First Semester Examination
2022/2023 Academic Session

February 2023

EEE443 – Digital Signal Processing
(Pemprosesan Isyarat Digit)

Duration : 2 hours
(Masa : 2 jam)

Please check that this examination paper consists of **TWELVE (12)** pages of printed material including appendix before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA BELAS (12)** muka surat yang bercetak termasuk lampiran sebelum anda memulakan peperiksaan ini.]*

Instructions : This paper consists of **FOUR (4)** questions. Answer **FOUR (4)** questions.

*[**Arahan** : Kertas ini mengandungi **EMPAT (4)** soalan. Jawab **EMPAT (4)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

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1. (a) Signal $y[n]$ is a periodic discrete-time signal, with period N_y equals to 10 samples. It is obtained by sampling analog signal $x_a(t)$. Signal $x_a(t)$ is given as:

Isyarat $y[n]$ ialah isyarat masa diskret berkala, dengan tempoh N_y bersamaan 10 sampel. Ia diperoleh dengan mensampel isyarat analog $x_a(t)$. Isyarat $x_a(t)$ diberikan sebagai:

$$x_a(t) = 0.5\cos(0.25\pi t + 0.1\pi)$$

(i) Suggest two sampling periods (T_1 and T_2) that can be utilized to sample $x_a(t)$ to produce $y[n]$.

Cadangkan dua tempoh pensampelan (T_1 dan T_2) yang boleh digunakan untuk sampel $x_a(t)$ bagi menghasilkan $y[n]$.

(20 marks/*markah*)

(ii) Based on T_1 from the answer in part (i), produce the expression for $y[n]$. Then, identify whether T_1 produces aliasing or not.

Berdasarkan kepada T_1 daripada jawapan dalam bahagian (i), hasilkan ungkapan $y[n]$. Kemudian, kenal pasti sama ada T_1 menghasilkan aliasing atau tidak.

(15 marks/*markah*)

(iii) Based on T_2 from the answer in part (i), produce the expression for $y[n]$. Then, identify whether T_2 produces aliasing or not.

Berdasarkan kepada T_2 daripada jawapan dalam bahagian (i), hasilkan ungkapan $y[n]$. Kemudian, kenal pasti sama ada T_2 menghasilkan aliasing atau tidak.

(15 marks/*markah*)

(b) A linear time invariant system is shown in Figure 1.

Sistem masa tak-varian lelurus ditunjukkan dalam Rajah 1.

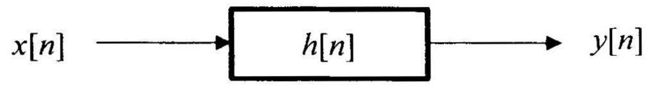

Figure 1
Rajah 1

Given the input $x[n] = \{4, -1, 5, 2\}$ and the impulse response $h[n] = \{3, 0, 4\}$, evaluate:

Diberikan masukan $x[n] = \{4, -1, 5, 2\}$ *dan sambutan dedenyut* $h[n] = \{3, 0, 4\}$, *nilaikan:*

(i) The output of the system, $y[n]$.

Keluaran sistem tersebut, $y[n]$.

(20 marks/*markah*)

(ii) The normalized cross-correlation, $\rho_{xy}[n]$.

Korelasi silang ternormal, $\rho_{xy}[n]$.

(30 marks/*markah*)

2. An input signal $w[n] = (-0.5)^n u[n]$ is given into a linear time-invariant system described by the following difference equation:

Isyarat masukan $w[n] = (-0.5)^n u[n]$ *diberikan kepada sistem masa tak-varian lelurus yang diterangkan oleh persamaan perbezaan berikut:*

$$y[n] = 0.4y[n-1] - 0.03y[n-2] + 0.5w[n]$$

To assist the analysis of the system, some of the common z-transform pairs are provided in Appendix A.

Untuk membantu analisa sistem tersebut, beberapa pasangan jelmaan-z yang biasa disediakan dalam Lampiran A.

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(a) From the difference equation, derive the equation for the system function, $H(z)$.

Daripada persamaan perbezaan, terbitkan persamaan untuk fungsi sistem, $H(z)$.

(10 marks/*markah*)

(b) Determine the impulse response $h[n]$ of the system.

Tentukan sambutan dedenyut $h[n]$ of sistem tersebut.

(20 marks/*markah*)

(c) Using Appendix A1, determine the z-tranform for input signal $w[n]$.

Menggunakan Lampiran A1, tentukan jelmaan-z isyarat masukan $w[n]$.

(10 marks/*markah*)

(d) Determine the expression of $Y(z)$ (i.e., the z-transform of $y[n]$) as a summation of fractions.

Tentukan ungkapan $Y(z)$ (iaitu jelmaan-z untuk $y[n]$) sebagai hasil tambah pecahan-pecahan.

(30 marks/*markah*)

(e) Determine the steady-state response, $y_{ss}[n]$.

Tentukan keluaran keadaan-mantap, $y_{ss}[n]$.

(10 marks/*markah*)

(f) Determine the transient response, $y_{tr}[n]$.

Tentukan sambutan fana, $y_{tr}[n]$.

(10 marks/*markah*)

(g) Determine the output of the system, $y[n]$.

Tentukan keluaran sistem, $y[n]$.

