ECONOMIC AND ECONOMIC-STATISTICAL DESIGNS OF VARIABLE SAMPLE SIZE AND SAMPLING INTERVAL COEFFICIENT OF VARIATION CHART AND DEVELOPMENT OF VARIABLE SAMPLE SIZE MULTIVARIATE COEFFICIENT OF VARIATION CHART FOR SHORT RUNS

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ARL	Average run length
ARL_0	In-control average run length
ARL_1	Out-of-control average run length
ASI_0	In-control average sampling interval
ASI_1	Out-of-control average sampling interval
ASS	Average sample size
ASS_0	In-control average sample size
ASS_1	Out-of-control average sample size
ATS	Average time to signal
ATS_0	In-control average time to signal
ATS_1	Out-of-control average time to signal
cdf	Cumulative distribution function
CUSUM	Cumulative sum
CV	Coefficient of variation
CV^2	Squared of the CV statistic
DOE	Design of experiments
EARL	Expected average run length
$EARL_0$	In-control expected average run length
$EARL_1$	Out-of-control expected average run length
EATS	Expected average time to signal
$EATS_0$	In-control expected average time to signal

$EATS_1$	Out-of-control expected average time to signal
EWMA	Exponentially weighted moving average
HWMA	Homogeneously weighted moving average
KOSPI	Korean Composite Stock Price Indexes
LCL	Lower control limit
LWL	Lower warning limit
MCV	Multivariate coefficient of variation
RS	Run Sum
SDRL	Standard deviation of the run length
SH	Shewhart
SPC	Statistical Process Control
SPR	Short production run
SQC	Statistical Quality Control
SSMGR	Side-sensitive modified group runs
Syn	Synthetic
TARL	Truncated average run length
TSDRL	Truncated standard deviation of the run length
tpm	Transition probability matrix
UCL	Upper control limit
UWL	Upper warning limit
VSI	Variable sampling interval
VSS	Variable sample size
VSSI	Variable sample size and sampling interval

LIST OF NOTATIONS

n	Fixed sample size of the SH CV and SH MCV charts
n^*	Optimal sample size of the SH CV chart
n_0	Initial sample size
n_1	Small sample size
n_1^*	Optimal small sample size for the VSSI CV chart
n_2	Large sample size
n_2^*	Optimal large sample size for the VSSI CV chart
h	Fixed sampling interval of the SH CV and SH MCV charts
h^*	Optimal sampling interval of the SH CV chart
h_0	Initial sampling interval
h_1	Short sampling interval
h_1^*	Optimal short sampling interval for the VSSI CV chart
h_2	Long sampling interval
h_2^*	Optimal long sampling interval for the VSSI CV chart
K	Control limit's parameter of the VSSI CV chart
K^*	Optimal control limit's parameter of the VSSI CV chart
W	Warning limit's parameter of the VSSI CV chart
W^*	Optimal warning limit's parameter of the VSSI CV chart
γ	Population CV or population MCV
γ_0	In-control population CV or in-control population MCV

γ_1	Out-of-control population CV or out-of-control population MCV
$\hat{\gamma}$	Sample CV or sample MCV
$\hat{\gamma}_0$	In-control sample CV or in-control sample MCV
μ_0	In-control mean
σ_0	In-control standard deviation
τ	Size of a CV or MCV shift
$\tau_{\min}$	Lower bound of τ
$\tau_{\max}$	Upper bound of τ
τ'	Incorrect shift size
α	Type-I error probability
α'	Warning limit's parameter of the VSS MCV SPR chart
p	Number of quality characteristics monitored simultaneously
P(I)	Probability of getting a signal within the I inspections
$\mathbf{P}$	Transition probability matrix with the transient and absorbing states
$\mathbf{b}$	Initial probability vector for CV chart
$\mathbf{q}$	Initial probability vector for MCV chart
$\mathbf{I}$	Identity matrix
$\mathbf{Q}$	Transition probability matrix with the transient states
$\boldsymbol{h}$	Vector of sampling intervals
$\mathbf{1}$	Vector with all elements unity
I	Number of samples scheduled for inspection during the short production run

β	Type-II error probability
M	Probability of an out-of-control signal on the SH CV chart
δ	Non-centrality parameter for the non-central F distribution
$f_\tau(\tau)$	Probability density function of τ
$F_{\hat{\gamma}}(\cdot)$	Cumulative distribution function of $\hat{\gamma}$
$U(\cdot)$	Uniform distribution
$F_{\hat{\gamma}}^{-1}(\cdot)$	Inverse cumulative distribution function of $\hat{\gamma}$
$F_t\left(\cdot \left\| n-1, \frac{\sqrt{n}}{\gamma}\right.\right)$	Cumulative distribution function of a noncentral t random variable with n – 1 degrees of freedom and non-centrality parameter $\frac{\sqrt{n}}{\gamma}$
$F_t^{-1}\left(\cdot \left\| n-1, \frac{\sqrt{n}}{\gamma}\right.\right)$	Inverse cumulative distribution function of a noncentral t random variable with n – 1 degrees of freedom and non-centrality parameter $\frac{\sqrt{n}}{\gamma}$
$F_F(\cdot \mid p, n-p, \delta)$	Cumulative distribution function of a noncentral F distribution with p and $n-p$ degrees of freedom and noncentrality parameter δ
$F_F^{-1}(\cdot \mid p, n-p, \delta)$	Inverse cumulative distribution function of a noncentral F distribution with p and $n-p$ degrees of freedom and noncentrality parameter δ
C	Cost function
C^*	Minimum cost
b	Fixed cost per sample

c	Cost per unit sampled
m	Expected time to sample and interpret one unit
s	Expected number of samples taken before an assignable cause occurs
C_0	Expected in-control quality cost
C_1	Expected out-of-control quality cost
λ	Process failure rate
Y	Cost of a false alarm
ϕ_1	= 1 if production continues during search = 0 if production stops during search
ϕ_2	= 1 if production continues during repair = 0 if production stops during repair
D	Cost of removing an assignable cause
T_0	Expected search time for a false alarm
T_1	Expected time to find the assignable cause
T_2	Expected time to repair the process
C^*_{SH}	Minimum cost of the SH CV chart
C^*_{VSSI}	Minimum cost of the VSSI CV chart

REKABENTUK EKONOMI DAN EKONOMI-BERSTATISTIK CARTA PEKALI VARIASI DENGAN SAIZ SAMPEL DAN SELANG PENSAMPELAN BERUBAH-UBAH DAN PEMBANGUNAN CARTA PEKALI VARIASI MULTIVARIAT DENGAN SAIZ SAMPEL BERUBAH-UBAH UNTUK LARIAN PENDEK

