MODELING THE MODIFIED INTERNAL RATE OF RETURN (MIRR) FOR LONG-TERM INVESTMENT STRATEGY BY THE ASSUMPTION OF GAMMA DISTRIBUTION

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by

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TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

Page

LIST OF SYMBOLS

K	Investment period
C_k	Investment annual contribution
a_k	Number of the annual contributions
D_k	Investment dividends
$F(K)$	Investment terminal fund
$u_{k,1}$	The of purchasing the share units at the beginning
$u_{k,2}$	The date that the annual financial report is released
E_k	Share unit purchased by the investor
B_k	Amount of the uninvested balance
$P_{u_{k,1}}$	Is the share price on the date $u_{k,1}$
$S^{(1)}$	Share units before the share issuance event
$S^{(2)}$	Share units after the event
Spl	Share split
Bns	Bonus share
Tre	Treasury share dividend
g	The share accumulation function
NPV	Net present value of a stock investment
r	Modified internal rate of return
i	Stocks invested in
t	Starting year of the investment
X_{itk}	The transformed modified internal rate of return $(1+r)$ at investment period k starts at year t and of stocks i
Γ	Gamma function
α	Shape parameters
θ	Scale parameter

γ	Growth rate parameters
f_X	Probability density function of X
t, δ	Some real random variables
m_{K_1}	First moment
m_{K_2}	Second moment
μ	The mean of a random variable X also can be written as $E(X)$
σ^2	The variance of a random variable X also can be written as $Var(X)$
L_K	Likelihood function
l_K	Log-likelihood function
$\hat{\alpha}$	Estimated shape parameters
$\hat{\theta}$	Estimated Scale parameter
$\hat{\gamma}$	Estimated growth rate parameters
δ , ω	Sets of unknown parameters
$MAE_{(\hat{\alpha},\hat{\theta},\hat{\gamma})}$	The mean absolute error of the estimated parameters
$RMSE_{V_i}$	The root means square error of the variance

LIST OF ABBREVIATIONS

NPV	Net present value
IRR	Internal rate of return
MIRR	Modified internal rate of return
GD	Gamma distribution
GGD	Generalized gamma distribution
MM	Method of moment
MLE	Maximum likelihood estimator
E(X)	The mean of *X*
Var(x)	The variance of *X*
SA	Simulated annealing
MCMC	Markov chain Monte Carlo
RS	Rejection sampling
MH	Metropolis Hastings
ARS	Adaptive Rejection sampling
ARMS	Adaptive Rejection Metropolis sampling
MAE	Mean absolute error
RMSE	Root mean square error

PEMODELAN KADAR PULANGAN DALAMAN YANG DIUBAHSUAI (MIRR) BAGI STRATEGI PELABURAN JANGKA PANJANG DENGAN ANDAIAN TABURAN GAMMA

ABSTRAK

Penyelidikan ini bertujuan untuk membangunkan model bagi Kadar Pulangan Dalaman Terubahsuai (MIRR) dalam strategi pelaburan jangka panjang menggunakan pengagihan gamma. MIRR menawarkan penyelesaian kepada masalah berbilang nilai Kadar Pulangan Dalaman (IRR) yang dihadapi apabila menggunakan model tradisional seperti Nilai Kini Bersih (NPV) dan IRR. Kajian ini meneroka penggunaan taburan gamma, yang memberikan fleksibiliti yang lebih besar berbanding dengan taburan normal dan eksponen yang biasa digunakan dalam kewangan. Untuk memodelkan MIRR dalam tempoh pelaburan lanjutan, pelbagai parameter kewangan, termasuk harga saham, dividen dilabur semula, pecahan saham, terbitan bonus dan dividen saham perbendaharaan, diambil kira. Anggaran parameter bentuk dan skala taburan gamma adalah agak mudah menggunakan kaedah momen. Walau bagaimanapun, menganggarkan ketiga-tiga parameter secara serentak (bentuk, skala dan pertumbuhan) melalui fungsi kemungkinan maksimum adalah kompleks dari segi pengiraan. Pendekatan alternatif seperti algoritma Simulated Annealing (SA), yang memaksimumkan fungsi log-kemungkinan, dan anggaran MCMC Bayesian dipertimbangkan. Kajian itu menganalisis data daripada 62 perniagaan hartanah Malaysia yang disenaraikan secara terbuka merangkumi tempoh dari 2008 hingga 2019. Tempoh pelaburan yang berbeza antara satu hingga lapan tahun dipertimbangkan. Penemuan menunjukkan bahawa taburan gamma memberikan kesesuaian yang baik untuk memodelkan MIRR yang diubah dalam tempoh pelaburan jangka panjang. Dengan menggunakan kaedah yang dicadangkan, penyelidikan berjaya menganggarkan parameter taburan gamma dan mengesahkan kesesuaiannya untuk menangkap pengagihan pulangan ke atas aset kewangan. Pengagihan gamma muncul sebagai pilihan yang sesuai untuk memodelkan MIRR dalam strategi pelaburan jangka panjang. Ia menawarkan fleksibiliti yang lebih besar berbanding dengan taburan normal yang biasa digunakan. Kajian ini

