First Semester Examination
2022/2023 Academic Session

February 2023

EUM 113 – Engineering Calculus
(Kalkulus Kejuruteraan)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **EIGHT (8)** pages of printed material including appendix before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN (8)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions : This paper consists of **FOUR (4)** questions. Answer **ALL** questions.

*[**Arahan** : Kertas ini mengandungi **EMPAT (4)** soalan. Jawab **SEMUA** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. a) Solve

Selesaikan

(i) $\lim_{\substack{had \\ x \to 1}} f(x)$

given

jika diberi

$$f(x) = \begin{cases} 1 - 5x, & x < -6 \\ 17, & x = -6 \\ x^3, & -6 < x < 1 \\ 1, & x = 1 \\ 2 - x, & x > 1 \end{cases}$$

(1 marks/*markah*)

(ii) $\lim_{\substack{had \\ x \to 1}} \frac{x^3}{1-\sqrt{x}}$

(1 marks/*markah*)

(iii) $\lim_{\substack{had \\ x \to \infty}} \frac{e^{2x}}{1+x^2}$

(1 marks/*markah*)

(iv) $\lim_{\substack{had \\ (x,y) \to (0,0)}} f(x,y)$

given

jika diberi

$$f(x,y) = \begin{cases} \frac{x^2y^2}{\sqrt{x^2+y^2}} \; if \; (x,y) \neq (0,0) \\ 0 \quad if \; (x,y) = (0,0) \end{cases}$$

(2 marks/*markah*)

(v) $\lim_{(x,y)\to(0,0)} had\ f(x,y)$

Given

Jika diberi

$$f(x,y) = \frac{(x+y)^2}{x^2+y^2}$$

(2 marks/*markah*)

b) The velocity of a toy bus travelling along a horizontal surface with negligible friction is estimated as $v(t) = \frac{e^{2t}}{1+t^2}$. Find the distance travelled using Simpson's $\frac{1}{3}$ rule for $t = 0$ to 2 s. Use $h = 0.25$.

Halaju sebuah bas mainan yang bergerak di sepanjang permukaan mengufuk dianggarkan sebagai $v(t) = \frac{e^{2t}}{1+t^2}$ *dengan mengabaikan geseran. Dapatkan jarak pergerakan dengan menggunakan petua Simpson's* $\frac{1}{3}$ *bagi* $t = 0$ *to 2 s. Gunakan* $h = 0.25$.

(7 marks/*markah*)

c) Determine if the Mean Value Theorem is applicable to $f(x)$ in the following given closed interval. If so, find all possible values of c.

Tentukan sama ada Teorem Nilai Min dapat digunakan bagi $f(x)$ *dalam selang tertutup yang diberikan berikut. Jika demikian, cari semua nilai* c *yang mungkin.*

$$f(x) = \frac{x^3}{1-\sqrt{x}}, \qquad [0,4]$$

(3 marks/*markah*)

d) A rectangular fish tank with cover has a volume of $250\ \mathrm{m}^3$. The top cover and bottom of the tank is specially designed with acrylic material costing 40 cents per square m. While the sides are made from glass material costing 20 cents per square m. Use Lagrange multipliers to find the dimensions of the tank so that the cost of the materials is minimized.

Sebuah tangki ikan dengan penutup mempunyai isipadu sebanyak $250\ \mathrm{m}^3$. *Penutup dan dasar tangki direka khas dengan kos bahan akrilik sebanyak* 40 *sen setiap meter persegi. Manakala sisi-sisinya dibuat dengan kos bahan gelas senyak 20 sen setiap meter persegi. Gunakan pendarab Lagrange untuk mendapatkan dimensi tangka itu supaya kos bahan-bahan adalah minimum.*

(8 marks/*markah*)

2. a) Solve $(x^2 - yx^2)dy + (y^2 - xy^2)dx = 0$ using separation of variable method.

Selesaikan $(x^2 - yx^2)dy + (y^2 - xy^2)dx = 0$ *dengan menggunakan kaedah pemisahan pembolehubah.*

(5 marks/*markah*)

b) Evaluate $y(2.2)$ using Euler's method from the equation $\frac{dy}{dx} = -xy^2$ with $y(2) = 1$ by considering the value of *n* as 4.

Nilaikan $y(2.2)$ *using Euler's method from the equation* $\frac{dy}{dx} = -xy^2$ *with* $y(2) = 1$ *by considering the iteration, n as 4.*

(10 marks/*markah*)

c) Solve the non-homogenous differential equation below.