ABSTRAK

Carta kawalan adalah salah satu alat kawalan proses berstatistik yang sangat berguna dan telah diaplikasikan untuk pemantauan proses dalam pelbagai bidang. Carta kawalan tradisional adalah tidak berkesan dalam pemantauan proses apabila proses yang dipantau tidak mempunyai min dan varians proses yang tak bersandar antara satu sama lain. Dalam keadaan sedemikian, pekali variasi (CV) digunakan dalam pemantauan proses, yang mana nisbah sisihan piawai kepada min dipantau. Carta CV dengan saiz sampel dan selang pensampelan berubah-ubah (VSSI CV) telah dibuktikan lebih berkesan daripada carta CV Shewhart (SH CV) dalam penyelidikan sedia ada tetapi hanya daripada segi prestasi berstatistik. Oleh itu, objektif pertama tesis ini adalah untuk mengkaji prestasi ekonomi dan ekonomi-berstatistik carta VSSI CV. Dengan meminimumkan kos melalui rekabentuk ekonomi dan ekonomi-berstatistik, carta VSSI CV boleh digunakan dalam keadaan yang lebih ekonomik. Prestasi ekonomi dan ekonomi-berstatistik carta VSSI CV dikaji dengan menggunakan contoh berangka, yang mana perbandingan dengan carta SH CV dilakukan. Keputusan menunjukkan bahawa carta VSSI CV adalah lebih baik daripada carta SH CV, daripada segi prestasi ekonomi dan ekonomi-berstatistik. Kajian dengan penggunaan saiz anjakan yang salah juga dijalankan untuk mengkaji kesan penetapan saiz anjakan

yang salah ke atas kos optimum. Berkenaan perkara yang berlainan, fenomena larian pengeluaran pendek (SPR) adalah amat biasa dalam industri yang melibatkan fleksibiliti yang tinggi dan kepelbagaian bahagian atau proses dalam pembuatan. Hanya wujud penyelidikan yang terhad tentang carta kawalan multivariat untuk larian pengeluaran pendek dalam penyelidikan sedia ada. Oleh itu, objektif kedua tesis ini adalah untuk mencadangkan dua carta adaptif satu-sisi untuk pemantauan CV multivariat (MCV) dalam larian pengeluaran pendek dengan mengaplikasikan saiz sampel berubah-ubah (VSS). Dua carta adaptif satu-sisi yang dicadangkan adalah carta VSS MCV SPR sisi-atas dan sisi-bawah, masing-masing untuk pemantauan anjakan meningkat dan menurun dalam proses MCV. Dengan menggunakan pendekatan rantaian Markov, parameter optimum carta VSS MCV SPR yang dikira untuk meminimumkan nilai panjang larian purata terpenggal (TARL) di luar kawalan untuk pelbagai saiz anjakan dipaparkan. Carta VSS MCV SPR yang dicadangkan mempunyai prestasi yang lebih baik daripada carta bukan adaptif Shewhart (SH) MCV SPR kerana carta yang terdahulu mempunyai nilai TARL di luar kawalan yang lebih kecil daripada carta yang terkemudian untuk saiz anjakan yang sama. Kepekaan carta VSS MCV SPR daripada segi kebarangkalian untuk memberikan isyarat dalam I pemeriksaan juga dikaji, yang mana carta VSS MCV SPR mengatasi carta SH MCV SPR. Suatu contoh aplikasi untuk carta VSS MCV SPR diberikan.

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ABSTRACT

Control charts are one of the most useful statistical process control tools that have been adopted for process monitoring in numerous fields. Traditional control charts are ineffective in process monitoring when the process being monitored does not have the process mean and variance that are independent of one another. Under such a circumstance, the coefficient of variation (CV) is used in process monitoring, where the ratio of the standard deviation to the mean is monitored. The variable sample size and sampling interval CV (VSSI CV) chart was shown to be more effective than the Shewhart CV (SH CV) chart in the literature but only in terms of the statistical performance. Thus, the first objective of this thesis is to investigate the economic and the economic-statistical performance of the VSSI CV chart. By minimizing the cost via the economic and economic-statistical designs, the VSSI CV chart can be implemented more economically. The economic and economical-statistical performance of the VSSI CV chart is studied using numerical examples, where comparisons with the SH CV chart are made. The results show that the VSSI CV chart outperforms the SH CV chart, in terms of both economic and economic-statistical performance. A study on the misspecification of the shift size is also conducted to study the effect of wrongly specifying the shift size on the optimal cost. On a different matter, the short production runs (SPR) phenomenon is common in industries that involve high flexibility and a variety of different parts or processes in manufacturing. Only a limited amount of researches on multivariate control charts in short production runs exist in the literature. Hence, the second objective of this thesis is to propose two one-sided adaptive charts for monitoring the multivariate CV (MCV) in short production runs by varying the sample size (VSS). The proposed two one-sided adaptive charts are the upper-sided and lower-sided VSS MCV SPR charts, for monitoring increasing and decreasing shifts, respectively, in the process MCV. By using the Markov-chain approach, the computed optimal parameters of the VSS MCV SPR charts in minimizing the out-of-control truncated average run length (TARL) value, for various shift sizes, are presented. The proposed VSS MCV SPR charts outperform their non-adaptive Shewhart (SH) MCV SPR counterparts, as the former have a smaller out-of-control TARL value than the latter, for the same shift size. The sensitivity of the VSS MCV SPR charts, in terms of the probability of signaling an alarm within *I* inspections is also studied, where the VSS MCV SPR charts surpass their corresponding SH MCV SPR counterparts. An example of implementation for the VSS MCV SPR charts is provided.

CHAPTER 1

INTRODUCTION

1.1 Introduction to Statistical Quality Control

Statistical Quality Control (SQC) is defined as the application of statistical approaches in monitoring and sustaining the product and service quality across a wide range of industries. The SQC techniques are extremely vital in managing the process variations which are present in most production processes as these variations will eventually affect the quality of the finished product.

Statistical Process Control (SPC) is one of the main tools in SQC. SPC is a collection of statistical methods used in analysing random samples taken from a process to ensure that the process quality meets predetermined requirements (Benneyan *et al.*, 2003). The production process will be allowed to continue if the predetermined requirements are met. Otherwise, the production process will be temporarily stopped until the cause of process variations, also known as assignable cause, is eliminated. Some examples of assignable causes are untrained workers, tool failure or failure to comply with the standard procedure. Thus, numerous SPC techniques have been used by different organizations all around the world to ensure that process quality is maintained.

Design of Experiments (DOE) is another statistical tool in SQC. DOE is a technique that systematically vary the input factors in a process to determine the effect of the input factors on the output parameters (Politis *et al.*, 2017). DOE is commonly used as an off-line tool in the early phases of a manufacturing process to manage the process quality.