menyerlahkan kepentingan menganggarkan parameter taburan gamma dengan tepat dan mencadangkan kaedah alternatif seperti Simulated Annealing dan anggaran MCMC Bayesian untuk mengatasi cabaran pengiraan. Penyelidikan menyumbang kepada pemahaman teknik penilaian pelaburan dan menyediakan rangka kerja yang berharga untuk menilai prestasi projek pelaburan jangka panjang.

MODELING THE MODIFIED INTERNAL RATE OF RETURN (MIRR) FOR LONG-TERM INVESTMENT STRATEGY BY THE ASSUMPTION OF GAMMA DISTRIBUTION

ABSTRACT

This research aims to develop a model for the Modified Internal Rate of Return (MIRR) in long-term investment strategies using the gamma distribution. The MIRR offers a solution to the problem of multiple Internal Rate of Return (IRR) values encountered when using traditional models like Net Present Value (NPV) and IRR. The study explores the use of the gamma distribution, which provides greater flexibility compared to the normal and exponential distributions commonly used in finance. To model the MIRR over an extended investment period, various financial parameters, including stock price, reinvested dividends, stock splits, bonus issues, and treasury share dividends, are taken into account. The estimation of the shape and scale parameters of the gamma distribution is relatively straightforward using the method of moments. However, simultaneously estimating all three parameters (shape, scale, and growth) through the maximum-likelihood function is computationally complex. Alternative approaches such as the Simulated Annealing (SA) algorithm, which maximizes the log-likelihood function, and Bayesian MCMC estimation are considered. The study analyzes data from 62 publicly listed Malaysian property businesses spanning the period from 2008 to 2019. Different investment durations ranging from one to eight years are considered. The findings demonstrate that the gamma distribution provides a good fit for modeling the transformed MIRR over a long-term investment period. By utilizing the proposed methods, the research successfully estimates the parameters of the gamma distribution and validates its suitability for capturing the distribution of returns on financial assets. The gamma distribution emerges as a suitable choice for modeling the MIRR in long-term investment strategies. It offers greater flexibility compared to the commonly used normal distribution. The study highlights the importance of accurately estimating the parameters of the gamma distribution and suggests alternative methods like Simulated Annealing and Bayesian

MCMC estimation to overcome computational challenges. The research contributes to the understanding of investment evaluation techniques and provides a valuable framework for assessing the performance of long-term investment projects.

CHAPTER 1. INTRODUCTION

1.1 Study Background

This chapter provides an overview of investment terminology and modelling, with a focus on the modified internal rate of return (MIRR) and the various studies conducted to develop and model it. The potential of using the gamma distribution to model and analyze real-life phenomena is also explored, along with common methods for estimating gamma parameters. Additionally, the generalized gamma distribution (GGD) is highlighted as an important generalization of the gamma distribution, with its statistical properties discussed. Finally, the chapter delves into the background of using simulation and optimization methods to solve data modelling problems.

1.2 Investment

Investment is a fundamental concept in finance, which involves allocating resources to generate future returns (Reilly et al., 2016). It is an important tool for individuals, companies, and governments to accumulate wealth, finance projects, and create economic growth. The study of investment covers a wide range of topics, including asset pricing, portfolio management, risk management, and behavioural finance (Renneboog et al., 2008). Investment can be made in a variety of asset classes, including stocks, bonds, real estate, commodities, and alternative investments. The goal of investing is to maximize returns while minimizing risks (Cumming et al., 2014).

According to the Modern Portfolio Theory, investment decisions should be based on the trade-off between risk and return. The risk-return trade-off suggests that

higher returns are typically associated with higher levels of risk (Lettau & Ludvigson, 2010). Therefore, investors must balance their desire for higher returns with their risk tolerance. The field of investment has evolved significantly over time, with new theories, models, and technologies emerging to help investors make more informed decisions (Virlics, 2013). For example, the emergence of index funds and exchange-traded funds (ETFs) has made it easier for investors to diversify their portfolios at lower costs. The rise of robo-advisors has also made it easier for individual investors to access professional investment advice and portfolio management services.

