MATHEMATICAL MODELLING OF STEADY BÖDEWADT FLOW AND HEAT TRANSFER IN NANOFLUID AND HYBRID NANOFLUID OVER A PERMEABLE AND RADIALLY STRETCHING DISK

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by

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LIST OF SYMBOLS

C	stretching rate
C_f	skin friction coefficient
C_p	specific heat at constant pressure
$\mathbf{F}$	body force
F, G, H	dimensionless velocity functions
k	thermal conductivity of fluid
Nu	Nusselt number
p	pressure
Pr	Prandtl number
(r, φ, z)	cylindrical coordinates
Re	Reynolds number
S	suction parameter
S_c	critical value
t	time
T	temperature of fluids
$\mathbf{u}$	velocity vector
u, v, w	nondimensional velocity components
w_o	velocity of wall transpiration
(x, y, z)	Cartesian coordinates
α	thermal diffusivity
ε	convergence tolerance
η	similarity variable
θ	dimensionless temperature function
λ	dimensionless stretching parameter
μ	dynamic viscosity

ν	kinematic viscosity
ρ	density of fluid
τ	wall shear stress
ϕ	volume fraction of nanoparticles
Ω	angular velocity
∇	gradient

Subscript

nf	nanofluid
f	base fluid
s	nanoparticle
s_1	the first nanoparticle
s_2	the second nanoparticle

LIST OF ABBREVIATIONS

BVP	Boundary Value Problem
CFD	Computational Fluid Dynamics
IVP	Initial Value Problem
ODE	Ordinary Differential Equation

LIST OF APPENDICES

PEMODELAN MATEMATIK BAGI ALIRAN DAN PEMINDAHAN HABA BÖDEWADT MANTAP DALAM NANOBENDALIR DAN NANOBENDALIR HIBRID KE ATAS CAKERA TELAP DAN REGANGAN JEJARI

ABSTRAK

Tesis ini mengkaji ciri-ciri aliran dan pemindahan haba Bödewadt dalam nanobendalir dan nanobendalir hibrid dengan kehadiran kuasa sedutan pada plat regangan jejari cakera telap. Aliran Bödewadt ialah aliran berputar yang disebabkan oleh sumber berputar yang diletakkan jauh dari cakera bawah. Dalam kajian ini, peranan nanobendalir dalam meningkatkan pemindahan haba disiasat secara teori. Jenis nanopartikel yang dipertimbangkan ialah kuprum, perak, dan aluminium oksida manakala air sebagai bendalir asas. Dengan menggunakan pemboleh ubah keserupaan, persamaan pembezaan separa diturunkan kepada persamaan pembezaan biasa tak linear yang kemudiannya diselesaikan secara berangka menggunakan kaedah kotak Keller. Jumlah sedutan yang mencukupi diperlukan supaya penyelesaian persamaan pemindahan haba wujud bagi kes cakera tidak bergerak. Kehadiran sedutan dan cakera regangan menyebabkan ketebalan lapisan sempadan momentum dan haba berkurangan. Kedua-dua nanobendalir dan nanobendalir hibrid membantu dalam meningkatkan pekali geseran kulit tempatan dan nombor Nusselt tempatan lebih baik daripada cecair likat. Walau bagaimanapun, peningkatan pemindahan haba berlaku hanya apabila pecahan isipadu zarah nano cukup kecil dengan kehadiran sedutan. Kehadiran zarah nano yang kedua pada nanobendalir hibrid mempamerkan peningkatan pemindahan haba yang lebih baik berbanding dengan nanobendalir. Sama seperti kes nanobendalir, jumlah sedutan dan cakera regangan yang mencukupi diperlukan untuk meningkatkan kadar pemindahan haba.

MATHEMATICAL MODELLING OF STEADY BÖDEWADT FLOW AND HEAT TRANSFER IN NANOFLUID AND HYBRID NANOFLUID OVER A PERMEABLE AND RADIALLY STRETCHING DISK

ABSTRACT

This thesis studies the flow and heat transfer characteristics of steady Bödewadt flow in nanofluid and hybrid nanofluid with the presence of suction on the radial stretching permeable disk. Bödewadt flow is a rotating flow that is induced by a rotating source that is placed far away from the bottom disk. In this study, the role of nanofluid in heat transfer enhancement are theoretically investigated. The nanoparticles that considered in this study are copper, silver, and aluminium oxide while the base fluid is water. By using the similarity variables, the governing partial differential equations are reduced to nonlinear ordinary differential equation which are then numerically solved using the Keller's Box method. Adequate amount of suction was needed for the similarity solution of heat transfer equation to exist in the case of stagnant disk. The presence of suction and stretching disk caused the momentum and thermal boundary layer thickness to reduce. Both nanofluid and hybrid nanofluid helps in increasing the local skin friction coefficient and local Nusselt number better than viscous fluid. However, the heat transfer enhancement occurs only when volume fraction of nanoparticle is sufficiently small due to the presence of suction. The presence of the second nanoparticles in hybrid nanofluid showed that the heat transfer enhancement is better compared to nanofluid. Same as the case of nanofluid, sufficiently amount of suction and stretching disk need to be applied in order to enhance the heat transfer rate.

