AN ERROR ANALYSIS AND THE IMPACT OF CONTINGENT TEACHING IN SOLVING PROBLEMS INVOLVING MEASURES OF DISPERSION

NG CHIEW LENG

UNIVERSITI SAINS MALAYSIA

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by

NG CHIEW LENG

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ABSTRAK

Kepentingan pembelajaran statistik telah dibincangkan secara meluas. Walau bagaimanapun, tidak banyak usaha yang telah dibuat untuk memahami kesukaran pelajar dalam pembelajaran sukatan serakan yang merupakan komponen utama dalam pembelajaran statistik. Dengan menggunakan kaedah campuran penyelidikan multifasa, kajian ini dibahagikan kepada tiga fasa. Fasa pertama yang melibatkan 85 pelajar sekolah menengah dengan berdasarkan persampelan kluster bertujuan untuk meneliti dan mengkategorikan jenis kesalahan yang dilakukan oleh pelajar dalam menyelesaikan masalah yang melibatkan sukatan serakan. Seterusnya, temuduga mendalam berdasarkan ‘contingent teaching’ model dijalankan pada fasa kedua bersama dengan sepuluh pelajar berprestasi rendah untuk memahami sebab-sebab kasalahan. Dalam fasa ketiga, kajian ini memberi tumpuan kepada keberkesanan “contingent teaching” dalam pencapaian para pelajar dalam menyelesaikan masalah mengenai sukatan serakan. Kajian menunjukkan bahawa kesalahan pelajar dalam menyelesaikan masalah yang berkaitan dengan sukatan serakan adalah dissebabkan kekurangan pengetahuan perbendaharaan kata statistik, pengetahuan simbol yang lemah, pembelajaran secara hafalan, dan kelemahan dalam penaakulan statistik serta kebolehan statistik. Kajian ini mendedahkan bahawa “contingent teaching” adalah berkesan (nilai $\boldsymbol{t} = \mathbf{4}.62$ dengan $\rho < 0.05$) untuk memahami pemikiran konseptual

pelajar dan dapat membantu mereka untuk mengurangkan kesilapan dalam penyelesaian soalan mengenai sukatan serakan. Pihak berkaitan dan perancang kurikulum dalam bidang statistik harus melingkungi pembelajaran perbendaharaan kata dan simbol statistic, serta meningkatkan penaakulan statistik dan kebolehan statistik pelajar, bukannya pembelajaran secara hafalan di semua peringkat. Kajian ini menyumbang kepada ketandusan literatur dalam bidang pendidikan statistik dengan memberikan contoh daripada konteks pendidikan Malaysia.

AN ERROR ANALYSIS AND THE IMPACT OF CONTINGENT TEACHING IN SOLVING PROBLEMS INVOLVING MEASURES OF DISPERSION

ABSTRACT

The importance of statistical learning has been widely discussed. However, little effort has been made to understand the difficulties of students in learning measure of dispersion, which is the key component to statistical learning. Employing multiphase mixed method, the present was divided into three phases. The first phase which involved 85 high school students using cluster sampling was sought to first examine and categorize the kinds of errors students committed in solving problems involving measures of dispersion. Subsequently, in depth interviews based on the model of contingent teaching were carried out in the second phase with ten low performing students in order to understand the reasons why students have the errors in solving problems involving measures of dispersion. In the third phase, this study focused on determining if there is a significant difference in the achievement of the students in solving problems regarding measures of dispersion before and after going through contingent teaching. The findings indicated that students committed errors in solving problems related to measure of dispersion due to lacking statistical vocabulary knowledge, weak symbol sense, rote learning, low statistical reasoning and statistical thinking ability. This study revealed that contingent teaching is effective ($\boldsymbol{t}$-value $= \mathbf{4.62}$ with $\boldsymbol{\rho} < \mathbf{0.05}$) in understanding students' conceptual thinking and further helping them to reduce their error in measures of dispersion. Instructional designers and curriculum planners in the field of statistics should take into account statistical

vocabulary and symbols as well as pedagogy to improve students' statistical reasoning and thinking rather than rote learning at all levels. This study contributes to the dearth of literature in the field of statistics education by providing an example from the Malaysian educational context.

CHAPTER 1

INTRODUCTION

1.1 Background of the study

Learning statistics has become increasingly important over the last two decades. There's a lot of value placed on statistical techniques in today's professional fields, including medicine, finance and engineering as well as social science and more (Wackerly et al., 2007). Citizens with data literacy are critical thinkers who are capable of critically evaluating and understanding statistical information, as well as developing their own intuitions about data and making logical judgments and decisions (Rumsey, 2002; Utts, 2003).