In recent years, environmental, social, and governance (ESG) factors have also become an important consideration for investors. Research has shown that companies with strong ESG profiles tend to outperform those with weaker profiles in the long run. As a result, investors are increasingly integrating ESG considerations into their investment decisions. Investing in the financial market is a process that necessitates the formulation of a decent strategy aimed at making profits from the investment. Investors, whether as companies or individuals, must have a clear vision of the investment return and be aware of the risks associated with investing (Amel-Zadeh & Serafeim, 2018; Barko et al., 2021; Gary, 2019; Giese et al., 2021). As a result, examining and evaluating the firm's financial report in which they desire to invest is critical in providing them with the information they need to make a wise decision. The study of investment is a critical component of finance and economics. Understanding the risk-return trade-off, diversification, and the impact of ESG factors on investment decisions are all essential for investors to achieve their financial goals (Madhavan et al., 2021).

An asset or object purchased to generate income or appreciation is an investment. The term "appreciation" indicates the increase in value of an asset over

time. When someone purchases something as an investment, the purpose is not to consume it, but to use it to generate wealth in the future. Stocks, bonds, and Certificates of Deposits (CDs) are some types of investments to invest in for the average person. Stocks allow you to invest in a company's equity, which means you have a residual claim to the company's future profits and typically earn voting rights (depending on the number of shares you hold) to influence the company's decision. Bonds and CDs are debt investments in which the borrower invests money in a project that is projected to generate more cash flow than the interest owing to the investors. Any investment can follow two strategies (Bodie & Kane, 2020; Reilly et al., 2016).

1.2.1 Short-term investment strategy

Short-term investment strategies are a popular choice for investors with short-term goals, as they provide quick liquidity and a low-risk profile. Examples of short-term investment instruments include money market funds, certificates of deposit, Treasury bills, and commercial paper (Virlics, 2013). One of the primary advantages of short-term investments is their liquidity. Investors can easily access their funds without facing penalties for early withdrawals. Additionally, these investments usually come with minimal risks, providing a relatively stable source of income (Lettau & Ludvigson, 2010).

However, short-term investment strategies may not offer significant returns compared to long-term investments. They may also be susceptible to inflation, resulting in a decline in the value of returns over time (Asafo-Adjei et al., 2020). Investors should consider their goals, risk tolerance, and prevailing market conditions before embarking on a short-term investment strategy. Diversification can

also minimize risks by spreading the investment across multiple instruments (Hoskisson et al., 2018).

Short-term investment strategies offer quick liquidity and a low-risk profile, making them a popular choice for investors with short-term goals. However, they may not provide substantial returns and may be vulnerable to inflation, making it crucial to carefully consider the prevailing market conditions and diversify the investment portfolio. That means the investor should decide whether to retain the stock for a short time, which can be profitable but risky sometimes.

1.2.2 Long-term investment strategy

A long-term investment strategy is a method of investing that involves holding securities for an extended period, typically over several years or decades. This approach is often used by investors who are seeking to build wealth over time, such as those planning for retirement, funding education, or buying a home (Ahmad, 2020). Long-term investment strategies may include investing in stocks, bonds, mutual funds, exchange-traded funds (ETFs), real estate, and other assets. Such investments tend to provide higher returns than short-term investments, although they also come with more significant risks. For example, the stock market may experience volatility in the short-term, but over the long-term, it has historically delivered strong returns (Zhang, 2018). One of the primary advantages of a long-term investment strategy is the power of compounding. By reinvesting the earnings from an investment over time, investors can potentially achieve significant growth in their portfolio. Additionally, long-term investments may provide tax benefits, such as capital gains tax deferral on investments held for more than one year (Siegel, 2021). However, long-term investments require patience and discipline, and investors must

be prepared to weather market fluctuations and economic cycles. Diversification is also essential to minimize risks by spreading investments across different asset classes and sectors (Obaro et al., 2022).

Investors should consider their financial goals, risk tolerance, and investment horizon before embarking on a long-term investment strategy. Professional financial advice may also be beneficial in developing and implementing such a plan. A long-term investment strategy can provide a powerful path to wealth accumulation, with the potential for significant growth and tax benefits. However, it requires patience, discipline, and diversification to manage risks and navigate market fluctuations over time. Which means investing in stocks for a long time to avoid the risk and make a steady profit. This study focuses on stock investment with an assumption of long-term investment strategy, explained in the next chapter.

1.2.3 Stock

A stock, also known as an equity, represents ownership in a company and represents a claim on a portion of the corporation's assets and earnings. As a shareholder, an individual has the right to vote on company decisions and to receive dividends if the company distributes profits. It is typically issued by companies to raise capital for operations, expansion, or other purposes. They can be traded on stock exchanges, such as the New York Stock Exchange (NYSE) or NASDAQ, or through private sales. Stocks can be a critical component of individual investors' portfolios due to their potential for long-term growth and returns(Cillo et al., 2018). However, buying and selling stocks involves risks, including fluctuations in the stock price and the potential loss of invested capital. Therefore, investors need to conduct

thorough research and analysis of a company's financials and overall health before investing in its stock (Peloza, 2009).