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Wave of technologies and innovations hit differently on the twenty-first century where one of the focus in this century is making use the unique features of fluids namely nanofluids. Nanofluid consists of a base fluid with the presence of nano-sized particles (1-100 nm) suspended within the fluid. Nanofluid consists of one type of nanoparticle in whereas, hybrid nanofluid has more than one type of nanoparticle suspended in the base fluid. Technically, hybrid nanofluid is defined as the combination of nanofluid with another type of nanoparticle. Generally, the nanoparticles are metal or metal oxides that can increase the heat transfer rate due to the conduction and convection characteristics of the nanoparticles.

Over the past decade, several techniques have been used to improve the heat transfer performance. New technologies have created a demand regarding the usage of nanofluid in the industries that involve heat transfer where nanofluid is able to improve the efficiency of the existing heat transfer process (Saidur et al., 2011; Kaggwa and Carson, 2019). Hence, many classical Newtonian problems have been re-visited by many researchers to investigate the behaviour of flow and heat transfer in the presence of nanoparticles in viscous fluid. One of the types of flow that is recently received consideration attention is Bödewadt flow.

Bödewadt flow refers to a rotating fluid with a uniform angular velocity far from the stagnant disk. The secondary flow that occurs in this flow is directed radially inward and axially upwards due to friction at the thin layer near the surface disk. In 1940, Bödewadt obtained a similarity solution of the three-dimensional swirling flow induced by rotating fluid flow. The transformation of the model is similar to von

Kármán's who solved the swirling flow induced by a rotating disk in 1921. Presently, there are limited studies in Bödewadt flow of nanofluids and hybrid nanofluids. Also, none of them investigated the effect of suction as well as the combination effect of suction and stretching disk to such study.

It has to be mentioned that, as pointed out by Sahoo et al. (2015) and Rahman and Andersson (2017), suction plays an important role in the thermal Bödewadt flow due to the fact that similarity solution is only possible with the presence of suction. The same goes to the role of stretching disk which could increase the heat transfer which has been reported by Mustafa et al. (2015), Turkyilmazoglu (2015) and Sahoo and Poncet (2012). This is because the Bödewadt flow is radially inwards and axially upwards near the body surface, therefore imposition of sufficient suction or replacing the stagnant disk with stretching disk will "pull" the axial flow to the disk surface. It is also expected that increasing of suction or stretching disk will reduce the thermal boundary layer thickness, hence manage to control the flow.

In this study, the mathematical model of Tiwari and Das on the boundary layer flow and heat transfer of steady revolving Bödewadt flow in nanofluid and hybrid nanofluid is introduced. The governing equations are transformed into the nonlinear differential equations by using the similarity variables that are similar to those of conventional von Kármán flow. The resulting equations are then solved numerically using implicit finite difference which is known as the Keller's Box method. The effects of nanofluid parameters, suction and stretching rate to the Bödewadt flow in nanofluids and hybrid nanofluids are investigated. The plots for velocity components, temperature, skin friction coefficient and local Nusselt number are presented and discussed. The results that obtained will give better understanding of flow and heat transfer characteristics of Bödewadt flow in nanofluids and hybrid nanofluids, hence provide substantial improvements in designing better performance and reliability fluid dynamics devices.

1.2 Problem Statement

Recently, nanofluid has emerged as an important industrial fluid that with the ability to enhance the heat transfer rate. The heat transfer in Bödewadt flow is physically realistic with the presence of suction or stretching of the disk, however, the existing published literature on Bödewadt flow in nanofluids had ignored the effects of suction to its flow and heat transfer characteristics. It is expected that in the Bödewadt flow, imposition of suction or stretching of the disk will reduce both the momentum and thermal boundary layer thicknesses, hence reduce the chances of developing separation flow and increase the heat transfer rate. However, the implication of the nanoparticles together with suction as well as the effects of combination of suction and stretching disk on the Bödewadt flow in nanofluids is yet to know. Therefore, further investigation needs to be done to inspect the interaction of particle-fluid and also its effects to the flow control and heat transfer rate.

1.3 Research Questions

There are three research questions for this study:

i. How does the Bödewadt flow in nanofluid behaves compare with the existing Bödewadt flow of viscous fluids in describing the flow and heat transfer characteristics?

ii. What are the effects of nanofluid properties parameters, suction parameters and combination effect of suction and stretching parameters on the skin friction coefficient and heat transfer rate as well as fluid's velocity and temperature profiles?

iii. How is the performance of the flow and heat transfer characteristics if a different type of nanoparticle is added to the nanofluid?

1.4 Objectives

This study aims to theoretically investigate the problem of Bödewadt flow and heat transfer in nanofluids. The objectives are:

i. To compare the Bödewadt flow and heat transfer characteristic in nanofluid with the existing behaviour of the Bödewadt flow in viscous fluids.
ii. To investigate theoretically the effect of suction and stretching on the skin friction coefficient and heat transfer rate as well as fluid's velocity and temperature profile.
iii. To examine the performance of the flow and heat transfer characteristics due to the addition of another type of nanoparticle into the nanofluid (hybrid nanofluid).

1.5 Scope of Study

This study takes into consideration the steady incompressible and boundary layer Bödewadt flow of nanofluid model in laminar flow. Tiwari and Das model is used to model the governing equation of nanofluids. The fluid properties such as thermal conductivity, density, specific heat capacity, dynamic viscosity and thermal diffusivity are assumed to be constant. The energy dissipation is neglected, and the temperature of the surface is constant. The types of nanoparticle considered in this study are copper, silver and aluminium oxide nanoparticles and the base fluid is water.