To ensure a rapid and long-term change to Malaysia's education system, the Malaysian Ministry of Education launched the Malaysia Education Blueprint 2013-2025 in 2011. For example, students will be taught to think critically and hypothesize in the new math curriculum outlined in the Blueprint. Teachers are encouraged to use lab work, student-led inquiry, and ICT games as instructional tools to support project-based and inquiry-based learning. The Ministry of Education in Malaysia is revising the educational system in order to equip students with the knowledge, skills, and attitudes necessary to succeed in the modern workforce.

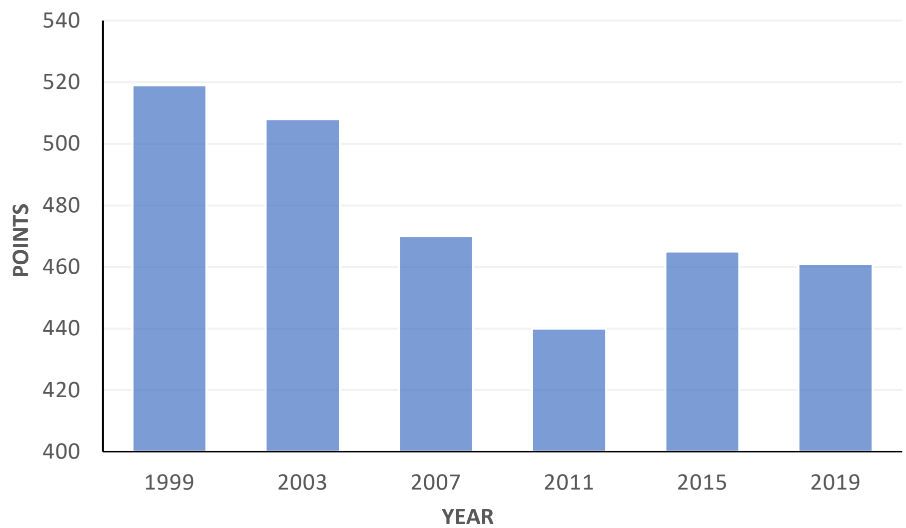
Quantitative information and numerical data can be found everywhere in today's increasingly data-driven and technological world. All students should learn statistics as part of their educational program in order to be able to provide good evidence-based arguments and to critically evaluate data-based claims (Watson, 2006).

Demand for high-level cognitive skills in the United States will rise by 9% in 2030, and Western Europe's demand will rise by 7% during the same period, according to the report (McKinsey Global Institute, 2018). An annual report from the McKinsey Global Institute identifies quantitative and statistical skills as one of the advanced cognitive skills that companies have prioritized to address through reskilling (McKinsey Global Survey, 2021).

It is critical that the school curricula be prepared and aligned with the development of the workforce in order to develop higher cognitive skills. The International Association for the Evaluation of Educational Achievement (IAEEA) conducts an annual international benchmarking study known as the Trends in International Mathematics and Science Study (TIMSS). Every four years, a survey is carried out. Since 1995, TIMSS has tracked changes in fourth- and eighth-grade math and science achievement. Data from TIMSS assessments include information on curricula and curriculum implementation, teaching practices, and school resources. Malaysia ranked 28th in TIMSS 2019 with an average student score of 461 points, a slight decrease from 465 points in 2015 (Ministry of Education Malaysia, 2019). In mathematics, the average eighth-grade student passed the low benchmark but fell short of the international intermediate benchmark. The low mathematics benchmark indicates that students have a limited grasp of mathematical principles and concepts. As shown in Figure 1.1, the overall performance of students in Malaysia has revealed a plummeting trend over the course of the last seven test cycles.

Figure 1.1

Malaysian students' mathematics scores in TIMSS (1999 – 2019)



According to the Preliminary Report of the Malaysian Education Blueprint 2013-2025, students will be taught to think mathematically and apply their mathematical knowledge in solving problems and making decisions (PADU, 2013). However, Program for International Student Assessment (PISA) in 2009 found that nearly 60% of Malaysian students didn't meet the minimum standard in mathematics required for students to participate effectively and productively in society. In Reading, Mathematics, and Science, only a tiny fraction of students (i.e. 0.1%) achieve proficiency at the OECD's highest level, a figure that lags far behind other countries in the group (where almost 8% perform at this level). Based on the PISA and TIMSS assessments in year 2009 and 2019 respectively, Malaysian students are lack in higher-order thinking skills as the test questions measured students' ability in solving non-routine questions (PADU, 2013).