1.2.4 Share

Shares of a corporation represent ownership in the company and are often referred to as stock or equity. As a shareholder, an individual holds a portion of the company and has the right to vote on corporate matters and receive a share of any profits, typically in the form of dividends(Sheu & Yang, 2005). However, not all companies pay dividends, and shareholders of such stocks rely on the expectation of capital appreciation from a rise in stock prices. This expectation is driven by the company's ability to generate profits and growth in earnings, leading to an increase in demand for its stock (Lazonick, 2014).

The concept of equity ownership dates back to the establishment of the first joint-stock companies in the 17th century, which allowed investors to own a portion of a company and share in its profits. Today, shares are typically bought and sold on stock exchanges or through electronic trading platforms (Davies, 1952). It is important for investors to conduct thorough research and analysis of a company's financials and overall health before investing in its stock. This includes assessing the company's management, competitive position, and potential risks and opportunities (Churet & Eccles, 2014). Shares of a corporation represent ownership in the company and can provide potential profits and growth for investors through dividends and capital appreciation. However, investing in stocks involves risks, and investors must conduct due diligence and assess a company's financials and overall health before investing (Bolton & Samama, 2013).

1.2.5 Cash flow

Cash flow is an important metric that measures the net balance of cash coming into and out of a firm during a specified period. This metric is vital for assessing a company's liquidity and financial health. Positive cash flow indicates that a corporation is generating more cash than it is spending, while negative cash flow suggests that a firm is spending more cash than it is generating (Lewellen & Lewellen, 2016).

According to financial experts, cash flow can be further classified into three distinct categories: operating cash flow, investing cash flow, and financing cash flow. Operating cash flow refers to the cash generated or consumed by a company's primary business activities, such as sales and expenses. Investing cash flow, on the other hand, reflects the cash used for investments in capital assets or the sale of those assets. Finally, financing cash flow is the cash inflow or outflow related to the company's financing activities, such as debt or equity issuance, dividend payments, and share repurchases (Dirman, 2020; Foerster et al., 2017). Understanding a company's cash flow position is crucial for investors and stakeholders alike, as it provides insights into a company's ability to meet its financial obligations, fund future growth, and create long-term value for shareholders. Cash flow is a crucial aspect of financial analysis, and there are three main types of cash flow (Harber, 2017).

1.2.5.1 Operating Cash Flow (OCF):

OCF is the cash generated or used by a company's primary operations, such as sales of products or services. OCF is an important indicator of a company's financial health because it shows how much cash a company has to pay its bills and

fund future growth. According to the study by (Ball et al., 2016), operating cash flow is positively related to firm value, and firms with higher operating cash flows are associated with higher market values.

1.2.5.2 Investing Cash Flow (ICF):

ICF reflects the cash used for investments in assets like property, plant, and equipment, as well as proceeds from the sale of those assets. ICF is important because it can indicate whether a company is investing in its future growth and profitability (Choi et al., 2018). As stated by (Ugwu & Oliver, 2021)investing cash flow is positively associated with firm value, which suggests that firms with higher investing cash flows are associated with higher market values.

1.2.5.3 Financing Cash Flow (FCF):

FCF reflects the cash raised or used for financing activities, such as issuing or repurchasing stock, paying dividends, or taking out or repaying loans. FCF is important because it can indicate how a company is funding its operations and growth (Bhandari & Adams, 2017). The study by Danila et al.(2020) indicated that financing cash flow is negatively related to firm value, which suggests that firms with higher financing cash flows are associated with lower market values. Understanding a company's cash flow is essential for investors to evaluate its financial health and make informed decisions about investing in the company (Blessing & Onoja, 2015; Daugaard, 2020; Lee & Lee, 2015; Palepu et al., 2020).

1.2.6 Rate of return.

The rate of return (R) is the net gain or loss on an investment over a specific period expressed as a percentage of the investment's initial cost(Jurek & Stafford,

2015). When computing the rate of return, it is necessary to consider the percentage change from the beginning to the end of the period. The rate of return is given by,

$$R = \frac{V_f - V_i}{V_i} \tag{1.1}$$

where *R* is the rate of return, V_f is the final value which includes the dividends and interest rate (investment gain), and V_i is the initial value (cost of investment).

1.2.7 Dividends

Dividends are a distribution of a portion of a company's profits to its shareholders and are typically paid out in two forms: cash or stock. The board of directors of a company decides on the dividend amount and declares it on the declaration date, but the shareholders who own the stock before the ex-dividend date are eligible to receive the payout (Hanafi et al., 2023).

Cash dividends are the most common form of dividend payment, and involve the company paying out cash to its shareholders. This form of dividend payout provides shareholders with a direct cash return on their investment in the company(Udeh et al., 2018). On the other hand, stock dividends, as explained by research by Nguyen et al. (2015), involves the company distributing additional shares of its stock to its shareholders instead of cash. This form of dividend payout increases the number of shares outstanding but does not affect the total value of the shareholder's investment. Companies must carefully evaluate their financial position before declaring dividends and choose the appropriate form of dividend payment that aligns with their financial goals and objective. Dividends can be declared in two Cash dividends and stocks dividends.