1.6 Significance of Study

Bödewadt flow has practical applications that involving mechanical rotor-stator system, turbo-machinery, vortex chambers and chemical mixing in chemical engineering Ellison and Cornet (1971). In the past few years, there are three common physical properties that being point out for the components that being used in electronic such as the reduction in size of the component, lighter in weight and the efficiency of the component. However, in achieving these three physical properties, the engineers had encountered other problems such as thermal management in devices and the compactness of heat exchanging devices (Saidur et al., 2011). The management of thermal in electronic devices should be controlled due to excessive heat may cause the device's response to be slower and may contribute to damage. Nanofluids can be applied in these applications that involve the heat transfer process in cooling or heating element. The present of nanofluid in large scale industry may help in decreasing the time taken to cool or heat their elements even though the cost of the production will be slightly increased. Therefore, investigation on the effects of different conditions, for example imposition of suction or replacing stagnant disk with the stretching disk, that will contribute to the efficiency and enhancement of heat transfer is important so that the industries may decide the best equipment setup in the manufacturing process. Since, investigation through experimental work is unfavourable because of the limited scope that can be covered and impractical for the problems that involve many parameters, hence, mathematical modelling approach is the best option especially during the initial investigation on these new problems.

1.7 Thesis Outline

Generally, this study on Bödewadt flow and heat transfer in nanofluid and hybrid nanofluid is about the effect of stretching disk with presence of suction. This thesis consists of five chapters which are the introduction of the thesis, literature review, methodology, result and discussion and last but not least, conclusion. Chapter 1 discusses the general overview of Bödewadt flow and numerical methods that have been used in past research. The problem statement of the study is being stated with research questions, objectives, scope of study and significant study are given. Next, the literature review on the latest development of the Bödewadt flow problem is given in Chapter 2. In Chapter 3, the Keller's Box method have been chosen to solve the research problem. The governing equations and the steps involve in the Keller's Box method of the study are being explained in this chapter. Next, the results and discussion part are where the evidence of this research are being explained with the obtained results. There are three sections in Chapter 4. The first section provided the validation of Keller's Box method and Matlab program coding that have been used in this study. Section 4.2 presented the numerical results for Bödewadt flow and heat transfer in nanofluid while Section 4.3 discussed the numerical results for the research problem in hybrid nanofluid. Lastly, Chapter 5 concluded the research study and summarized the accomplishment of the research's objectives. The recommendation for future research is also given in the last section of Chapter 5. Thus, this is literally the outline for the study of Bödewadt flow and heat transfer in nanofluid and hybrid nanofluid with the presence of suction acting on the stretching disk.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter contains the literature review that are divided into three sections. Section 2.2 gives the general review of Bödewadt flow and past research work of Bödewadt flow and heat transfer in both Newtonian and non-Newtonian fluids. Sections 2.3, on the other hand, presents the definition, properties, advantages and mathematical modelling of nanofluid and hybrid nanofluid. Various numerical methods that have been used by past researchers to solve the Bödewadt flow problems are presented in the last section.

2.2 Bödewadt Flow

Bödewadt flow is named after Bödewadt (Bödewadt, 1940) who was the first person to investigate such flow theoretically in 1940. It is the steadily revolving flow of a fluid which rotates with a constant angular velocity at a large distance above a stationary disk. Bödewadt applied the similarity transformation that similar to von Kármán's transformation (Kármán, 1921) that used in the study of swirling flow driven by a constantly rotating disk. The connection between Bödewadt flow and von Kármán flow can be seen from the rator-stator system that proposed by Batchelor in 1951. He showed that, under the large Reynolds number condition, two shear layers are formed in between a rotating disk and a stationary disk. As the boundary layer separated, the flow near the stator is being identified as Bödewadt flow while the flow near the rotor is known as von Kármán flow (Batchelor, 1951).

The schematic diagram for the Bödewadt flow is shown in Figure 2.1 (Schlichting, 1979). It can be seen that the flow is radially inwards near the boundary layer of the stationary disk. For the reason of continuity, this radially inward flow is compensated by an axially upward flow, away from the bottom surface. Its radial pressure gradient is being balanced by centrifugal forces where the fluid is swept upwards due to drawn to the axis of the rotation. This phenomenon can be seen as an analogy for the teacup effect (Schlichting, 1979). If we stir the tea vigorously and then leave it for a moment, the tea leaves will settle in a little heap near the center at the bottom. In fact, Bödewadt flow can be considered as a reversed of von Kármán flow. This is because of the axial velocity component of Bödewadt flow is directed away from the stationary disk rather than towards the rotating disk for the von Kármán flow. However, compared to the von Kármán flow, the velocity components in the Bödewadt flow exhibit more complex variation and has a thicker boundary layer (Rahman and Andersson, 2017).