Selesaikan persamaan pembezaan tak homogen berikut.

$$\frac{dy}{dx} = \frac{x + 2y - 5}{2x + y - 2}$$

(9 marks/*markah*)

3. a) Solve the initial-value problem.

Selesaikan masalah berkaitan nilai-awal.

$$y'' + y' - 6y = 0 \qquad y(0) = 1 \qquad y'(0) = 0$$

(5 marks/*markah*)

b) By using the Variation of Parameter method, find the particular solution for:

Menggunakan kaedah variasi parameter, dapatkan penyelesaian khusus bagi:

$$y'' - 4y' - 4y = (x + 1)e^{2x}$$

(10 marks/*markah*)

c) Solve the following initial value problem with Laplace transformation:

Selesaikan masalah nilai awal yang berikut dengan jelmaan Laplace:

$$2f''(t) - f'(t) - f(t) = \sin(t) - \cos(t).$$

Given

Diberi

$f(t = 0) = 0$ and $f'(t = 0) = 0$

(10 marks/*markah*)

4. a) A semi-circular lake is built encircling a city as indicated in shaded region R shown in Figure 1. In order to approximate the volume of the lake, Mathematicians apply the formula $V = \iint_R A(x,y)dA$.

Given $A(x,y) = e^{\sqrt{x^2+y^2}}$, evaluate the volume of the lake.

Suatu tasik berbentuk semi bulatan dibina mengelilingi sebuah bandar seperti yang ditunjukkan dalam rantau R pada Rajah 1. Untuk menggangarkan isipadu tasik itu, ahli-ahli Matematik mengaplikasikan formula $V = \iint_R A(x,y)dA$.

Diberi $A(x,y) = e^{\sqrt{x^2+y^2}}$, *nilaikan isipadu tasik itu.*

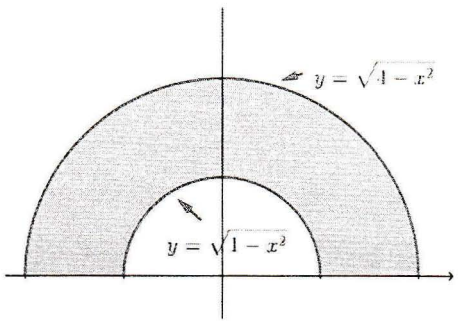

Figure 1
Rajah 1

(11 marks/*markah*)

b) The movement of a swinging pendulum having mass of bob *m* and length of rod *l* with tension *T* is schematically shown by Figure 2 and can be described by mathematical model:

Pergerakan bagi satu bandul berayun dengan jism bebola bandul m dan panjang rod l dengan tekanan T adalah ditunjukkan seperti dalam Rajah 2 dan boleh diterangkan dengan model matematik:

$$ml\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -mg\sin\theta$$

...7/-

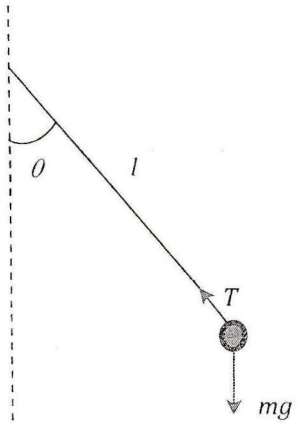

Figure 2

Rajah 2

Given the mass of bob is 0.5 kg with massless rod of length 0.7 m, and g = 9.8 ms^{-2}. The displacement from the equilibrium position is small enough such that sin $\theta \approx \theta$ is considered valid. The pendulum is released from stationary at the angle of $\theta = 4^{o}$ (note that at t = 0, also $\frac{d\theta}{dt} = 0$).

Diberi jisim bagi bebola bandul 0.5 kg dengan mengabaikan jisim rod yang panjangnya 0.7 m, dan g = 9.8 ms^{-2}. Sesaran dari posisi seimbang adalah sangat kecil yakni sin $\theta \approx \theta$ adalah sah. Bandul dilepaskan dari keadaan pegun pada sudut $\theta = 4^{o}$ (ambil perhatian bahawa pada t = 0, juga $\frac{d\theta}{dt} = 0$).

(i) What would be the time taken to reach the first zero displacement?

Apakah masa yang sepatutnya diambil untuk sampai sesaran kosong pertama?

(ii) Estimate the angle θ after 5.0 s.

Anggarkan sudut θ selepas 5.0 s.

(15 marks/*markah*)

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APPENDIX

LAMPIRAN

Question	Course Outcome (CO)	Programme Outcome (PO)
1	1	PO1
2	2	PO1
3	3	PO1
4	4	PO2