Second Semester Examination
2022/2023 Academic Session

July/August 2023

EUM 114 – Advanced Engineering Calculus

Duration : 3 hours

Please check that this examination paper consists of **FIVE (5)** pages of printed material including appendix before you begin the examination.

Instructions : This paper consists of **FOUR (4)** questions. Answer **FOUR (4)** questions.

1. Given the Fourier expansion for the function $f(x) = x(1-x)(1+x)$ in the interval $-1 < x < 1$

a) Identify whether the function is even, odd, or neither using the formula.

(3 marks)

b) Sketch the graph of $f(x) = f(x+2)$ periodic function for $-3 < x < 3$ range

(3 marks)

c) Find the Fourier expansion for the function $f(x) = x(1-x)(1+x)$ in the interval $-1 < x < 1$

(14 marks)

d) Based on (c), deduce the Fourier series to $1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots = \frac{3\pi^3}{(8)(12)}$

(5 marks)

2. a) In an experimental study of a musical instrument, it was found that the vibrations in organ pipe obey 1-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the following boundary and initial conditions:

(i). $u(0,t) = 0$($t \geq 0$)(the end $x = 0$ is closed)

(ii). $\frac{\partial u}{\partial x}(l,t) = 0$($t \geq 0$)(the end $x = l$ is open)

(iii). $u(x,0) = 0$($0 \leq x \leq l$)(the pipe is initially undisturbed)

(iv). $\frac{\partial u}{\partial t}(x,0) = v$ (constant) ($0 \leq x \leq l$)(the pipe is given an initial uniform blow)

By applying the above first three conditions, show that the general solution for
wave equation in this study can be written as

$$u(x,t) = \sum_{n=0}^{\infty} b_n \sin\left[\frac{\left(n+\frac{1}{2}\right)\pi x}{l}\right] \sin\left[\frac{\left(n+\frac{1}{2}\right)\pi ct}{l}\right]$$

With b_n as the coefficient corresponding to specific individual solution, where the expression for this coefficient may be obtained further through Fourier sine series expansion if condition (iv) is applied.

(12 marks)

b) The initial temperature of a solid rod of 1 unit length is given by $u_o(x,0) = \sin \pi x$ for $0 \le x \le 1$, where x is measured from one end of the rod to the other end. At time $t = 0$, the rod is cooled down by subjecting 0 °C at both of its ends. The temperature variation in the rod, $u(x,t)$ satisfies the 1-dimensional heat equation:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Given $c^2 = 0.2$, solve for values $u(x,t)$ at $t = 0.3$ numerically giving your answer at 4 decimal points using explicit formula by taking $\Delta x = 0.25$ and time step $\Delta t = 0.1$. The explicit formula for heat equation based on finite difference method is

given as

$$u(x_i, t_{j+1}) = \lambda u(x_{i-1}, t_j) + (1 - 2\lambda) u(x_i, t_j) + \lambda u(x_{i+1}, t_j)$$

Where $\lambda = c^2 \frac{\Delta t}{(\Delta x)^2}$

(13 marks)

3 a) Find the work done by the vector field $F(x, y, z) = 3x^2 i + (2xz - y) j + z\,k$ on a particle moving along the line segment that goes from (0,0,0) to (2,1,3).

(10 marks)

b) Let S be a surface defined by $\frac{3}{4}x + \frac{3}{2}y + z = 3$ with a unit vector N on plane S as shown in Figure 3.1. Use Stokes' theorem to find $\oint_c F \cdot dr$ where $F = (3yx^2 + z^3) i + y^2 j + 4yx^2 k$.

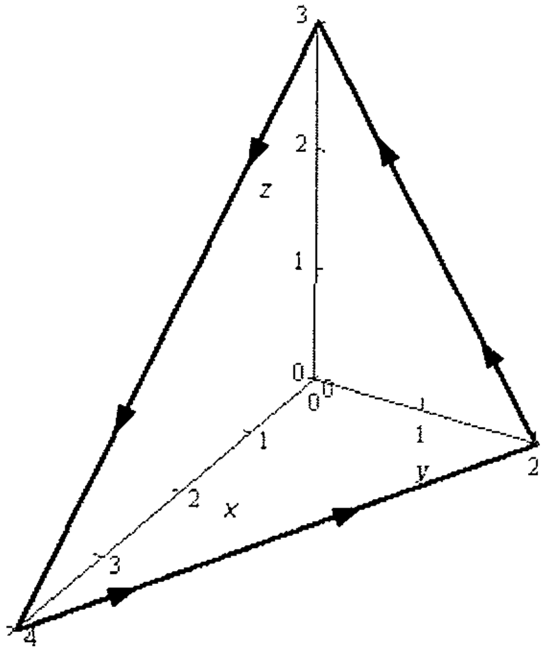

Figure 3.1

(15 marks)

4. The following matrix A has $[2 \quad 2 \quad 1]^T$ as an eigenvector

$$A = \begin{bmatrix} x & y & 2 \\ y & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

a) Find the values of x and y

(10 marks)

b) Then determine all the eigenvalues and corresponding eigenvectors of the matrix A.

(15 marks)

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APPENDIX

Question	Course Outcome (CO)	Programme Outcome (PO)
1	1	PO1
2	2	PO2
3	3	PO2
4	4	PO1