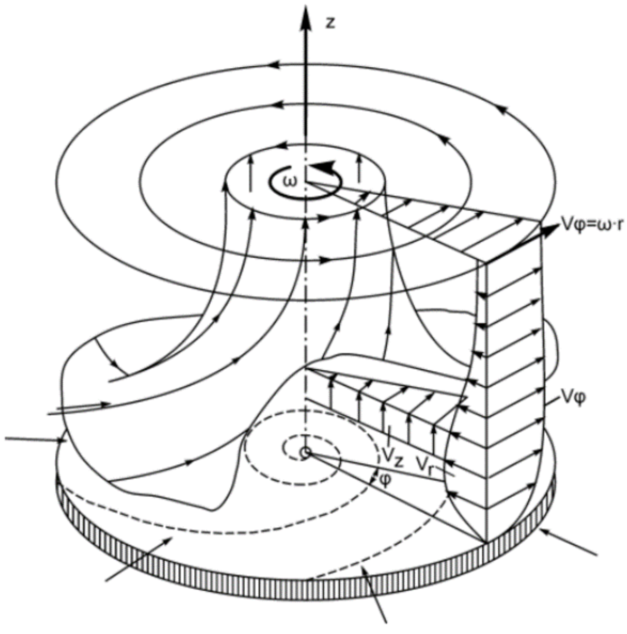

Figure 2.1 The schematic diagram of Bödewadt flow (Schlichting, 1979)

Traditional von Kármán flow and heat transfer has been a great deal of attention in the past decade. However, for its cousin – Bödewadt flow, exact solution of the Navier-Stokes equations in the vicinity of boundary layer was first presented by

Bödewadt (1940). The Bödewadt flow is characterized by the balance of radial pressure gradient and the centrifugal forces. Since then, many experiment studies and analyses have been carried out and focused more on the stability of the flow (Savaş, 1987; Faller, 1991; Lopez and Weidman, 1996; Lingwood and Alfredsson, 2000; Fernandez-Feria, 2002; MacKerrell, 2005).

2.2.1 Bödewadt Flow and Heat Transfer in Newtonian Fluids

Sahoo et al. (2014) studied the Bödewadt flow of viscous fluid with partial slip and concluded that slip condition has important effects on reducing both the axial velocity and the pressure. Also, they found that increasing of slip parameter will decrease the torque required to maintain the disk at rest. Turkyilmazoglu (2015) investigated the Bödewadt flow and heat transfer over a uniformly stretched stationary disk. An important outcome of his study is that both the momentum and thermal boundary layer thicknesses reduce even for moderate strength of stretching. Both Sahoo et al. (2014) and Turkyilmazoglu (2015) used the similarity transformation which is similar to the conventional von Kármán flow. Hence, the results that they obtained are similarity solutions.

Turner and Weidman (2017) studied the stability of Bödewadt flow over a stretching and shrinking disk using temporal and spatiotemporal stability analysis. They examined the involved instability properties and noticed that the flow is more stable for large stretching rate, moreover, shrinking case is more unstable than the stretching case. Recently, Rahman and Andersson (2017) commented that the similarity solution of heat transfer in Bödewadt flow of viscous fluid is physically realistic only with the presence of significant suction. Besides, Rahman and Andersson (2018) investigated the Bödewadt flow of a fluid-particle suspension with strong suction and reported that the particle-fluid interaction can cause the reduction of fluid motion.

The similarity solution of heat transfer in Bödewadt flow with the existence of nonlinear radiation mechanism over a rough slip disk has been presented by Mustafa et al. (2018). Analysis of entropy generation and heat transfer rate due to the presence of wall suction have been carried out. They found that wall suction and slip coefficients have remarkable effects on the solution profiles. Adequate wall suction can cause the reversal of axial flow, hence this “pulling” effect leads to physically plausible temperature profiles. Besides, the amplitude of the oscillatory velocity profile is suppressed when the slip coefficient increases. More recently, Rafiq and Hashmi (2019) considered suction in the numerical study of homogeneous-heterogenous reactions in Bödewadt flow. The existence of the similarity solution is possible only with the presence of sufficient amount of suction at the permeable disk.

2.2.2 Bödewadt Flow and Heat Transfer in Non-Newtonian Fluids

There are quite a number of papers that focused on the problem of Bödewadt flow in a type of non-Newtonian fluid, namely the Reiner-Revlin fluid. It is a visco-inelastic fluid that has applications in testing blood flow and also in the simulation of heat and chemical reaction (Ramesh et al., 2020). Sahoo (2011) has included the heat transfer analysis in his study of Bödewadt flow of an electrically conducting Reiner-Rivlin fluid with partial slip. On the other hand, the case of no slip condition is considered by Sahoo et al. (2012). The effect of partial slip on the Bödewadt flow of Reiner-Rivlin fluid also has been studied by Sahoo and Poncet (2012). Their studies showed that slip has prominent effects on the velocity fields where the velocity profiles reverse the direction away from the disk and approach the asymptotic value at a shorter distance from the disk as the slip parameter is incremented.

Later, Sahoo et al. (2015) extended the work by Sahoo and Poncet (2012) to include the effect of suction and injection on the studied problem and they found that suction dominates the oscillations in the velocity profiles while injection has opposite effects compare to the case of suction. Sahoo and Shevchuk (2019) considered the revolving flow of Reiner-Rivlin fluid over a stretchable disk with variation in power-law temperature. The study showed that stretchable surface influences the velocity profile, reduces the boundary layer thickness and increases the rate of heat transfer. The shrinking disk oppose the result of stretching where it increases the thickness of boundary layer.

On the other hand, Joshi et al. (2017) studied the Bödewadt flow of a ferrofluid, a colloidal fluid with nanoscale ferromagnetic particles in the presence of geothermal viscosity. Geothermal viscosity is the situation where the rotation of disk fluid is moving rapidly in the horizontal direction. They found that increasing the porosity of porous medium and rotation of the fluid will enhance the heat transfer rate.