1.2.7.1 Cash dividends

A cash dividend is a distribution of a company's earnings in the form of cash payments to shareholders, typically paid out of the company's current earnings or accumulated profits. Unlike other forms of dividends, such as stock dividends, which are paid in additional shares of the company, or property dividends, which are paid in assets, cash dividends are paid in cash to shareholders (Yusra et al., 2019).

Cash dividends can be a signal of the company's financial health and ability to generate consistent profits. They also provide a source of income for shareholders and can help attract and retain investors (Baker & Weigand, 2015). The amount and timing of cash dividends are determined by the company's board of directors, based on various factors such as financial performance, capital needs, and growth opportunities. Cash dividends are a common way for companies to reward their shareholders and distribute their profits, while also maintaining financial stability and attracting new investors (Dedman et al., 2017).

1.2.7.2 Stock dividends

A stock dividend is a form of dividend payment in which the company distributes additional shares to its shareholders rather than cash. This approach is frequently used by companies when they have a shortage of liquid cash(Rashid, 2018). The board of directors determines when a stock dividend should be declared and in what form it should be paid. Stock dividends can be advantageous to investors since they allow them to acquire more shares in the company without having to buy additional shares on the open market. Additionally, stock dividends can increase the liquidity of the stock by increasing the number of shares available for trading (Bostanci et al., 2018).

1.2.8 Investment Risk

Investment risk is defined as the possibility of incurring losses concerning the expected return on a particular investment (Holzmeister et al., 2020). It reflects the degree of uncertainty in achieving the investor's anticipated returns. Investment risk can arise from a variety of factors, including market fluctuations, economic instability, changes in government regulations, and company-specific issues, such as management or operational risks (Liu et al., 2017). Investors must assess the risk associated with an investment before committing funds to it to determine if the potential reward outweighs the risk involved (Guiso et al., 2018).

Overall, the study of long-term investment modelling is an important area of research for investors, financial institutions, and policymakers alike. By providing insights into the performance of different types of investments over long periods, these models can help investors make informed decisions about how to allocate their resources and manage risk over the long term. This capital investment analysis is done through investment moulding which can be done in many techniques including The net present value (NPV), Profitability index (PI), and Internal rate of return (IRR) to mention but a few.

1.2.9 Net present value (NPV)

The net present value (NPV) is one of the powerful methods that can model stock investment (N. Nguyen et al., 2021). As it is a profitability metric that calculates the present value of all predicted future cash flows generated by a project or investment, including the initial capital invested. The NPV model depicts the difference in the current value of cash inflows and outflows over time. The Net Present Value of a series of cash flows is determined by calculating the costs

(negative cash flows) and benefits (positive cash flows). The NPV on a long term-investment with multiple cash flows can be calculated as follows,

$$NPV = \sum_{t=0}^{n} e^{\frac{C_t}{(1+i)^t}} \tag{1.2}$$

where C_t is the net cash inflow and outflows within a specific period i determined in the formula for net present value (NPV). The discount rate or alternative investment return represents the rate at which the investment could earn. Additionally, the number of periods is also a crucial factor in NPV calculation. While NPV is a robust financial tool, its accuracy is heavily dependent on key assumptions, such as utilizing a uniform discount rate throughout the project, which can leave significant room for error. It can be challenging to avoid changes that impact the rate of return, making it a critical factor in accurate NPV calculations (Tao & Finenko, 2016).

1.2.10 Internal rate of return (IRR)

In the field of investment modelling, the internal rate of return (IRR) is widely utilized as a powerful tool(Dhavale & Sarkis, 2018). IRR is defined as the discount rate that sets the net present value (NPV) of a project equal to zero. Simply put, it is the expected annual compounded rate of return on a given investment or project. The IRR can be calculated using financial calculators, the XIRR function in Excel, or by equating the NPV to zero using the Goal Seek tools in Excel (Sabri et al., 2020; Sabri & Sarsour, 2019; Sarsour & Sabri, 2020a).

However, the IRR may not accurately reflect the profitability and cost of a project because it assumes that all positive cash flows from a project are reinvested at

the same rate as the project, rather than at the company's cost of capital (Bora, 2015; Lilford et al., 2018). Moreover, the process of calculating IRR may produce multiple values, making it less effective for use in investment modelling (Puška et al., 2018). Despite its limitations, the IRR remains a widely used tool in investment modelling due to its simplicity and ease of calculation.