Another common tool in SQC is acceptance sampling, which involves the inspection of some samples selected randomly from a group of products, and the decision to accept or reject the group of products is made according to the inspection results. The quality level of the entire group of products is determined according to the quality of the randomly selected samples. The downside of this approach happens when the selected samples fail the quality inspection and the entire batch of products is considered as rejected products. The reproduction of products to compensate for the rejected products will increase the cost of production and causes manufacturing waste. The use of SPC can help to avoid this problem by detecting the assignable causes whenever they are present throughout the production process and immediately cease the production process if assignable causes are detected. Hence, SPC is typically adopted in industries by practitioners compared to acceptance sampling in order to save production cost and reduce manufacturing waste (Montgomery, 2020).

The statistical tools mentioned above are frequently applied on two different types of data, i.e. attribute and variable data. Attribute data refer to qualitative data with quality characteristics that are countable and the outcome of an inspection can be classified into two groups, i.e. either conforming or non-conforming to product specifications. Attribute data only classify a product as good or bad, but they do not indicate how good or bad the product is (Lind *et al.*, 2017). On the contrary, variable data comprise measureable quality characteristics, such as dimensions, weight, length, etc.

In a production process, the size and structure of two similar items will not be identical as variations happen during the production process. Common causes and assignable causes of variations are two typical causes of variations. The common causes refer to the variations that happen naturally and cannot be eliminated entirely.

For instance, the changes in temperature or humidity caused by the environmental condition in a manufacturing area is uncontrollable and unavoidable. However, variations due to assignable causes are non-random and are correctable. These assignable causes can be determined accurately and removed immediately to prevent process deviation from the predetermined requirements.

The advantages of adopting SPC to monitor the process quality are obvious. SPC helps in reducing the cost of the process and improves the product quality by eliminating variations during the manufacturing process. This increases the efficiency of an industry in producing high quality products at a minimum cost. There are seven quality control tools developed by Dr. Kaoru Ishikawa in 1974, which are commonly used to monitor the process output. These seven quality control tools include the histogram, Pareto diagram, Ishikawa (fishbone) diagram, scatter diagram, check sheet, stratification and control chart. These tools are commonly called "the magnificent seven" (Montgomery, 2020).

The histogram is the most common graph used in illustrating the frequency distribution. According to Jaware *et al.* (2018), histogram is a frequency distribution indicating how frequent each parameter in a dataset occurs. The Pareto diagram is a bar chart, where the frequency is reflected by the length of the bars. The bars are arranged from longest to shortest, from left to right. Thus, through a Pareto diagram, practitioners can easily identify which situation should be prioritized (Jaware *et al.*, 2018). The Pareto diagram looks like a histogram but there is a significant distinction between them. The Ishikawa diagram is also called the cause-and-effect diagram. It is used in brainstorming sessions to identify possible causes and their relative effects (Jaware *et al.*, 2018). The scatter diagram is a graph that plots the numerical data of two different variables on two axes to identify their relationship. If the points form a line or a curve, the variables are deemed as correlated (Jaware *et al.*, 2018). A check sheet (defect concentration diagram) is a prepared list or form that is used to collect or analyze the data. A check sheet is applicable in a wide variety of fields. Stratification is the process of categorizing and sorting data into discrete categories (Jaware *et al.*, 2018). This tool is often used in conjunction with other tools, where it helps to separate data from different sources that are lumped together into distinct groups. A control chart is a graph that is used to monitor the process changes over time. It illustrates the relationship between the output of the quality characteristic of a sample and the sample number or time. Control chart is extremely useful in monitoring the process quality.

1.2 Control Charting Techniques

Dr. Walter A. Shewhart, known as the "Father of SQC", developed the control charting techniques in 1924, which are now commonly used in industries worldwide. A control chart is used to identify the presence of assignable cause(s) in a production process which lead to process variation. In a control chart, there are two different regions, which are the in-control and out-of-control regions. These two different regions are separated by the control limits, where the upper line is called the upper control limit (UCL) and the lower line is called the lower control limit (LCL). The center line in a control chart is the average value of the measured quality characteristic when the process is in-control. During the sampling process, when a sample point plots beyond the control limits, the control chart signals an out-of-control and the production process should be ceased immediately to identify and remove the assignable cause(s) before the production process is allowed to resume its operation. Figure 1.1 illustrates a basic control chart. According to Montgomery (2020), besides providing key information related to the process, control charting techniques are able to increase productivity, reduce the production of defective products and consequently minimize manufacturing waste.

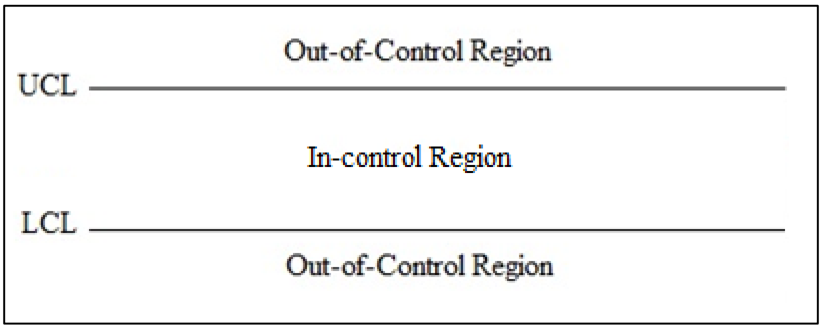

Figure 1.1 An illustration of a basic control chart

There are two different types of control charts, which are used to monitor either attribute or variable data. If practitioners need to monitor a quality characteristic whose data are presented in a continuous scale, the variable control charts are adopted. However, the attribute control charts are adopted when the data that need to be monitored are in the discrete form. The Shewhart (SH) $\bar{X}$ chart is used to monitor the process mean, while the range or *R* chart is used to monitor the process variance. The $\bar{X}$ and *R* charts are two basic control charts used in monitoring variable data. The *c* and *p* charts are two control charts that are frequently applied to monitor attribute data, where the former is based on the Poisson distribution while the latter is based on the binomial distribution. A process is considered as out-of-control when a sample point plots beyond the limits of the control chart, signifying that corrective actions are required to eliminate the assignable causes. If no out-of-control signal is detected by a control chart, the production process is allowed to continue until it ends (Montgomery, 2020).

1.3 Applications of Control Charts

The existence of a wide range of publications on control charts is a testament to the importance of control charting techniques in various disciplines. For instance, in manufacturing industries, Gitlow *et al.* (1989) explained the use of the *p*-chart in reducing the production of defective electric insulators, and the $\bar{X}$ and S charts in monitoring the stability of a plastic film operation. Ghute and Shirke (2008) implemented the synthetic chart in a spring manufacturing industry to monitor the multivariate mean vector, whereas Castagliola *et al.* (2011) illustrated the application of exponentially weighted moving average (EWMA) charts using data from a sintering process when producing mechanical components. Castagliola *et al.* (2013a) applied the variable sampling interval chart to monitor the coefficient of variation (CV) of the data from the process of hot chamber die casting during the production of zinc alloy (ZAMAK) components in a sanitary industry. Korzenowski *et al.* (2015) applied control charts for monitoring the multi-variety production systems in the production of gears, whereas Guo *et al.* (2015) considered the application of synthetic charts in the monitoring of process dispersion in the manufacturing and service sectors to maintain process stability. Abbas (2018) illustrated the application of the homogeneously weighted moving average (HWMA) chart using the semiconductor manufacturing data in monitoring the expansion of resist materials caused by the burning process. Recently, Amin *et al.* (2021) considered the inverse Gaussian regression model in developing memory control charts and implemented the chart in a yarn manufacturing industry.