2.2.3 Bödewadt flow and Heat Transfer in Nanofluids

Other than Reiner-Rivlin fluid and ferrofluids, nanofluids are considered in the study of Bödewadt flow problem. To date, there are only three published literature on Bödewadt flow and heat transfer of nanofluid over a stationary disk. Mustafa et al. (2015) and Khan et al. (2017) considered the same problem of Bödewadt flow and heat transfer in nanofluids over a radially stretching and stationary disk. The former formulated the problem using Tiwari and Das model while the latter used the Buongiorno model. They found that both skin friction coefficient and heat transfer rate from the disk are enhanced when the nanoparticle volume fraction is increased. Rafiq et al. (2019) pursuing the recent research in numerical study of Bödewadt slip flow on a convectively heated porous disk in nanofluid by focusing on the effect of solid volume fraction towards the heat transfer rate while suction and wall roughness is taken into consideration. The finding shows that the wall suction accelerates the heat transfer process of nanofluid.

So far, there is no research paper that can be found regarding the study of hybrid nanofluid in Bödewadt flow. Even though there are many studies that involve hybrid nanofluid, none of them involve in Bödewadt boundary layer flow. For example, there is study on hydromagnetic hybrid nanofluid in the presence of suction and stretching sheet along with the magnetic field in laminar boundary layer flow investigated by Devi and Devi (2016). The research explained about the effect of suction, effect of magnetic field and the effect of nanoparticle volume fraction but did not mention anything about the effect of stretching disk along with the presence of suction.

Recently, Ramesh et al. (2020) studied the von Kármán flow over a stretching disk in hybrid nanomaterial and solved the numerical problem by using the Runge-Kutta-Fehlberg method. Although Ramesh et al. (2020) claimed that they solved the Bödewadt flow problem, however, their mathematical model showed that they solved the von Kármán flow problem where the flow is induced by a rotating disk. As per concern, von Kármán flow has opposite characteristics compared to Bödewadt flow. Ramesh et al. (2020) found that the velocity and temperature curves reduce along the increasing value of stretching parameter. Besides that, they also observed that the heat transfer coefficient decelerates as the nanoparticles volume fraction increases.

Based on the above literature review, the Bödewadt flow and heat transfer in hybrid nanofluids that involves the effect of stretching with suction as their parameters in hybrid nanofluid in Bödewadt flow has not yet been studied.

2.3 Nanofluids

In fluid mechanics, the fluid flow can be defined as behaviour of fluids at rest or in motion. Fluid mechanics also can be defined as the interaction of fluid with solids or other fluid boundaries. There are many types of fluids, for examples, ideal fluid, real fluid, Newtonian fluid, non-Newtonian fluid, ideal plastic fluid, incompressible fluid and compressible fluid. However, under the circumstance of fluid mechanics, two types of fluid that being concerned are Newtonian fluid and non-Newtonian fluid. Newtonian fluid is the fluid that remain its viscosity constant although the shear stress is applied on it at constant temperature. Opposed to the Newtonian fluid, the viscosity of non-Newtonian fluid viscosity changed as the shear is applied on it. Nanofluid can be found in two states which is in Newtonian fluid and non-Newtonian fluid, depending on the type of base fluid. As an example, the base fluid for Newtonian nanofluid is water, while the base fluid for non-Newtonian nanofluid is non-Newtonian fluid, such as ethylene glycol (Al-Chlaihawi et al., 2022; Ali and Salam, 2020). Other example of non-Newtonian nanofluid is the suspension of nanoparticles in the base fluid which is a solution of carboxymethyl cellulose in water (Nasiri et al. 2020).

It has been a common practice that Newtonian fluids such as water, ethylene glycol and oils are the contribution factor to increase the heat exchange from device to the outer space. As the time goes by, such fluids may not support the thermal heat exchange due to excessive heat that produce by the devices. Nanotechnology has been introduced in the few past decades where the new generation of coolants are called “nanofluids” (Bandewar, 2017). The term “nanofluid” was proposed by Choi and Eastman (1995) at the ASME International Mechanical Engineering Congress and Exposition at San Francisco in 1995.

The particles in nanofluids are practically in nanometer size where the composition of this nanoparticles will affect the thermophysical properties of the fluid such as density, thermal conductivity, viscosity and heat transfer coefficient. This type of fluid has been used to enhance the heat transfer process in the application such as chemical production, power generation, air conditioning, automobiles and refrigeration process. For better performance of thermophysical properties and heat transfer, nanofluid has been introduced due to its potential in transferring heat for fluids. Addition of small volume fraction of nanoparticles in base fluid can increase the thermal conductivity of fluid, hence, cause the heat exchange of the devices become more efficiency when compared to the Newtonian fluid (Choi and Eastman, 1995; Trisaksri and Wongwises, 2007; Wang and Mujumdar, 2008; Saini et al., 2016). Further observation by Kumar et al. (2015) and Kleinstreuer and Feng (2011) regarding some important factors such as particle volume concentration, material of the particle, and size of particle have been identified to be able to control the thermal conductivity of nanofluid.

