAT in DHNN named 3 Satisfiability (3SAT). The structure of 3SAT

comprises three literals per clause. The core impetus of introducing 3SAT is to increase the interpretability of the logical structure in DHNN. However, systematic SAT has a smaller search space due to the low variety of logical combinations that can be obtained.

Author(s)	Detail of studies	Summary and Findings
Sathasivam (2010)	The work investigates the effectiveness of the relaxation method with HornSAT in DHNN.	The study proposed HornSAT in DHNN that constituted clauses with at most one positive literal and the literal can be repeated.
Kasihmuddin <i>et al.</i> (2017)	A hybrid approach of DHNN and Genetic Algorithm (GA) in doing 2 Satisfiability (2SAT).	The study proposed 2SAT in DHNN that decomposes two literals per clause. 2SAT managed to produce 90% global minimum solutions.
Mansor <i>et al</i> . (2017)	Suggest higher order of SAT in DHNN.	The study proposed 3SAT in DHNN to increase the interpretability of the logical structure in DHNN.

Table 2.2 Summary of Systematic Satisfiability in Discrete Hopfield Neural Network

2.3.2 Non-systematic Satisfiability

In 2020, Sathasivam suggested a new approach to SAT in DHNN that results in the development of a new class of SAT namely non-systematic SAT (Sathasivam *et al.*, 2020a). The work proposed Random 2 Satisfiability (RAN2SAT) which combines the first and second-order clauses in the logic. In order to ensure the RAN2SAT is compatible as a new symbolic rule in DHNN, RAN2SAT was embedded in the benchmark DHNN that utilized ES as a training algorithm. The performance of RAN2SAT was evaluated based on the capability of RAN2SAT to obtain optimal solutions in DHNN and the efficiency of RAN2SAT in representing the simulated data. This new approach is able to promote logical variation which is expected to enhance the search space of the optimal solutions. The study shows that RAN2SAT can successfully minimize the cost function that correlates with the inconsistencies of the network. However, the proposed logic still suffers in repetitive final neuron states which results in overfitting solutions. Karim et al. (2021) extend the work by proposing higher order of non-systematic SAT namely Random 3 Satisfiability (RAN3SAT). The study suggested three types of high-dimension decision combinations of RAN3SAT in DHNN. The first combination comprises the first and third order of clauses only, the second combination comprises the second and third order of clauses, and the third combination decomposes of the first, second, and third order of clauses. The behavior of all combination RAN3SAT was evaluated in benchmark DHNN. The performance of this combination was compared with various performance metrics in terms of synaptic weight management, neuron variation, and energy analysis. From the results, the second combination of RAN3SAT is the optimal combination because of the lower error, produces consistent global solutions, and increases neuron variations.

In another development, Alway *et al.* (2021) proposed Major 2 Satisfiability (MAJ2SAT) that consists of the first and second order of clauses. Interestingly, the MAJ2SAT emphasizes the ratio of the 2SAT clause in the logical structure. The work investigates the dynamic of logical structure with different ratios of 2SAT in benchmark DHNN. The majority of the element in the logical structure is vital in increasing the global solutions indicating the convergence of the DHNN model and enhancing the diversification of the retrieved neurons. Despite the successful result, the large number of 2SAT in the logical structure will increase the process of obtaining a consistent interpretation. Moreover, the same pattern of retrieving neuron will be