Table 1.1

Proficiency Levels in Mathematics Defined by PISA (PADU, 2013)

Proficiency level	Definition
Minimum level	Students lack knowledge of fundamental formulas, procedures, and conventions. They cannot use direct reasoning or interpret the results literally, but they are able to respond to clearly worded questions about familiar situations.
Advanced level	Students are able to comprehend and process information that is more complex through a series of steps. They demonstrate insight when selecting an effective solution to a problem and use other higher-order cognitive processes to explain or discuss the outcomes.

The study of statistics equips students with the knowledge and skills they need to make sense of the quantitative data they encounter on a daily basis (Garfield et al., 2008). Prior studies have found that students and adults alike have faulty reasoning and misconceptions when it comes to statistics, despite the fact that statistics is becoming increasingly important in today's world. (Garfield, 2003; Hirsch & O'Donnell, 2001; Konold et al., 1993). A series of studies conducted at four universities found that the majority of students lacked basic statistical knowledge (Clark et al., 2007; Mathews & Clark, 2007).

Students are at the beginning stages of developing an interest in and understanding of statistics in high school at the level of grades 7 to 12 (Watson, 2006). Therefore, it is essential to investigate the statistical reasoning skills of high school students and the root causes of various errors involving measures of dispersion. This study first investigated the errors committed by students in the statistical measures of dispersion. Secondly, the students’ errors in the measures of dispersion were classified.

Finally, the study investigated the effectiveness of contingent teaching as a scaffolding technique in helping students to eliminate their errors.

1.1.1 Context of the study

The sample in this study is two classes of 85 grade 11 students at a private Chinese high school in Georgetown, Penang. In the first phase, the 85 students were given a diagnostic test to find out the errors that students might have on the measures of dispersion. Students' errors on measures of dispersion were identified and categorized. In the second phase, ten students with weak performance (with pretest score lower than the mean mark of 10.67) were selected from the sample of 85 students to undergo an individual in-depth interview by the researcher. Each student was given the same diagnostic test and researcher questioned and interviewed the students in order to elicit their thinking on the errors. Along the way, contingent teaching was carried out during the test to help the students overcome their errors. The second phase of the study sought to achieve this objective by analyzing students' responses to test questions, which were indicative of the nature of students' difficulties, a microstructure of their understanding of the problem-solving process, and the reasons for their errors. This were accomplished through the application of contingent teaching during the interview, where appropriate scaffoldings were provided. These scaffoldings not only enabled a student to continue advancing towards the solution of the problems, but they also aid the researcher in comprehending the nature of perceived difficulties and the causes of errors. The interview of each students took about 1 hour.

Private Chinese high schools represent a small number of high schools in Malaysia. The number of private Chinese high schools differed among sources, ranging from 60 to 63, due to the ambiguous status of SM Chong Hwa Kuantan and whether branch campuses count as separate schools. In year 2020, UCSCAM adopted the "60+2+1" formula in describing the number of private Chinese high schools (Dong Zong, 2020). There were about 80,000 students studying at the private Chinese high schools in Malaysia as at August, 2022 (Dong Zong, 2022). The Curriculum Department of United Chinese School Committees' Association of Malaysia (UCSCAM) plans and coordinates the curriculum used in the private Chinese high schools. with reference to secondary education curricula around the world, particularly Malaysia's national secondary education curriculum and those of mainland China as well as Taiwan. They achieve this by analyzing secondary education curricula from around the world, specifically the national secondary education curricula of Malaysia, mainland China, and Taiwan. The curriculum design of the mathematics subject taught in private Chinese high schools enables students to acquire the fundamentals and fundamental skills of mathematics, laying the groundwork for advanced mathematics. Consequently, they acquire the ability to solve problems involving quantity, measure, and number, among others. In addition, mathematical evaluation fosters a scientific mindset and develops logical reasoning, not to mention skills such as inquiry, mathematical evaluation, regular pattern recognition, inference, and expression. Additionally, students can use digital technology for inquiry-based activities to enhance their cognition and creativity (Dong Zong, 2022a). UCSCAM publishes textbooks for use in the private Chinese high schools. Since 1975, the UCSCAM has administered the Unified Examination Certificate (UEC), a standardized test for the private Chinese high school students. Junior Middle (UEC-JML), Vocational (UEC-V), and Senior Middle (UEC-SM) are the three UEC levels (UEC-SML) (Dong Zong, 2022). The grade 11 students of the private Chinese high schools will learn about the measures of dispersion which involves calculation of quartile deviation, quartile range, median, variance and standard deviation in the senior level mathematics curriculum.