Compared to the conventional solid-liquid fluid, nanofluid has more advantages as the nanoparticle has a bigger surface area where more heat transfer can occur between the surface of fluids and particle. Besides that, nanofluids is predominant with Brownian motion where it shows that nanofluid has a higher stability compared to conventional solid-liquid fluid. Another advantage of nano size particle is that it can reduce the clogging of particle where system miniaturization can be promoted. Moreover, application of nanofluids in a system can reduce the pumping power as the heat transfer reached its equivalent phase faster. This advantage can be seen in heating, ventilating and air conditioning system. Although, clogging may still occur for nanofluids but it will happen after a long time where the nanoparticles undergo degradation process after a long term (Saidur et al., 2011).

The investigation on the viscosity and specific heat of nanofluid in the view of wetting characteristic of nanofluid has also been carried out by Gopalan and Narayan (2011). Due to the tremendous ability in heat transfer enhancement, nanofluid has appeared as a pioneering coolant in the industry of heat treatment. Heat treatment process is a common process in manufacturing industry, for example the mechanical process of quench hardening is usually being applied in this industry. For the next generation of heat transfer fluids, nanofluids have been selected due to its properties in enhancing heat transfer performance and improving wetting characteristics which are completely different from pure liquids. Thus, nanofluid is extremely useful and preferable in the industry of heat treatment due to its special characteristic (Gopalan and Narayan, 2011).

2.3.1 Hybrid Nanofluids

Combination of different nanofluid has been recognised to be one of the mechanisms that helps in order to acquire the desired heat transfer rate. Hybrid nanofluids is the nanofluids by mixing two or more nanoparticles. More precisely, hybrid nanoparticles consist of two or more types of materials or metal in nanometer size that are mixed together to be synthesized into hybrid nanofluids (Sundar et al., 2017).

Hybrid nanofluids can also be described as a type of fluid that is in the homogenous phase where two or more metals that have different chemical bond are being mixed simultaneously. Comparison between this hybrid nanofluid and unitary nanofluid showed that hybrid nanofluid contributes better to heat transfer rate, heat generation and thermal radiation. Hence, the advantage of hybrid nanofluid is, it gives better heat transfer performance compared to conventional heat transfer fluids such as ethylene glycol, water and oil. Research done by Abbasi et al. (2013) proved that 0.1% volume fraction of hybrid nanofluid gave out 20.68% of thermal conductivity in heat transfer process.

Over the past decade, investigation regarding hybrid nanofluid has gained momentum and researchers found that hybrid nanofluid helps in enhancement of heat transfer rate. Many properties should be taken into accounts in conducting hybrid nanofluids studies such as better thermal network, the aspect ratio of both nanoparticles and the synergistic effects. Proper hybridization of hybrid nanofluids also may give promising heat transfer enhancement. There are three types of hybrid nanofluid which are water-based nanofluids, oil-based nanofluids and ethylene glycol-based nanofluids (Sarkar et al., 2015).

Similar to unitary nanofluids, hybrid nanofluids also face some challenges such as application of hybrid nanofluids in long-term stability, the cost in preparation of hybrid nanofluid where the synthesis of nanoparticles needs to be considered, types of nanoparticles and the complication that may occur in production process. The main obstacle in hybrid nanofluids is the stability of the hybrid nanofluid's performance where different method in synthesizing the nanoparticles it may lead to different type of results (Che Sidik et al., 2017). The changes of nanoparticles from hydrophobic to hydrophilic cause some changes in the characteristic and may affect the result and affect the performance of the hybrid nanofluids (Babar and Ali, 2019). Despite the inadequate understanding regarding the heat transfer mechanism and variability of reported result, hybrid nanofluid has been emerged to be one of promising mechanism used in heat transfer fluid. Examples of hybrid nanofluid are alumina nanoparticles and titania nanoparticles with water as base fluid, zinc oxide nanoparticles and silver nanoparticles with water as base fluid, and single-walled carbon nanotubes and alumina nanoparticles with ethylene glycol as base fluid.

2.3.2 Types of Nanoparticles

Nanoparticles or nanostructured materials are technologies advancement research area current active subject in due to its physiochemical characteristics such as melting point, wettability and thermal conductivity. There are four categories in classification of nanomaterials which are carbon-based nanoparticles, inorganic-based nanoparticles, organic-based nanoparticles, and composite-based nanoparticles. The nanomaterials that are categorized as carbon-based nanoparticles contain carbon. It is a novel class of materials that are widely used in biomedical fields including the delivery of therapeutics, biomedical imaging, biosensors, tissue engineering and cancer therapy. Inorganic-based nanoparticles, on the other hand, are metal and metal oxide nanomaterials. Inorganic nanoparticles are non-toxic, hydrophilic, biocompatible and highly stabile compared to organic materials. The organic-based nanoparticles consist of nanomaterials formed from organic materials excluding carbon materials. For instance, dendrimers, cyclodextrin, liposome and micelle. The composite-based nanoparticles are multiphase with one phase on the nanoscale dimension that can be other combination of nanoparticles with other nanoparticles or nanoparticles combined with large or bulk-typed materials. The composites may be any combinations of carbon-based, metal-based, or organic-based nanomaterials with any form of metal, ceramic, or polymer bulk materials (Jeevanandam et al., 2018). In this study, inorganic-based nanoparticles have been chosen where this category of nanoparticles is classified as metal and metal oxide. Examples of metal are copper, gold and silver while examples of metal oxide are alumina, silica and titania. (Sarkar et al., 2015; Jeevanandam et al., 2018).