Control charts are also used in the healthcare industry. Woodall (2006) demonstrated the use of control charts in healthcare. Marshall and Mohammed (2003) evaluated the variation in antibiotic prescriptions by multiple doctors with the aid of control charts. By using real data, Mohammed *et al.* (2008) illustrated the workings of four common types of control charts frequently used in healthcare, i.e. the X-MR, p, c and u charts. The X-MR chart was recommended for monitoring the deviation in the patients' blood pressure, in order to provide appropriate treatment immediately once a deviation is detected. The p chart was used to illustrate the performance of the healthcare staff over a specified time period, while the c chart was used to monitor the number of emergency admissions into an emergency ward on a specified day. Moreover, Mohammed *et al.* (2008) showed how the u chart was used to monitor the number of falls that occur in a hospital department over a specific period. Koetsier *et al.* (2012) applied the SH charts in the Plan-Do-Study-Act cycle to monitor the process and output data from the clinical information system, in studying the effect of using SH charts in the healthcare quality improvement process. Alencar *et al.* (2017) illustrated the use of the cumulative sum (CUSUM) chart in monitoring the number of patients above 65 years old who were admitted to a hospital due to respiratory disorders. More recently, Mahmood *et al.* (2021) studied the mortality due to the COVID-19 pandemic in Pakistan by monitoring three phases of variations using control charts.

Control charts are also playing an important role in finance. Walter *et al.* (1990) employed control charts in accounting, while Dull and Tegarden (2004) applied control charts in monitoring the financial reporting of several companies. Scordaki and Psarakis (2005) used actual data to demonstrate the implementation of different types of control charts in monitoring the performance of sales personnel in their weekly and monthly sales to ensure that the sales of these personnel are consistent with one another. Meanwhile, Golosnoy and Schmid (2007) implemented the EWMA chart to monitor the global minimum variance portfolio weights in the financial sector. Ryu and Shin

(2012) monitored the Korean Composite Stock Price Indexes (KOSPI) using control charts, in order to determine the timing to enter and exit the stock market. Dumičić and Žmuk (2015), and Žmuk (2016) investigated the use of control charts to aid the decision-making process when trading in the stock market in Croatia, where the trading signals were obtained using control charts. Their findings showed that the application of EWMA and CUSUM charts is more efficient in monitoring long term trading compared to short term trading. Recently, Qiao and Han (2021) studied the historical data of the Chinese stock market using the CUSUM multi-chart which will help organizations to avoid losses when trading in the market. Their study showed that the optimal CUSUM multi-chart is the most suitable chart to be used when post-changes in the stock market are unknown.

Control charts are also applied in other types of industries. For example, in the aerospace industry, Beabout (2003) incorporated the *p* and Pareto charts to facilitate the maintenance process of aircrafts. In addition, Montgomery (2020) showed the benefits of adopting the $\bar{X}$ and R charts, where the profit in the aerospace manufacturing industry can be increased by reducing rework cost, as the aforementioned charts enable the problems faced by the company's supplier to be identified in time. Zhou *et al.* (2008) combined the SH and CUSUM charts to assess the quality of water resources and minimize contamination of karst aquifers. Aliverdi *et al.* (2013) showed the use of control charts in the construction sector, where the performance of a project was evaluated by monitoring the cost and duration of the project. In the education sector, Bakir *et al.* (2015) illustrated the students' grade point average (GPA) in a college using the SH sign control chart to track the stability of the academic performance of students. In the area of renewable energy, Cambron *et al.* (2018) used the EWMA chart to monitor the wind turbine generators for detecting unusual variation in the generator or the components in the generator. More recently, Aslam *et al.* (2021) proposed the new CUSUM $\bar{X}$ control chart to monitor the stock price of a petroleum company and to forecast the weather condition in Pakistan.

1.4 Problem Statements

Control charts have been adopted in a wide range of applications. The traditional control charts, such as the SH charts monitor the process mean or variance. For the aforesaid charts to function correctly, the mean and variance of the process should be independent of one another. Unfortunately, in some sectors, such as healthcare and finance, the process mean and variance are not independent of one another. As the independence assumption between the process mean and variance does not hold in the above-mentioned sectors, traditional control charts for the mean and variance are inefficient in process monitoring in such situations. To overcome this problem, Kang *et al.* (2007) introduced the SH CV chart, where the proportional relation between the process mean and standard deviation is considered. However, the SH CV chart is less sensitive in detecting small and moderate shifts in the process CV. To boost the sensitivity of the SH CV chart, Khaw *et al.* (2017) proposed an adaptive CV chart with the variable sample size and sampling interval (VSSI) strategy, where small and moderate CV shifts can be detected quicker than the SH CV chart. Moreover, the performance of a control chart should also be evaluated from the economic aspect, as the cost of implementing a control chart in process monitoring is also a major consideration among practitioners. Hence, this thesis investigates the VSSI CV chart from the economic and economic-statistical point of view, as such study is not available in the literature, and compare the findings obtained with that of the basic SH

CV chart. This investigation is in line with the first objective of this thesis, explained in Section 1.5.

In some process monitoring situations, more than one correlated quality characteristic needs to be monitored simultaneously. It is misleading to implement univariate control charts to monitor each of the quality characteristics separately and ignore the correlation among the characteristics. The multivariate CV (MCV) control chart was proposed by Yeong *et al.* (2016) to enable multivariate process CV to be monitored. Khatun *et al.* (2019) extended the MCV chart to short production runs which can be used in low volume production but adaptive MCV charts for short production runs (SPR) do not exist in the current literature. As the traditional SH chart is not efficient in monitoring small and moderate shifts, coupled with the superb performance of adaptive charts reported in numerous publications (see Amdouni *et al.*, 2015 and Yeong *et al.*, 2017a, to name a few), this thesis proposes the variable sample size (VSS) MCV chart for short production runs and compares its performance with the basic SH MCV SPR chart. The development of the VSS MCV SPR chart is in line with the second objective of this thesis in Section 1.5.

1.5 Objectives of the Thesis

The objectives of this thesis are as follows:

i) To conduct an investigation on the economic and economic-statistical designs of the VSSI CV control chart by analyzing the effect of statistical constraints, sensitivity analyses and misspecification of the input parameters on the chart's economic performance.

ii) To develop the VSS MCV control chart for short production runs, as short runs based adaptive MCV charts do not exist in the literature.