1.1.2 Measures of dispersion

Dispersion is the degree to which a statistical distribution is stretched or compressed. Common measurements of dispersion include the variance, the standard deviation, and the interquartile range. To be able to draw conclusions, students must comprehend the fundamental ideas of statistics and develop an understanding for statistical reasoning. This has been demonstrated by past investigations (Ben-Zvi & Garfield, 2008; Pfannkuch, 2005; Wild & Pfannkuch, 1999). Psychologists and educators have recorded the numerous errors made by students and adults when reasoning about data and chance in real-world circumstances and contexts. Variation plays a crucial role in students' comprehension and application of chance (Metz, 1997), and Moore (1990) states that variation is the first step in recognising the relationship between statistics and probability. Past research on statistical reasoning indicates that both variation and distribution are essential to the study of statistics (Chance et al., 2004; Reading & Shaughnessy, 2004). Because of its essential position in statistical reasoning, the ability to reason about variation has implications for every area of statistics (Wild & Pfannkuch, 1999).

According to Wild and Pfannkuch, understanding variation in data requires an understanding of the following concepts: (1) variation is an observable reality; (2) some variation can be explained; (3) other variation cannot be explained based on current knowledge; (4) random variation is how statisticians model unexplained variance; (5) this unexplained variance can be produced in part or in whole by random sampling; (6) randomness is a convenient human construct that is used in statistical analysis; and (7) randomness is a convenient In addition, it was discovered that children are unable to learn these concepts through typical statistics instruction (Wild & Pfannkuch, 1999).

Statistical variation is fundamental to every aspect of statistical problem-solving and lies at the core of statistics (Shaughnessy, 1997; Snee, 1999). Reading & Shaughnessy (2004) found that there is an obvious conceptual misunderstanding in students' knowledge of variation. Delmas and Liu (2005) investigated the conceptualization of standard deviation among students in an introductory statistics course. In a computer-based environment, students learned to correlate the variance of values relative to the mean with the standard deviation. The analysis of student perceptions and techniques for classifying and contrasting was revealed by the data. The finding showed that students frequently transitioned from simplistic, one-dimensional conceptions of the standard deviation that disregarded variation around the mean to mean-centered conceptions that correlated frequency (density) and departure from the mean. Meletiou-Mavrotheris & Lee (2002) had undertaken a study on how different teaching methods affect students' grasp of sampling variability. The study investigated on how a teaching pedagogy focusing on data and variation centered on students’ experience could promote understanding of the stochastic nature of statistical concepts. Watson et sl. (2007) conducted in-depth interview to explore the statistical understanding of 73 school students in 6 contextual settings which included probability sampling, representation of temperature change, beginning inference, independent events, the relationship of sample and population, and description of variation. Peters (2011) presented a framework that encapsulates the complexity of reasoning about variation in a manner indicative of comprehensive understanding and describes reasoning as a combination of design-centric, data-centric, and modelling perspectives. Integrating reasoning about variation across perspectives and four elements demonstrates robust comprehension which include variational disposition, variability in data for contextual variables, variability in the relationship between data and variables, and sample size effects on variability. Further investigations entailed expository writing presents viewpoints needed to understand variation by Gould (2004), who states that variability is worthwhile of study in its own right, that the study of variability yields insights that would have been overlooked if only the data's trend were considered.

Variation is a difficult concept to teach and comprehend, hence the majority of students struggle with it (Sánchez et al., 2011). The curricula of many countries, including Malaysia, do not include the study of measures of dispersion until high school, although research results indicate the possibility of developing intuitive notions of variations in earlier grades. Students only learn statistics course in high schools and undergraduate levels which include measures of dispersion such as range, interquartile range, and standard deviation (Sánchez et al., 2011). Motivated by the disparity between the significance of variation in statistics and the paucity of research on related areas in Malaysia (Saidi & Siew, 2022), this study investigated the types and causes of errors made by students when attempting to solve problems involving measures of dispersion, as well as the efficacy of contingent teaching in assisting students to overcome these difficulties.