The nanoparticles can be in spherical, cylindrical or rod type, platelet, blade or brick type shapes (Ali and Salam, 2020). Raza et al. (2016) investigated the effects of different shapes of nanoparticles in a stretching and slip channel filled with Cu-Water nanofluid. They found that a cylindrical shape and a reduced particle size is in favor of heat transfer process. Kim et al. (2015) found that brick shaped nanoparticles possess higher thermal conductivity than platelet and blade shaped nanoparticles.

The use of copper nanoparticle, Cu has become a trendy due to its unique physical and chemical properties. It helps in improving the thermal, catalytic and electrical properties, and thus it can be used in various applications such as in heat transfer for industrial cooling and heating. Besides, the use of copper nanoparticles in the process of metal ion reduction, electrical or thermal conductivity give a big impact towards this production. From the experiment data by Garg et al. (2008), copper nanoparticle in ethylene glycol nanofluid has increased the thermal conductivity of the fluid twice as predicted by Maxwell model. Study of convective heat transfer enhancement using copper nanoparticles has been carried out by Gurav et al. (2014) shows that the addition of copper nanoparticles has increased the heat transfer characteristic of the base fluid (water). However, the measured of increase in thermal conductivity this increment seemed to be lower than being predicted by Eastman et al. (2001) and Garg et al. (2008). Moreover, the viscosity of the fluid has increased four times larger than being predicted by Einstein law of viscosity due to the present of copper nanoparticle in the fluid effects the viscosity of the solution (Garg et al., 2008). Besides reducing the thermal conductivity, copper nanoparticle has been used as disinfectant for wastewater disinfection since it displays higher antibacterial activity against bacteria (Khan and Jameel, 2018).

For over centuries, the interest towards the silver nanoparticles, Ag have been growing due to their dichroic character as glass when it was being integrated. The integration of silver nanoparticles has enhanced the solar cells where it helps in trapping the light for solar cells. This property of silver nanoparticles has helped in the application of inks, microelectronics, medical imaging and waste management (Seyhan et al., 2017). Besides, silver is a nontoxic and inorganic compound, hence, it is popular with its application as antibacterial agents in wound treatment, sterilization, water purification, food sanitation and antibacterial textiles. From the experimental results, the decrease in size and increasing concentration of the nanoparticles cause the increase in thermal conductivity of the nanofluids. Besides, silver nanoparticles are expected to have a better thermal property which is suitable for heat transfer applications (Iyahraja and Rajadurai, 2015). Other than being famous for antimicrobial and antifungal agent, silver nanoparticles also involve in automotive industries where the energy performance of the automotive radiators is being observed.
An analysis regarding the presence of silver nanoparticles in the coolant of an automotive radiator has been analyzed by Cordeiro Junior and Nogueira (2020).

Aluminium oxide, Al_2O_3 also known as alumina, is one of the types of nanoparticles that not only able to increase the efficiency of heat transfer coefficient but also improves the magnetic forces and cause less radiation (Jain et al., 2020). The presence of aluminium oxide nanoparticles in thermal energy storage technology helps in increasing the heat transfer system where it shortens the melting and freezing time of the devices. It also forms a stable phase change material where the nanoparticles act as heat transfer fillers. The time required for melting and freezing has decreased about 21% to 23% when aluminium oxide nanoparticles is applied (Ke and Li, 2018).

2.3.3 Nanofluids Models

Mathematical model is a description of a system using mathematical relation to explain and solve real-life problems. There are two commonly used mathematical models for nanofluids. The first model was proposed by Buongiorno in 2006. Buongiorno's model is a model where the slip velocity is the sum of the nanofluid relative velocity and base fluid velocity, and the sum value is not equal to zero. Buongiorno has considered seven slip mechanisms in nanofluid model which are Brownian diffusion, thermophoresis, Magnus effect, gravity settling, fluid drainage, inertia and diffusiophoresis (Malvandi and Ganji, 2015). However, among the seven slip mechanisms, the significant influence in Buongiorno model are Brownian diffusion and thermophoresis while others are too small and can be ignored. Tiwari and Das had proposed another nanofluid model in 2007. In this model, the nanoparticles and its base fluids are in thermal equilibrium and no slip condition between the nanoparticles and the base fluids. The significant influence in this Tiwari and Das model is the volume fraction of the nanoparticles (Salleh et al., 2019).

Moreover, these two models of nanofluid have different approaches. The Buongiorno model studies the convective transport of nanofluid which involves the Brownian diffusion and thermophoresis. It is called the two-phase model as the Buongiorno model analysed the heat transfer process separately between based fluid and the nanoparticles where based fluid and nanoparticles are not in the same thermal equilibrium state. On the other hand, Tiwari and Das model is known as the single-phase model where the base fluid and nanoparticles are in thermal equilibrium state and the velocity flows in the same local velocity.

In this study, the Tiwari and Das model is used to model the problem of Bödewadt flow and heat transfer in nanofluid and hybrid nanofluid as the considered problem is a single-phase flow where the nanoparticles and base fluid are in thermal equilibrium.

2.4 Numerical Methods in Solving Bödewadt Flow Problem

In the areas of mathematics and computer sciences, numerical analysis method has been used in solving complicated problems of continuous variables. It creates, analyses and implements algorithm that can be used to solve problems that occur in many applications such as engineering, natural sciences, business and medicine. Thus, techniques that are used to obtain the approximate solutions are called numerical methods.