1.2.11 Modified Internal Rate of Return (MIRR)

In projecting stock market performance in short- and long-term investments, stock price plays a critical role in guiding investors' investment selections. The product of the number of share units held and the current stock price is used to determine the value of an investment. Therefore, it is necessary to investigate investment performance, as they primarily measure the fluctuations that occur in the stock market In projecting stock market performance in short- and long-term investments, stock price plays a critical role in guiding investors' investment selections. The product of the number of share units held and the current stock price is used to determine the value of an investment. Therefore, it is necessary to investigate investment performance, as they primarily measure the fluctuations that occur in the stock market (Siegel, 2021). As a result, many investment evaluation methodologies have been combined to calculate the rate of return, such as net present value (NPV) and internal rate of return (IRR) or modified IRR (MIRR) methods, in capital budgeting based on discounted cash flow (DCF) to determine the investment feasibility (Focacci, 2022; Gul, 2018; Kulakov & Blaset, 2020; Lifland, 2015; Mubashar & Tariq, 2019).

The rate of return calculation can be an iterative approach aimed at finding the root, such as the Heuristics Feature Selection with Newton Raphson and Gradient Descent Algorithm (Handoyo et al., 2022), Fast Newton-Raphson (Ahmadi

et al., 2021), and Enhanced Newton-Raphson Algorithm (Pascual et al., 2019). However, there are certain issues with employing these strategies because they do not account for all relevant aspects that influence investment return, making their methodologies ineffective for determining stock performance. As a result, several researchers have created alternate investment appraisal methodologies, called the modified internal rate of return (MIRR), to address this problem defined the MIRR as the rate at which the NPV equals zero, that is, where the present value of the invested fund's terminal value (future value of cash inflows assumed to be reinvested at the firm's required rate of return) equals the present value of the investment outlays (cash outflows over the investment period) when discounted at the firm's required rate of return.

Sabri & Sarsour (2019) proposed an effective approach to compute the modified internal rate of return (MIRR) on long-term investments by considering significant financial data such as stock price, reinvested dividends, and share issuances like share splits and bonus issues. Their stock investment model provided a more accurate picture of the investment's rate of return by demonstrating the computation of MIRR using the method of yearly annuity mode contributions. However, the study overlooked the value of the treasury share (share payback) that some corporations use instead of cash dividend payout, which affects the investor's share unit and determines the rate of return. Investment modelling is crucial since it determines the type of information that influences an investor's judgment. Therefore, it is more beneficial to use an appropriate statistical model to distribute and analyze the return. However, as pointed out by the cited studies, this aspect has been overlooked, and it is essential to consider it.

1.3 Statistical Models for Modified Internal Rate of Return (MIRR)

The Modified Internal Rate of Return (MIRR) is a method used to estimate the profitability of an investment, taking into account the time value of money and reinvestment rate assumptions. It is a useful tool for evaluating investment opportunities as it provides a more accurate measure of return than the traditional Internal Rate of Return (IRR) method. However, analyzing MIRR requires an appropriate statistical model that can handle the complex financial data involved. One statistical model that has been used for analyzing MIRR is the Gamma distribution. The Gamma distribution is a two-parameter distribution commonly used to model continuous random variables with positive skewness. It has been used in finance to model various financial variables, including the distribution of asset returns, the time to default, and the size of insurance claims (Brunner et al., 2022; de Véricourt & Gromb, 2018; Gamm, 2018; Gholami & Mirzazadeh, 2018; Gollier, 2016; Zahro & Caraka, 2017).

Gamma distribution has been explored in modelling in finance modelling. This includes intelligent financial planning decisions(Blanchett & Kaplan, 2018), Corporate Bond Market (Zhang, 2022), Portfolio formation (Cremers et al., 2003), Asset return distributions and the investment horizon (Levy & Duchin, 2004), asset return distributions, risk management, portfolio selection, and option pricing (Fabozzi et al., 2005), risks and rewards (Madan et al., 2020), Sequence-of-returns Risk (Cotton, 2015) and The Valuation Model for a Risky Asset When Its Risky Factors (Tsai & Chiang, 2016). Most of these studies found that the Gamma distribution provided a better fit to the data than other commonly used distributions such as the Normal and Lognormal distributions. Therefore, the Gamma distribution

has been employed in this study due to its useful statistical model for analyzing the MIRR, as it can handle the positive skewness of financial data and provide a good fit to the data. Other statistical models such as the Beta distribution and the lognormal distribution have also been used for analyzing the MIRR and may be worth exploring depending on the nature of the financial data involved.

1.4 Problem Statement

Investors make decisions based on various factors, including economic and behavioural analysis. However, investment modelling studies that evaluate all financial aspects of the stock market are necessary to provide a more realistic picture of investment returns and aid in appropriate decision-making. Previous studies focused on short-term investment strategies and overlooked the risk associated with such investments. Sabri and Sarsour (2019) long-term stock investment model calculated the MIRR using an annual annuity-style strategy of contributions, providing a more accurate representation of investment returns. However, their model overlooked the evaluation of treasury share dividends that are commonly used by companies in the Malaysian stock market. This study aims to expand the MIRR calculation by considering treasury share dividends, which can significantly impact the investment rate of return. Additionally, this study recommends adjusting the MIRR by adding one to negate any negative returns.