1.6 Organization of the Thesis

Chapter 1 presents an overview of SQC and the associated tools in quality control. The control charting techniques are discussed and the applications of control charts in various sectors are provided. The problem statements and objectives of this thesis are enumerated and the organization of this thesis concludes the chapter.

Chapter 2 gives a brief overview of the different types of CV control charts, which include univariate, multivariate, adaptive, non-adaptive and short production runs CV type charts. The economic and economic-statistical designs of CV charts are also reviewed. Moreover, detailed discussions of the basic SH CV and SH MCV charts, which include the charts' properties, statistics and control limits, are presented.

In Chapter 3, the economic and economic-statistical performances of the VSSI CV chart, in terms of minimizing the cost function, are investigated. The VSSI CV chart was proposed by Khaw *et al.* (2017). The properties of the VSSI CV chart are presented. Comparisons of the economic and economic-statistical performances of the SH CV and VSSI CV charts based on the average time to signal (ATS) and expected ATS (EATS) criteria are conducted. The sensitivity analyses of the economic and economic-statistical designs based SH CV and VSSI CV charts are conducted and compared. Lastly, a study on the effects of shift size misspecification is presented.

In Chapter 4, a new adaptive MCV chart for SPR using the VSS technique, called the VSS MCV SPR chart, is developed. The VSS MCV SPR chart is proposed to improve the statistical performance of the existing SH MCV SPR chart. The properties of the proposed VSS MCV SPR chart and the chart's optimization procedure in minimizing the out-of-control truncated average run length (TARL) value is provided. Consequently, the TARL performances of the VSS MCV SPR and SH

MCV SPR charts are compared. An example of application is provided to show the working of the VSS MCV SPR chart.

The findings of this thesis are summarized in Chapter 5. The important contributions of the thesis are highlighted and ideas for future extensions are suggested at the end of this chapter.

CHAPTER 2

LITERATURE REVIEW ON COEFFICIENT OF VARIATION CONTROL CHARTS

2.1 Introduction

In SPC, control charts are very useful in process monitoring for attaining quality improvement. Control charts are used in various industries, especially in the manufacturing and service sectors, to monitor a production process, in order to increase process stability and productivity. Control charts are now commonly used by management and practitioners in process monitoring due to the advancement in computer technology which allows real-time data collection and analysis.

By using control charts, practitioners can monitor one or more process variables, in determining whether the quality characteristics of the process are "in-control" or "out-of-control". A process classified as “in-control” runs with only chance causes of variation, while a process that runs in the presence of assignable causes is classified as an “out-of-control” process. When a process is “out-of-control”, quick actions are needed to identify and remove the assignable cause(s), in order to minimize the production of defective products.

After the SH $\bar{X}$ chart was introduced, modifications and enhancements of various types of control charting methods were made by researchers. The Hotelling’s T^2 chart, developed by Hotelling (1947) was the first control chart for monitoring multivariate quality characteristics. On similar lines, the SH $\bar{X}$ chart was extended to memory type charts by Roberts (1959) and Page (1954) who proposed the EWMA and CUSUM charts, respectively. Compared with the SH $\bar{X}$ chart, these memory charts are more sensitive and have better statistical performance in detecting small shifts.

The use of the SH, EWMA or CUSUM chart in detecting process shifts requires the assumption of a constant process mean and process standard deviation when the process is in-control. Nevertheless, some processes in manufacturing and service industries do not have constant mean and standard deviation, even though the process is in-control. To overcome this problem, monitoring the proportional relationship or ratio of the process standard deviation to the process mean, called the CV, using a CV chart was introduced by Kang *et al.* (2007). Since then, various types of more robust and effective CV charts have been developed. Section 2.2 discusses the various types of CV charts in the literature.

2.2 Types of CV Control Charts

In this section, various types of CV charts in the literature will be reviewed. Section 2.2.1 reviews the non-adaptive univariate and multivariate CV charts, while Section 2.2.2 reviews the adaptive univariate and multivariate CV charts. The adaptive CV charts were developed to improve the sensitivity and efficiency of their corresponding non-adaptive CV counterparts. The univariate and multivariate CV charts discussed in Sections 2.2.1 and 2.2.2 assume that the process being monitored has an infinite production horizon. Section 2.2.3 reviews the univariate and multivariate CV charts for process monitoring in a finite production horizon.

2.2.1 Non-Adaptive Univariate and Multivariate CV Control Charts

The SH chart is a common example of a non-adaptive chart which plots the sample statistics sequentially over time, as new samples are collected. Two straight lines plotted on the chart, i.e., the upper and lower control limits function as decision lines in determining whether a process under monitoring is in-control or out-of-control.

If sample statistics plot within the control limits, the process is in-control, while the process is deemed as out-of-control, otherwise. The control charting parameters of non-adaptive charts are always kept constant from the beginning until the end of process monitoring.

In a clinical chemistry-control process where the process mean and process standard deviation are not constant, Kang *et al.* (2007) proposed the first Shewhart-type CV chart for monitoring the CV with rational subgroups. According to Kang *et al.* (2007), the benefit of monitoring the CV in physical and chemical processes is evident when the quality characteristic needs to be controlled and the process standard deviation is proportional to the process mean. The EWMA CV chart was proposed by Hong *et al.* (2008) to improve the efficiency of the standard CV chart in detecting small shifts. Castagliola *et al.* (2011) introduced a method for monitoring the CV by employing two one-sided EWMA charts of the CV squared, called the EWMA CV^2 chart. The EWMA CV^2 chart outperforms the EWMA CV chart as the former gives smaller out-of-control average run length $\left(\mathrm{ARL}_1\right)$ values. Calzada and Scariano (2013) suggested the synthetic chart for monitoring the CV (called the Syn CV chart). The Syn CV chart integrates the basic CV and conforming run length charts. The ARL_1 values produced by the Syn CV chart are lower than the corresponding ones of the basic CV chart but slightly larger than that of the EWMA CV^2 chart. Additionally, Castagliola *et al.* (2013b) introduced the SH CV chart with supplementary runs rules, in which different types of runs rules schemes were considered. You *et al.* (2016) suggested a side sensitive group runs chart, while Teoh *et al.* (2017) proposed a run sum (RS) chart to monitor the CV. The RS CV chart is based on the concept of dividing the interval between the chart's limits into several regions and assigning scores to each of these regions. The RS CV chart signals when the accumulated scores exceed a triggering score. Haq *et al.* (2020) developed enhanced EWMA charts using an auxiliary information based estimator to monitor the process CV, where the proposed charts surpass the existing EWMA charts for monitoring the CV. Saha *et al.* (2021) developed the side-sensitive modified group runs CV (SSMGR CV) chart and showed that the SSMGR CV chart outperforms all existing CV charts. Yeong *et al.* (2021a) introduced the side-sensitive Syn CV chart and the results based on the average run length (ARL) criterion show that the side-sensitive Syn CV chart is able to detect CV shifts quicker than the existing SH CV and Syn CV charts for almost all shift sizes and having comparable performance with the EWMA CV chart, except in detecting very small shifts. Yeong *et al.* (2021b) evaluated the Syn CV chart based on the median run length (MRL) criterion, where the Syn CV chart was shown to outperform the EWMA CV chart. Figure 2.1 summarizes the literature on univariate CV charts.