1.2 Statement of the problem

Variation is one of the fundamental statistical concepts taught in secondary school in order to achieve statistical literacy (Drew et al., 2022). According to Garfield and Ben-Zvi (2008), grasping the notions of spread or variability of a data set is a crucial part of comprehending the concept of distribution and is necessary for drawing statistical inferences. Past research shows that students have many misconceptions about central tendency, variability, and distribution (Ciancetta, 2007; Chan & Ismail, 2013; Ismail & Chan, 2015) and have trouble with statistical reasoning (Chan et al., 2014). The lack of emphasis on variation in traditional school mathematics curricula and textbooks may contribute to students' inability to understand variation in data and chance, as well as teachers' inadequacy in teaching statistical topics. Educators and students may be able to calculate variance and standard deviation, but they may not fully comprehend the concepts. A misconception and incomplete understanding of the measures of dispersion many cause errors in students solving statistical problems and further limit students’ understanding of learning more advance statistical topics (Delmas & Liu, 2005). Keeping a misconception is a natural part of learning and obtaining more right concepts (or expert comprehension) may require students to maintain two or more competing conceptions simultaneously (Smith et al., 1994). To rectify these misconceptions, teachers must learn what students know and believe. The perception that many teachers lack experience with statistics (Shaughnessy, 2007) adds urgency to the need for teachers and students to gain a grasp of measures of dispersion by explaining the causes of students' errors.

Some of the past studies conducted in Malaysia revealed that Malaysian students had difficulties in the learning of measures of dispersion. Chan and Ismail (2013) found that Malaysian 10th graders had misconceptions in reasoning about variance. Almost 41.5% of 10th graders surveyed answered a standard deviation question incorrectly. Chan and Ismail (2013) used a modified statistical reasoning test from Garfield and Ben-Zvi (2008) to assess students' descriptive statistics misconceptions. One of the five questions required students to compare standard deviation for two histograms and explain their reasoning. Only 10 out of 412 students (2.42%) answered correctly. The researchers concluded that tenth-grade students in Malaysia have poor descriptive statistics reasoning skills. Chan et al. (2014) revealed that in a study involving high school students who took a test on average, weighted mean, central tendency, and standard deviation, a high percentage (53.64%) of students mistook standard deviation for mean while others (9.17%) thought same frequency equals same standard deviation. In the study, students were required to compare standard deviation on two different sets of data presented on two different histograms, In another study conducted by Saidi and Siew (2022) to assess high school students' statistical reasoning, attitudes towards statistics, and statistics anxiety, the level of students' statistical reasoning was found to be low, particularly the construct of analysing and interpreting data.

Students' understanding of measure of dispersion, how it grows, and how they might use it to compare two or more distributions are largely unknown. It is difficult to calculate and explain the standard deviation as a method for measures of dispersion which could be due to the fact that students lack simple models and metaphors for the standard deviation (Reading & Shaughnessy, 2004). Misconception of measures of dispersion may prevent students from learning advanced statistics such as sample distribution, inference, and p-values (Chance et al., 2001; Saldahna & Thompson, 2002). Many studies were conducted on students' misconceptions in mathematics, but there is little research about the errors of students in solving the problems involving measures of dispersion. Problems involving the measurement of dispersion require a combination of procedural, conceptual, and practical knowledge to be solved. Successfully solving these types of problems requires a comprehension of statistical concepts and principles, as well as the terminology and procedures (i.e., equations, formulas, rules, and their interrelationships) commonly used to represent them. In addition to procedural knowledge, students must also have conceptual knowledge of statistics in order to solve a statistical problem (American Statistical Association, 2005). Crooks et al. (2019) added that conceptual comprehension of statistics necessitates an understanding of the 'why' as well as the 'how' of statistics. Students can surmount difficulties with problems involving measures of dispersion by gaining a deeper understanding of the types of errors they make when solving typical problems. This information is crucial for both the diagnosis of a student's misconceptions and the creation of individualized adaptive instruction. The purpose of this study was thus to analyze the errors of students in solving the problems involving the measures of dispersion in the first phase and to find out whether contingent teaching as a scaffolding technique help students to eliminate those errors in phase 2 and phase 3.