While several studies analyze financial data, there is still a lack of literature on the use of statistical distributions in financial analysis. The gamma distribution is a useful tool for analyzing investment rate of return, as it describes continuous variables with a positive sign and time intervals between occurrences logically with positive and continuous variables. As the MIRR is a continuous and positive variable

calculated over periods, it is appropriate to be distributed using the gamma distribution. This study is the first to use gamma distribution and generalized gamma distribution to analyze the MIRR on long-term investment strategies and study its behaviour over different investment periods. There are established methods to estimate the shape and scale parameters of the gamma distribution, estimating three or more parameters of the generalized gamma distribution using the same methods is not efficient. There is a literature gap in addressing the challenges of employing extended and generalized distributions in modelling real-world problems. Additionally, few studies have looked into the methods for estimating the parameters of the generalized gamma distribution. Simulated annealing (SA) and other optimization approaches have been employed to overcome modelling difficulties. In this study, SA is used to estimate the parameters of the generalized gamma distribution and compared with Bayesian estimation using the MCMC' ARMS algorithm. The performance of SA is measured to support the argument for its use as a parameter estimator for extended distributions.

Despite the increasing number of studies that evaluate different financial aspects of the stock market to help investors make informed decisions, there is still a significant gap in the literature regarding the use of statistical distributions in financial analysis. This gap is particularly evident when it comes to the application of the gamma distribution to analyze the investment rate of return. The current study aims to address this gap by utilizing the gamma distribution and generalized gamma distribution to analyze the MIRR in long-term investment strategies. Specifically, the study aims to investigate the behaviour of MIRR over different investment periods by using the properties of the gamma distribution.

Furthermore, while there are well-known methods to estimate the shape and scale parameters of the gamma distribution, such as the moment method and the maximum likelihood estimator, there is still a lack of research on how to estimate three or more parameters of the generalized gamma distribution using the same methods. To address this research gap, the study employs simulated annealing as a method to estimate the parameters of the generalized gamma distribution and compare its performance with Bayesian estimation using the MCMC' ARMS algorithm

1.5 Research Objectives

The main objective of this study is to perform investment modelling using the modified internal rate of return (MIRR) on the gamma and generalized gamma distributions. The objectives of this study can further be divided into the following specific objectives:

1. To analyze the modified internal rate of return (MIRR) on long-term investments by considering treasury share dividends, which may considerably affect the investment rate of return.
2. To investigate the behavior of MIRR over different periods of investment using gamma distribution and generalized gamma distribution.
3. To estimate the parameters of the generalized gamma distribution using simulated annealing (SA) method and Bayesian estimation using the MCMC' ARMS algorithm and compare their performance.

1.6 Justification of the Study

This study is justified by the need for a more comprehensive and accurate approach to investment modelling that can aid investors in making informed

investment decisions. Current studies in this field tend to focus on either economic or behavioural analysis perspectives, but there is a gap in research that examines every financial aspect of the stock market that can impact investment returns. This study aims to address this gap by expanding the calculation of the Modified Internal Rate of Return (MIRR) to include treasury share dividends, which is an important factor that is often overlooked in current investment models.

Moreover, while there is a wide range of literature on statistical analysis in finance, there is still a lack of research on the use of statistical distributions in financial analysis. The Gamma distribution is a useful distribution for analyzing investment rate of return, as it is appropriate for continuous variables with positive values that are calculated over some time. This study is the first to apply Gamma distribution and Generalized Gamma distribution to analyze the MIRR on a long-term investment strategy and to use their properties to examine its behavior over different investment periods.

Furthermore, there is a gap in the literature regarding the challenges of employing extended and generalized distributions in modeling real-world problems in general, and the number of studies on the methods for estimating the parameters of the generalized gamma distribution is quite modest. This study aims to fill this gap by utilizing the Simulated Annealing (SA) method and Bayesian estimation using the MCMC' ARMS algorithm to estimate the parameters of the generalized gamma distribution and compare their performance.

This study seeks to contribute to the field of investment modelling by providing a more comprehensive and accurate approach to calculating the MIRR and

utilizing statistical distributions, as well as by addressing the challenges associated with using extended and generalized distributions in modelling real-world problem

1.7 Significance of the Study

This study has several significances. First, it aims to improve the calculation of the Modified Internal Rate of Return (MIRR) by incorporating all financial elements that can affect investment returns, including stock price, reinvested dividends, and share issuance events such as stock splits, bonus issues, and treasury share dividends. By doing so, the study will provide a more reliable and realistic picture of investment returns that can help investors make more informed decisions.