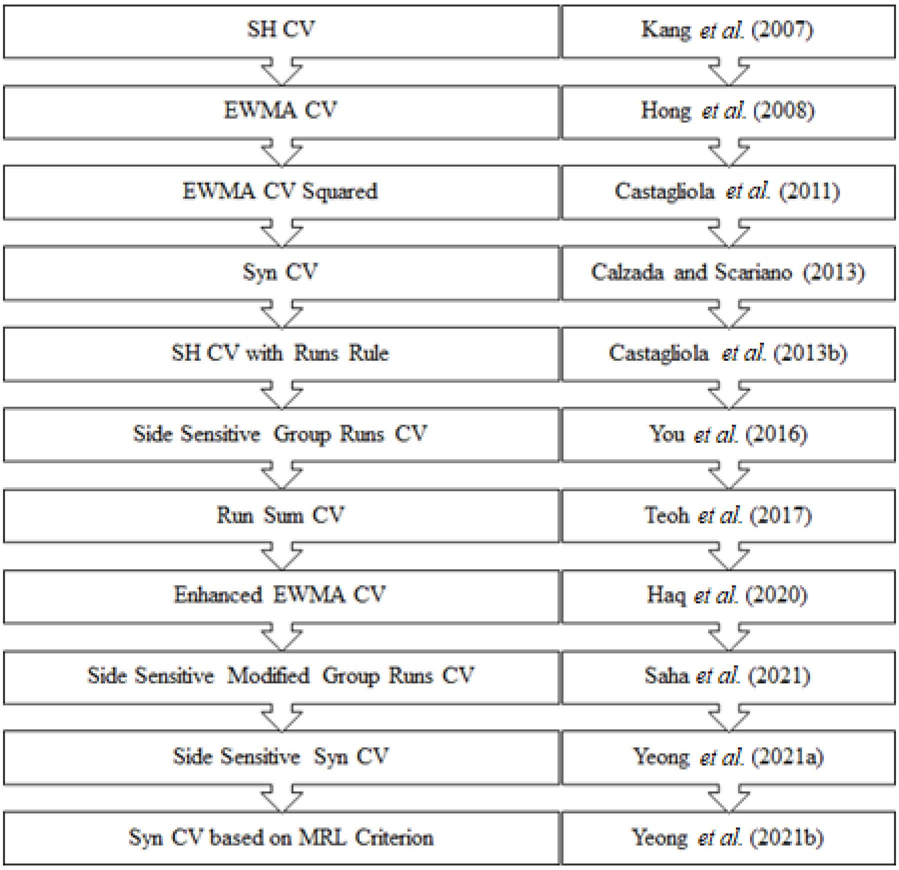

Figure 2.1 A summary on the literature of univariate CV charts

Most process monitoring scenarios in practical applications involve multiple correlated variables. Hence, process monitoring methods for monitoring more than one correlated variables are required. The Hotelling's T^2 chart is the first multivariate control chart, introduced by Hotelling (1947), for monitoring two or more correlated variables simultaneously. The first control chart for monitoring the MCV (called the SH MCV chart) was proposed by Yeong *et al.* (2016), where two one-sided charts were employed to detect increasing and decreasing shifts in the process MCV. Lim *et al.* (2017) developed a RS chart to monitor the MCV, called the RS MCV chart, which outperforms the basic MCV chart. Giner-Bosch *et al.* (2019) proposed the EWMA chart for monitoring the MCV. This chart is known as the EWMA MCV chart and it detects small shifts quicker than the SH MCV and RS MCV charts. Chew *et al.* (2020a) proposed the use of runs rules in monitoring the MCV using the Markov chain approach and compared the former's performance with the SH MCV chart. It was shown that the run rules based MCV chart outperforms the SH MCV chart in detecting small and moderate shifts. Lee *et al.* (2020) investigated the Syn MCV chart based on the MRL performance and the findings showed that the MRL criterion is preferable to the ARL criterion in evaluating the Syn MCV chart's performance. The Syn MCV chart has better performance than the existing SH MCV chart based on the MRL criterion. Recently, Ayyoub *et al.* (2021) studied the upward SH MCV and EWMA MCV charts with measurement errors using the linear covariate error model. Figure 2.2 shows a summary of the literature on MCV charts.

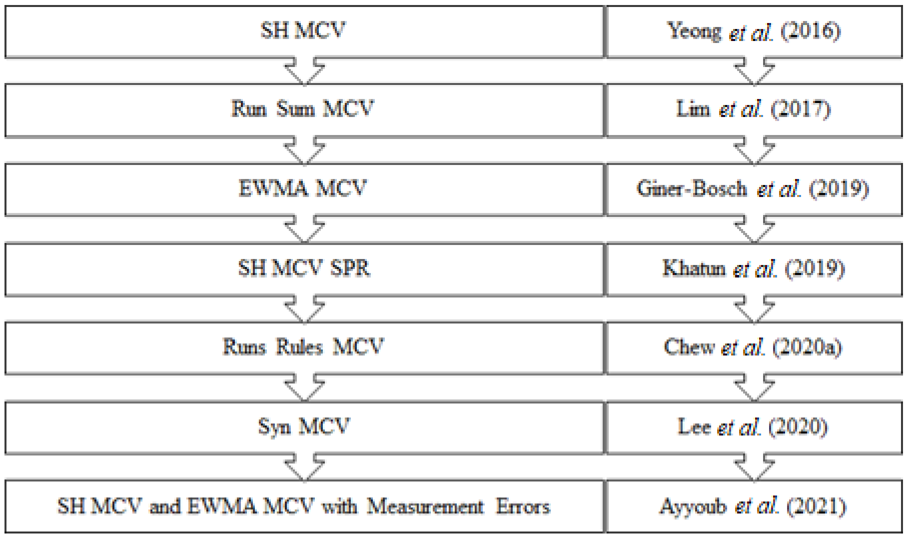

Figure 2.2 A summary on the literature of MCV charts

2.2.2 Adaptive Univariate and Multivariate CV Control Charts

All the CV type charts discussed in the previous section are non-adaptive charts, where fixed chart parameters are used. Control charts with fixed chart parameters adopt the same values of the parameters throughout the process monitoring, regardless of the quality of the process observed from the recent samples. This phenomenon causes inefficiency in detecting process shifts and will delay the detection of small and moderate process changes. To circumvent this problem, adaptive control charts are introduced to improve the performance of their non-adaptive counterparts, where the former allows the charting parameters, i.e. the sample size, sampling interval and action limits to be varied.