There are many numerical methods that have been used by researchers to solve the Bödewadt flow problem. Runge-Kutta method is one of the classical methods that is effective and widely used in solving initial value problem (IVP), hence, it needs to be used together with other method, for example, shooting method, in order to solve boundary value problem (BVP). It is also called as four-order Runge-Kutta method as it needs four values of function for each step iteration. The ordinary differential equation is then solved into the first-order differential equation where its initial values is solved to acquire the computation formula.

In solving nonlinear BVP, shooting method can be used where this method transforms the BVP into an IVP where the initial parameter needs to be established. Thus, applying Runge-Kutta method first helps in solving the IVP. The shooting method has limited advantage as good initial guesses is required at the single point while the Runge-Kutta method has a big disadvantage where extensive computational is required though the accuracy is still comparable with other numerical method. The

Runge-Kutta method with shooting technique has been used by Joshi et al. (2017) and Ramesh et al. (2020) to solve the Bödewadt flow problems.

Finite difference, finite element and finite volume methods are some of the numerical methods that have been used intensively by many authors to solve the BVP. Finite difference method is the most common method to solve the BVP. This method reposes the principle of discretization where the derivatives in the differential equations are substituted by the finite divided differences. The resulting set of discretized equations then solved by simple algebraic arithmetic.

On the other hand, finite element method subdivides a shape into finite-sized elements of geometrically simple shapes which are called finite-element mesh. The systems of partial differential equation are being solved within each element where the solved equations from each elements assembled and form a set of matrix equation system. For finite element method, the results are known in every point of the grid (elements).

Next, finite volume method is also known as control volume method where the basic ideas of this method is the same as finite element and finite difference method which is to acquire a set of discrete equations. To obtain the set of discrete equations, calculation area needs to be divided into a series of overlapping control volume where the control volume is contained in each grid point. Then, for each integral of control volume, the differential equation needs to be solved. This finite volume method can be assumed as the intermediate between the finite element method and the finite difference method. (Christara and Jackson, 2005).

From the literature review, a group of researchers have been using the finite differences method in solve the Bödewadt flows problem, for instance, Sahoo (2011), Sahoo and Poncet (2012), Sahoo et al. (2014), Sahoo and Shevchuk (2019) had study the Bödewadt flow of viscous fluid and Reiner-Rivlin fluid subject to different boundary conditions. They integrated the nonlinear equation by using the finite difference method into two parts where the semi-infinite domain is replaced by finite domain. This method has been chosen as it converges and gave more accurate solving problems and more stable than other method.

Next, other numerical method that can be found is Keller's Box method where this method is often used in solving boundary layer problems especially in fluid mechanics. This method is designed to solve the algorithm in large size of matrix. In Keller's Box method, the differential equations are reduced to the system of first order differential equations by applying implicit finite difference schemes. This implicit finite difference scheme is conjunction with Newton's linearization to produce a tridiagonal matrix which the entries are expressed in blocks. The advantages of Keller's Box method are, it is simple, easy to program and efficient. The Keller's Box method is the most common method that have been used by many researchers in solving boundary flow problem. Mustafa et al. (2015) applied this method in the Bödewadt flow and heat transfer problem. Other researchers such as Sahoo et al. (2014) has applied this Keller's Box method to study the Bödewadt flow with Navier slip boundary conditions while Mustafa et al. (2015) present the Bödewadt flow and heat transfer of nanofluids over stretching stationary disk by using this Keller's Box method.

Recently, a BVP solver which is known as bvp4c in MATLAB software has gained interest among the researchers to solve the boundary value problems efficiently. This method is convenient and easy to use in solving sophisticated problems. The bvp4c solver is a fourth-order method that solve the boundary value problems where the condition of the problem can be involved with two-point condition, multipoint boundary conditions, singularities of solution or any unknown parameters. Surprisingly, this bvp4c method is a finite-difference code that applies the three stage Lobatto IIIa formula. Since this method rely on the iteration schemes, the effectiveness of this method will ultimately rely on the ability of the user to provide the initial guesses for the solution. Examples of researchers that being used bvp4c method are Rahman and Andersson (2017) and Rafiq and Hashmi (2019), where they studied the problem of Bödewadt flow and heat transfer in Newtonian fluid and non-Newtonian fluid, respectively.

There is another technique called the spectral technique that has been used to solve the classical Bödewadt flow problem over a uniformly stretching disk by Turkyilmazoglu (2015). The Chebyshev collocation method had been used to solve the velocity and temperature profile in computational procedure. This method is conveniently illustrated by single first-order for linear differential equation as it gives essential points without any unnecessary complication. However, it needs more work to solve the non-linear differential equations. Moreover, this method is reliable for IVPs but not for the convergence in boundary problems. Taking into consideration, this method does provide solution in a useful but it involves more work than conventional methods.

2.5 Motivation of Study

Motivated from the past research work by Mustafa et al. (2015), Turkyilmazoglu (2015), Rahman and Andersson (2017) and Ramesh et al. (2020), the present study aims to investigate the problem of Bödewadt flow and heat transfer in nanofluids with the presence of suction at uniform stretching disk. In this study, water is chosen as base fluid since many of the nanofluids preparation used water as based