Students' errors and misconceptions while learning mathematics are considered as rich vehicles for uncovering their conceptions, ways of thinking, and learning difficulties (Ashlock, 2010; Nesher, 1987). An error is an erroneous response to a question (Hadjidemetriou & Williams, 2002) and caused by insufficient mastery of basic facts, concepts and skills (Legutko, 2008). A misconception is a student conception that produces a systematic pattern of error which may be due to misapplication of a rule, over- or under-generalization, or an alternative conception of the situation (Drews, 2005). Indeed, errors and misconceptions have potential to open a window onto the way how students internalize concepts and skills (Ryan & William, 2007). Errors, according to Luneta (2008), are the manifestation of misconceptions. Errors are also considered to be symptoms or indicators of a misconceptions. Swan (2001) asserts that errors may result from negligence or a misunderstanding of symbols or text. These definitions imply that majority of the errors are the result of misconceptions, and that these misconceptions are fundamental concepts that the learner ought to have grasped (Luneta, 2013). Since error analysis often reveals students' misconceptions and errors, diagnosing and breaking down students' work has become essential for teachers (Bush & Karp, 2013; Egodawatte & Stoilescu, 2015; Schnepper & McCoy, 2013). In light of the fact that analysing students' errors as informal knowledge is more productive and beneficial for learning than labelling them incorrect or failure (Smith et al., 1994), this study aimed to identify the errors made by students and the reasons for making such errors when solving tasks involving measures of dispersion.

This study also aimed to focus on the conceptual thinking of the low achieving students when solving tasks involving measures of dispersion. Mathematical difficulties for low achieving students were well-documented. Baker et al. (2002) found that low-achieving pupils are often recognised by teacher reports and standardised or informal test scores below the 50th percentile, but they are not labelled with learning difficulties. Gray et al., (2000) found that low achieving students who struggle in mathematics may have trouble recalling basic concepts. Academically these tend to assume mathematics is more about doing than thinking. Grey et al., (2000) also found that these students are sensitive to their circumstances and lack self-reflection. These students struggle more in solving mathematical problems and these difficulties may lead them to use less sophisticated strategies and commit more errors. Repeated failures and struggles to keep up with the class may demotivate students and make them passive learners. Despite their obstacles, low achieving students can enhance their math abilities. In supportive learning environments such as small group instruction, Karsenty et al. (2007) and Chazan (2000) show that low achieving students may demonstrate mathematical reasoning orally. Teachers' judgements regarding the educational strategy to employ are heavily influenced by their students' prior levels of achievement (McKown & Weinstein, 2008). A student-directed and self-regulated learning environment where the teacher acts as a guide to facilitate the students' learning process has been shown to be more effective for high-achieving students (Yoon, 2009). Low achievers, on the other hand, exhibited less learning motivation, self-control, and selfmanagement, indicating insufficient preparedness for self-directed learning; therefore, teachers were advised to be more involved in the learning processes of low achieving students (Abraham et al., 2011) This study followed a similar approach, predicated on the premise that even low-achieving students can make progress in statistics, and motivated by the desire to build on students' existing abilities as opposed to focusing solely on their deficiencies.

Many past studies on mathematics misconceptions used traditional one-tier achievement tests and aimed to identify rather than eliminate the misconceptions (Ay, 2017). Pesman (2005) states that the wrong answers given in the one-tier tests can be considered as misconceptions although they are not, which could be due to lack of knowledge, wrong information in the question or faulty thinking during the test. Pesmen asserts that tests include more than two tiers are more appropriate to be able to detect the misconceptions. In addition, qualitative data collection through interview or observation is one of the most appropriate ways to determine students' misconceptions and errors since they provide in depth information about students' knowledge. A contingent teaching approach necessitates the formulation of carefully crafted hints or escalating levels of difficulty, which may vary based on student responses. During the course of contingent teaching, scaffolding strategies provide custom-made support for the development of new skills of each individual student and are readily disassembled when no longer required. Even though the primary purpose of scaffolding is to help students channel their thinking, the same scaffolding can also serve as a tool for understanding learning disabilities, as the nature and method of scaffolding used by a student will reflect their thought process. Scaffolding helps students develop and refine the problem-solving skills they need, although its efficacy is highly context-dependent (Hegde & Meera, 2012).

The aim of the research on errors in mathematics should move beyond just determining them. First of all, effective instructional methods were required to prevent the arising of the errors and eliminate the present errors. Before this, it is also important to detect the errors using correct tools. Kingsdorf & Krawec (2014) assert that the implementation of error analysis has primarily been procedural in nature; therefore, qualitative analysis, such as in-depth interviews, has supported conceptual analysis. These findings support the objective of this research that mathematical errors should extend beyond merely identifying them. First of all, it is essential to detect the errors using the appropriate strategies. Then, effective instructional methods were required to prevent the occurrence of errors and eradicate those that had already occurred. Therefore, the design of this study administered the pre- and post-tests to first identify students' errors in measures of dispersion, followed by in-depth interviews conducted with the selected low-achieving students to elicit their thinking, and lastly the determination of contingent teaching as an effective method for assisting students in eliminating the errors.