The first adaptive control chart was suggested by Reynolds *et al.* (1988), where the variable sampling interval (VSI) approach was incorporated into the $\bar{X}$ chart. The sampling interval of the VSI $\bar{X}$ chart is varied according to the location in which the value of the sample mean falls on the chart and the chart offers great improvement in detecting mean shifts compared to the non-adaptive SH $\bar{X}$ chart. Subsequently, Prabhu *et al.* (1993) and Costa (1994) developed the adaptive VSS $\bar{X}$ chart that varies the sample size, instead of the sampling interval. The sample size of the VSS $\bar{X}$ chart is varied according to where the sample mean plots on the chart and its performance is superior to the SH $\bar{X}$. Later on, Prabhu *et al.* (1994) and Costa (1997) incorporated the VSSI approach on the $\bar{X}$ chart, where the sample size and sampling interval of the chart are varied according to the location of the plotted sample mean on the chart. The VSSI $\bar{X}$ chart has better performance than the SH $\bar{X}$, VSI $\bar{X}$ and VSS $\bar{X}$ charts.

For a CV type chart, the first adaptive chart was developed by Castagliola *et al.* (2013a), where the VSI method was used to monitor the CV. The proposed chart is called the VSI CV chart and it outperforms the standard CV chart in terms of the ATS and standard deviation of the time to signal (SDTS) performance criteria. Castagliola *et al.* (2015a) and Yeong *et al.* (2017a) used the VSS method to monitor the CV, where the proposed chart is known as the VSS CV chart. Subsequently, Khaw *et al.* (2017) adopted the VSSI approach to monitor the CV. Their proposed VSSI CV chart responses quicker to moderate and large shifts compared to the typical non-adaptive CV charts, as well as the VSI and VSS CV charts. On similar lines, Yeong *et al.* (2018) developed the variable parameters (VP) CV chart which varies all the chart's parameters, i.e. the sample size, sampling interval and action limits, in monitoring the CV. The VP CV chart detects small and moderate shifts quicker than the VSI, VSS and VSSI CV charts.

Khaw *et al.* (2018) extended the study on univariate adaptive CV charts to monitor the MCV, which includes the use of the VSI, VSS and VSSI methods on the MCV chart and concluded that the VSSI MCV chart has the best performance. However, the VSSI MCV chart proposed by Khaw *et al.* (2018) is only for detecting upward shifts in the process MCV. Hence, Chew and Khaw (2020) proposed the downward VSSI MCV chart to monitor downward process MCV shifts, and the results show that the proposed chart's performance is better than that of the existing downward MCV charts. Figure 2.3 shows a summary of the literature on adaptive CV and MCV charts.

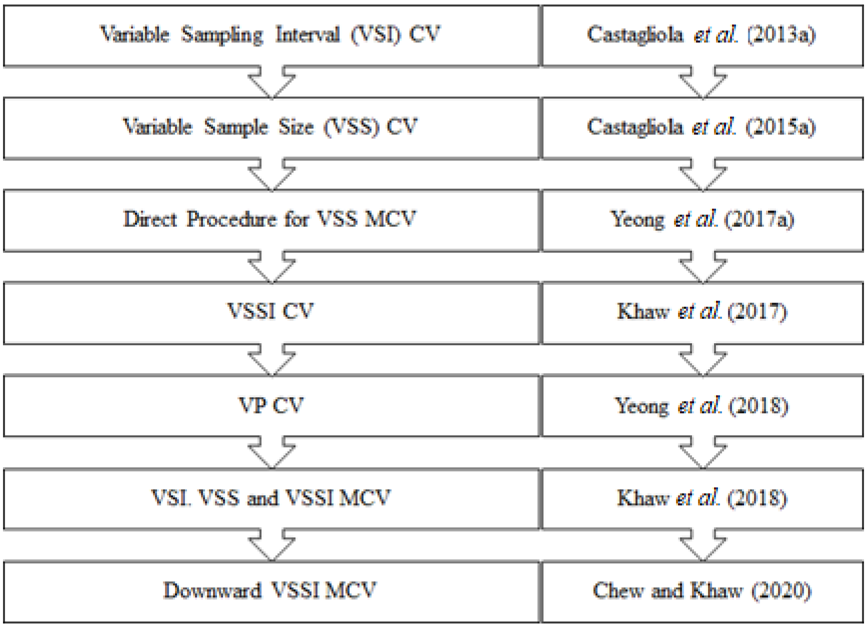

Figure 2.3 A summary on the literature of adaptive CV and MCV charts

2.2.3 Univariate and Multivariate CV Control Charts for Short Production Runs

All the control charts discussed prior to this section assume that process monitoring involves an infinite production horizon. However, there are instances where a process being monitored lasts for only a few hours or days, such as in the assembly of electronic boards in a semiconductor industry. This type of process is known as short production runs or finite horizon. The first control chart designed for a finite horizon was proposed by Ladany (1973), who presented the methodology of an economic optimization of the *p*-chart in short production runs. Subsequently, Del Castillo and Montgomery (1993, 1996) introduced the $\bar{X}$ charts for short production runs, while Tagaras (1996), Tagaras and Nikolaidis (2002) and Nenes and Tagaras (2007) studied several Bayesian type $\bar{X}$ charts for short production runs.

Monitoring the CV using one-sided Shewhart charts in a finite horizon process was first investigated by Castagliola *et al.* (2015b). As an adaptive strategy is usually used to improve the efficiency of control charts, Amdouni *et al.* (2015) and Amdouni *et al.* (2017) suggested the VSS and VSI charts, respectively, to monitor the CV in short production runs. Khatun *et al.* (2019) extended the study on MCV charts by proposing the one-sided MCV charts for monitoring the process MCV in a finite production horizon. Chew *et al.* (2020b) adopted the runs rules schemes in monitoring the MCV in a finite production horizon and the findings show that the run rules based MCV chart outperforms the SH MCV chart in detecting small and moderate shifts. More recently, Khaw *et al.* (2021) introduced the one-sided 4-of-5 run rules MCV chart for short production runs where this chart is superior to the existing short production runs based MCV charts in detecting small and moderate shifts. Figure 2.4 summarizes the literature on CV and MCV SPR charts.