Second, the study focuses on long-term investment strategies and aims to demonstrate the attractiveness of this approach to investors. By analyzing data from 62 publicly traded Malaysian property companies over the period from 2008 to 2019, with investment periods ranging from one to eight years, the study aims to show that long-term investment strategies can reduce risk and increase profits.

Third, the study employs advanced statistical techniques, including the Gamma and Generalized Gamma Distributions, and optimization algorithms, including the Simulated Annealing (SA) and Bayesian estimation using the MCMC' ARMS algorithm, to analyze the transformed MIRR and estimate the parameters of the GGD. This contributes to the literature on the use of statistical distributions in financial analysis and provides a new method for estimating the parameters of the GGD.

Overall, this study can help investors make more informed decisions by providing a more reliable picture of investment returns and demonstrating the attractiveness of long-term investment strategies. The use of advanced statistical

techniques and optimization algorithms also contributes to the literature on financial analysis and provides new methods for estimating the parameters of statistical distributions

1.8 Scope of the Study

The primary objective of this study is to investigate the effectiveness of various statistical techniques such as gamma distribution, GGD, SA algorithm, and ARMS algorithm in modelling the modified internal rate of return (MIRR) for investment portfolios. The study is conducted on 62 shariah compliance of publicly traded Malaysian property companies between 2008 and 2019. The MIRR values extracted from these companies are used to compare the accuracy of the statistical models and determine which one provides the best fit

1.9 Limitations of the Study

The present study has several limitations that should be acknowledged. Firstly, due to the limited availability of research on investment modelling using the modified internal rate of return, it was challenging to draw upon the existing literature. Secondly, the study only focuses on one financial sector, the Malaysian property sector, as obtaining financial data from different sectors can be difficult and time-consuming. Additionally, the study period was limited to 2008-2019 to avoid any impact of the Covid-19 pandemic on the Malaysian financial market. Lastly, the study only considers investment periods of up to eight years, as a longer period may result in insufficient data to work with

1.10 Study Outline

This Thesis is divided into seven chapters and proposes novel models based on the gamma distribution to improve the fitting of financial datasets, such as MIRR

data, insurance claims data and other real-world datasets. The organization of the paper is as follows:

Chapter 2 provides an extensive literature review of investment modelling frameworks and statistical distributions. The review covers investment and return, modified internal rate of return (MIRR) as an investment model, and the gamma distribution approach for real-life data. The review also discusses the generalized gamma distribution model, parameter estimate methods, and simulated annealing algorithms to estimate the parameters of the GGD model.

Chapter 3 describes the research methodology and introduces a long-term investment strategy framework that models the MIRR affected by several financial elements such as share accumulation and investment cash flows. This chapter also presents the calculation of the MIRR and the transformed modified internal rate of return.

Chapter 4 focuses on the gamma distribution model of the transformed modified internal rate of return MIRR. The chapter discusses the classic numerical technique, maximum likelihood estimation (MLE), and methods of moments (MM) as methods for estimating unknown gamma distribution parameters.

Chapter 5 proposes a new model called the generalized gamma distribution (GGD) to suit the MIRR data. By introducing the growth rate parameter into the previous model, the two-parameter gamma distribution has been generalized. The chapter also discusses the data mathematical properties and maximum likelihood estimation. Simulated annealing and Bayesian approaches are used to estimate the parameters of the suggested model.

Chapter 6 presents a simulation analysis based on the proposed GGD model. The GGD is used to model simulated data, and the SA algorithm along with the ARMS algorithm is used to estimate the model parameters.

Finally, Chapter 7 summarizes the paper's primary aims, conclusions, and recommendations for further research.

CHAPTER 2. LITERATURE REVIEW

2.1 Introduction

Investment modelling is an important field in finance that involves predicting the future values of financial assets. The modified internal rate of return (MIRR) is a popular approach for evaluating investment opportunities, especially for long-term investment strategies. The selection of an appropriate probability distribution to model financial data is a crucial aspect of investment modelling. The gamma distribution, characterized by two parameters, shape, and scale, is a widely used distribution for financial data modelling due to its flexibility and ability to model skewed and heavy-tailed data. Recent studies have focused on modelling the MIRR using the gamma distribution and other statistical distributions, such as the generalized gamma distribution and the skewed generalized t-distribution. Various optimization methods, including maximum likelihood estimation, method of moments, simulated annealing, and adaptive rejection metropolis sampling, have been used to estimate the parameters of the gamma distribution. This literature review provides an overview of these studies and their contributions to the field of investment modelling.

2.2 Review of the investment modelling

Investment modelling is a crucial field in finance that deals with predicting the future values of financial assets. One popular approach for evaluating investment opportunities is to use the modified internal rate of return (MIRR), which is a modification of the internal rate of return (IRR) that accounts for the reinvestment rate and capital cost. The MIRR is particularly useful for evaluating long-term