1.3 Research objectives

The objectives of this study are:

1. To examine and categorize the kinds of errors students committed in solving problems involving measures of dispersion.
2. To understand the reasons why students have the errors in solving problems involving measures of dispersion.
3. To determine if there is a significant mean difference in the achievement of Grade 11 students in solving problems regarding measures of dispersion before and after going through contingent teaching

1.4 Research questions

1. What are the kinds of errors students committed in solving problems involving measures of dispersion?
2. What are the reasons the students have the errors in solving problems involving measures of dispersion?
3. Is there a significant mean difference in the achievement of Grade 11 students in solving problems regarding measures of dispersion before and after going through contingent teaching?

1.5 Null Hypothesis

To answer the above research question (3), the following null hypothesis was evaluated:

H_0 : There is no significant mean difference in the achievement of Grade 11 students in solving problems regarding measures of dispersion before and after going through contingent teaching.

1.6 Significance of the study

This study, which focus primarily on measures of dispersion, will contribute to the present body of knowledge regarding the learning of higher order thinking skills by students. In addition, the outcome of the research will aid in determining the nature of students' challenges and errors when addressing issues requiring measures of dispersion. This will provide a key to improving student achievement and contribute to the theoretical understanding of statistical learning.

Secondly, the findings of this study will inform the educators about the effectiveness of probing learners as a scaffolding technique. The teacher and student engage in a conversation using probing to stimulate the student's higher-order, critical thinking about a task at hand. Probing the students will enable them to think about the tasks relationally. As Skemp (1976) emphasizes that relational understanding is the only thing that will ever be sufficient for a student to improve. Even though many schools in Malaysia have brought in new ideas and innovations, there are lack of professional development programs to prepare teachers for their new role. The findings of the present study can contribute to the development of such program. It will help teachers not only understand how scaffolding works, but also determine what each student needs and how they comprehend concepts.

The current findings can also be applied to student development. The quantity and quality of information a teacher obtains from students depends not only on the teacher and whether diagnostic tools are used in interactions with students, but also on the students' willingness and capacity to discuss their knowledge and concerns. Future research and development may therefore find instructing students on how to better convey what they already know and what questions they have to teachers to be a highly beneficial topic. Finally, since this study demonstrates that determining a student's level of understanding is a crucial component of contingent teaching, future research on scaffolding should focus not only on the assistance a teacher provides, but also on how to determine the methods of preceding diagnosis on how much a student already knows and understands.

1.7 Limitation of the study

There are a few of limitations involved with this study. Due to time and expense constraints, a random sample cannot be taken from each school. Given the small sample size, i.e., from only one school, the external validity of the findings could be questioned. Consequently, the findings cannot be generalized to all schools in the nation. However, as the primary purpose of the study is not to generalize the results, keeping the number of participants small will allow the researcher to examine the students' errors and the scaffolding process, in great depth.

Since there is no universally accepted instrument for measuring scaffolding, its measurement is the most challenging aspect of study. Extensive scaffolding research conducted over the past decade has provided light on its appearance, but its utility and method remain unknown. Assessing the qualities of scaffolding will remain difficult. Sometimes it is unclear whether an interaction is contingent or independent. This study video recorded the interview process and the video observation were taken for analysis of scaffolding interactions.

Thirdly, this study focuses solely on errors in measures of dispersion, but other statistical subjects such as mean is included. In light of the aims of the study, which are to elicit students' thought processes in committing errors and to determine the efficacy of scaffolding, it is more acceptable to focus on only measures of dispersion so that the information acquired is comprehensive.

1.8 Assumptions

This study was conducted with the assumption that all students who provided answers and feedback from the survey acted honestly and described their true feeling on difficulties towards solving of problems involving measures of dispersion.

1.9 Definition of terms

1.9.1 Errors

Error derives from the Latin verb "errare," which means to err. The definition of an error is a simple lapse in care or concentration, which nearly everyone commits on occasion. In mathematics, an error is the departure from a problem's correct solution. An error is a mistake made when solving a mathematical problem algorithmically, procedurally, or using any other method. Errors may be found in incorrectly answered problems where the process that generated the answers was flawed (Richard & Tim, 1978). Errors are systematic, persistent, and pervasive errors made by students in a variety of contexts (Nesher, 1987). Errors are consequently mistakes made by learners as a result of a lack of concentration control or a faulty memory, and they indicate a lack of knowledge.