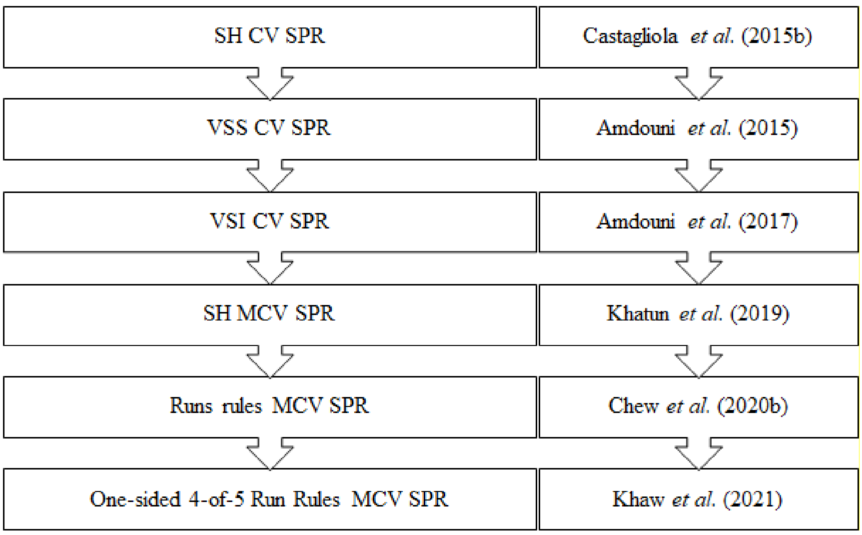

Figure 2.4 A summary on the literature of CV and MCV SPR charts

2.3 Economic and Economic-statistical Designs of CV Control Charts

According to Saniga (1989), the selection of optimal parameters of a control chart is grouped into four main categories, namely, heuristic, statistical, economic and economic-statistical designs. Heuristic designs are mostly applied in industries due to their simplicity but they may lead to poor statistical and economic performances (Vommi and Seetala, 2007). In statistical design, the chart's parameters are adjusted to minimize the out-of-control run length value, based on a desired in-control run length performance (Woodall, 1985).

However, pure statistical designs might not always be the most cost-effective option (Surtihadi and Raghavachari, 1994). For example, in some situations, false alarms or failure to indicate an out-of-control signal can be prohibitively expensive. According to Surtihadi and Raghavachari (1994), an economic design is preferable when the economic effects of the design are considerable. Vommi and Seetala (2007) mentioned that economic designs are usually used to increase profit or to minimize the cost of producing each unit of a product. The first control chart that was economically designed was introduced by Duncan (1956) and this concept was enhanced by Lorenzen and Vance (1986). Pure economic designs could result in a poor statistical performance, as too frequent false alarms are being triggered by the control chart at hand (Woodall, 1986). To overcome this problem, Saniga (1989) suggested the economic-statistical designs, where statistical constraints, such as the ARL and ATS are incorporated into the economic designs of control charts. As cost is a very important factor in business, control chart parameters should be determined in such a way that the cost of implementing the chart is minimized without sacrificing the chart's statistical performance (Vommi and Seetala, 2007).

In investigating the cost of implementing CV type control charts, Yeong *et al.* (2015) was the first to study the economic and economic-statistical performances of a CV chart. Yeong *et al.* (2017b) studied the economic and economic-statistical performances of the Syn CV chart and compared its performances with that of the standard CV, CUSUM CV and EWMA CV charts. The Syn CV chart was found to be the most cost effective chart among all the non-adaptive CV charts considered. Ng *et al.* (2022) extended the investigation of the cost aspect of CV charts by investigating the economic and economic-statistical designs of the MCV chart, where a comparison between the two designs was enumerated. However, a study on the economic and economic-statistical performances of adaptive CV charts does not exist in the current literature, hence, this thesis will fill this gap by investigating the economic and economic-statistical performances of the VSSI CV chart. Figure 2.5 shows a summary on the economic and economic-statistical designs of CV charts in the literature.

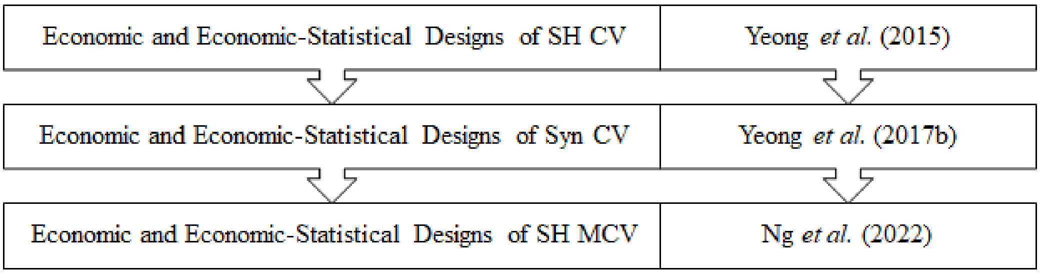

Figure 2.5 A summary on the economic and economic-statistical designs of CV charts

2.4 Univariate CV Control Chart

The CV is defined as the ratio of the process standard deviation to the process mean. For the univariate case, if μ and σ refer to the population mean and standard deviation, respectively, of the random variable X, then the CV of X is defined as

$$\gamma = \frac{\sigma}{\mu}. \tag{2.1}$$

For a random sample $\{X_1, X_2, ..., X_n\}$ of size *n*, the sample mean, $\bar{X}$ is computed as

$$\bar{X} = \frac{1}{n}\sum_{k=1}^{n} X_k, \tag{2.2}$$

while the sample standard deviation, *S*, is calculated as

$$S = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(X_k - \bar{X}\right)^2}\ . \tag{2.3}$$

The sample CV of *X*, $\hat{\gamma}$, is defined as (Khaw *et al.*, 2017)

$$\hat{\gamma} = \frac{S}{\bar{X}}. \tag{2.4}$$

According to Iglewicz *et al.* (1968), $\sqrt{n}/\hat{\gamma}$ follows a noncentral t distribution with $(n-1)$ degrees of freedom and noncentrality parameter $\sqrt{n}/\gamma$. The cumulative distribution function (cdf) of $\hat{\gamma}$ is (Iglewicz *et al.*, 1968)

$$F_{\hat{\gamma}}\left(x \middle| n, \gamma\right) = 1 - F_t\left(\frac{\sqrt{n}}{x} \middle| n-1, \frac{\sqrt{n}}{\gamma}\right), \tag{2.5}$$

while the inverse cdf of $\hat{\gamma}$ is

$$F_{\hat{\gamma}}^{-1}\left(\alpha \middle| n, \gamma\right) = \frac{\sqrt{n}}{F_t^{-1}\left(1-\alpha \middle| n-1, \frac{\sqrt{n}}{\gamma}\right)}. \tag{2.6}$$

Note that $F_t\left(\cdot \middle| n-1, \frac{\sqrt{n}}{\gamma}\right)$ and $F_t^{-1}\left(\cdot \middle| n-1, \frac{\sqrt{n}}{\gamma}\right)$ denote the cdf and inverse cdf of the noncentral *t* distribution with *n* – 1 degrees of freedom and noncentrality parameter