1.9.2 Misconceptions

Leinhardt et al. (1990) defined misperception as learners' inaccurate, repeated, and explicit knowledge. Nesher (1987), on the other hand, defined misconception as "a line of thinking that leads to a series of errors all stemming from an incorrect underlying premise, as opposed to random, unconnected, and non-systematic errors". Neshers considers misconceptions to be systematic errors that recur whenever the same type of problem is presented. A misconception is a learner-constructed conceptual structure that makes sense in light of the student's current knowledge but is not consistent with conventional mathematical knowledge. A misconception is a student's conception that results in a pattern of systematic errors.

1.9.3 Error analysis

Error analysis, also known as error pattern analysis, is the study of errors in learners' work with the purpose of determining the causes of these reasoning errors. The multifaceted activity dates to the work of Radatz in 1979. Error analysis focuses on the pervasive errors made by students due to a lack of conceptual or procedural understanding (Ketterlin-Geller & Yovanoff, 2009). Error analysis entails not only the analysis of learners' correct, partially correct, and incorrect solution-finding steps, but also the investigation of best practices for remediation (McGuire, 2013)

1.9.4 The model of contingent teaching

The model of contingent teaching was developed by Van de Pol et al. (2011). It integrates the four essential aspects of the scaffolding process and provides a step-by-step operationalization of the theoretical concept of scaffolding. In accordance with the model of contingent instruction, teachers must first determine what students already know. This can be accomplished, for instance, by asking diagnostic questions or reading student work. Second, teachers can confirm with students that they comprehend their diagnosis. The teacher can then offer conditional support to the student based on the information gathered. Finally, the teachers can assess the students' new (potential) understanding or the extent of their learning.

1.9.5 Scaffolding techniques

Scaffolding can be defined as "the process that enables a child or novice to solve a problem, complete a task, or achieve a goal that would be unattainable without assistance" (Wood et al., 1976). Wood et al. (1976) defined scaffolding as an interactive system of exchange in which the tutor operates with an implicit theory of the learner's acts to recruit his attention, reduces degrees of freedom in the task to manageable limits, maintains 'direction' in problem solving, marks critical features, controls frustration, and demonstrates solutions when the learner is able to recognize them. Scaffolding refers to the provision of temporary assistance for the completion of a task that learners might not otherwise be able to complete. This assistance can be provided in a variety of ways, such as through modelling and the posing of questions for various subjects and ages.

1.9.6 Low achieving students

Baker et al. (2002) identified low-achieving students as those who scored below 50 percentile in a standardized or informal test, but they are not labelled with learning difficulties.

1.9.7 Statistical literacy

Statistical literacy is the ability to understand and use the language and tools of statistics. This includes knowing what basic statistical terms mean, how to use simple statistical symbols, and being able to recognize and understand different ways of showing data (Garfield & Gal, 1999; Rumsey, 2002). It is a key skill that people in information-rich societies are expected to have, and it is often pushed as a result of schooling and as a part of adults' numeracy and literacy (Garfield & Ben-Zvi, 2008).

1.9.8 Statistical reasoning

Statistical reasoning is the manner in which individual reason with statistical concepts and make sense of statistical data. Statistical reasoning may involve making connections between concepts or combining data and chance concepts. Understanding and being able to explain statistical processes and interpreting statistical results also constitute statistical reasoning (Garfield, 2002).

1.9.9 Statistical thinking

Statistical thinking is the professional mode of thought employed by statisticians (Wild & Pfannkuch, 1999). It is the knowledge of how and why to apply a specific method, design, measurement, or statistical model. It also requires a comprehensive understanding of the theories underlying statistical processes and techniques; and being aware of the limitations and constraints of statistics and statistical inference. Understanding the use of statistical models to simulate random phenomena and the production of data to estimate probabilities also constitutes statistical reasoning. In addition, statistical reasoning involves recognizing how, when, and why existing inferential tools can be applied, as well as being able to comprehend and apply the problem's context to plan and evaluate investigations and draw conclusions (Chance, 2002).

1.9.10 Variation and Variability

Recent research indicates that the terms variation and variability are sometimes interchangeable, but a closer examination reveals a distinction (Reading & Shaughnessy, 2004). According to multiple dictionaries, variation is a noun used to describe the act of varying or changing a condition, and variability is a noun form of the adjective variable, which means that something is likely or susceptible to vary or