(10 marks/*markah*)

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3. (a) Using a bilinear transformation procedure, design a Butterworth bandpass IIR filter that fulfills the following specifications:
   - Sampling Frequency is 8000 Hz
   - Passband Attenuation is 2 dB in frequency range 800 Hz ≤ f ≤ 1000 Hz
   - Stopband Attenuation is 20 dB in frequency range 0 Hz ≤ f ≤ 400 Hz and f ≥ 2000 Hz

   In the proposed design, please determine:

   Menggunakan prosedur jelmaan bilinear, reka penapis IIR laluan jalur Butterworth yang memenuhi spesifikasi berikut:
   - *Frekuensi Pensampelan ialah 8000Hz*
   - *Pelemahan Jalur-laluan ialah 2dB dalam julat frekuensi 800 Hz ≤ f ≤ 1000 Hz*
   - *Pelemahan Jalur-henti ialah 20dB dalam julat frekuensi 0 Hz ≤ f ≤ 400 Hz dan f ≥ 2000 Hz*

   Dalam rekabentuk yang dicadangkan, sila tentukan:

   (i) The estimated filter order, n.

   Anggaran tertib penapis, n.

   (20 marks/*markah*)

   (ii) The analog filter transfer function, $H(s)$ when the filter uses the low pass prototype of $H_P(s) = \frac{1}{s+1}$.

   Fungsi pemindahan penapis analog, H(s) apabila penapis menggunakan prototaip laluan rendah bagi $H_P(s) = \frac{1}{s+1}$.

   (20 marks/*markah*)

   (iii) The digital filter transfer function, H(z) of the Butterworth bandpass filter.

   Fungsi pemindahan penapis digital, H(z) untuk penapis laluan jalur Butterworth tersebut.

   (20 marks/*markah*)

To assist the calculation, you may use the related formulas given in the appendix.

Untuk membantu pengiraan, anda boleh gunakan formula-formula berkaitan yang dibekalkan dalam lampiran.

(b) Using window design method, design a linear phase FIR band-stop filter with the desired frequency response as follow:

Menggunakan kaedah rekabentuk tetingkap, reka penapis jalur-henti FIR fasa lelurus dengan sambutan frekuensi yang dikehendaki seperti berikut:

$$0.99 \le H(e^{j\omega}) \le 1.01 \quad \text{for } 0\,Hz \le |f| \le 500\,Hz$$
$$H(e^{j\omega}) \le 0.0003 \text{ for } 1000\,Hz \le |f| \le 1500\,Hz$$
$$0.99 \le H(e^{j\omega}) \le 1.01 \quad \text{for } 2000\,Hz \le |f| \le 2500\,Hz$$

Given that the sampling frequency is at 5000 Hz. In the proposed design, please determine:

Diberi bahawa frekuensi pensampelan adalah pada 5000 Hz. Dalam rekabentuk yang dicadangkan, sila tentukan:

(i) The most appropriate window function to be used.

Fungsi tetingkap yang paling sesuai digunakan.

(10 marks/*markah*)

(ii) The length of the filter coefficients, M.

Panjang pekali penapis, M.

(10 marks/*markah*)

(iii) The general equation of the impulse response, $h[n]$.

Persamaan umum bagi sambutan dedenyut, $h[n]$.

(20 marks/*markah*)

To assist the calculation, you may use the related formulas given in the appendix.

Untuk membantu pengiraan, anda boleh gunakan formula-formula berkaitan yang dibekalkan dalam lampiran.

4. (a) Consider the signal flow graph of IIR filter in Figure 4(a). Produce the flow graph into a block diagram as a cascade second-order sections in direct form II format.

Pertimbangkan graf aliran isyarat penapis IIR dalam Rajah 4(a). Hasilkan graf aliran kepada gambar rajah blok sebagai keratan tertib kedua lata dalam format langsung II.

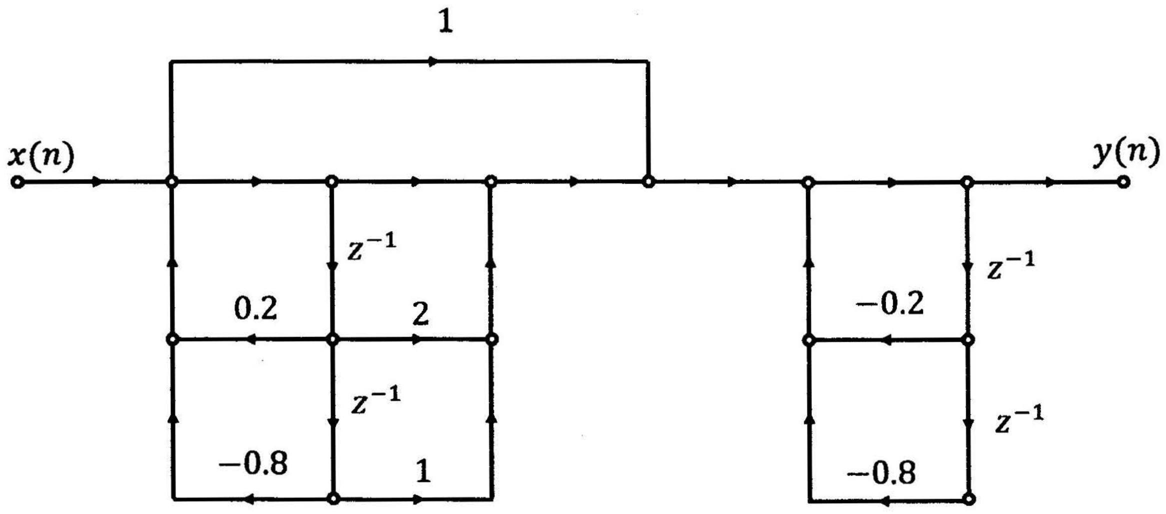

Figure 4(a)
Rajah 4(a)

(30 marks/*markah*)

(b) Given the IIR digital filter transfer function as below. Implement the system as a parallel network of first-order direct form II structure.

Diberikan fungsi pemindahan penapis digital IIR seperti di bawah. Laksanakan sistem tersebut sebagai rangkaian selari struktur bentuk langsung II tertib pertama.

$$H(z) = \frac{4 + \frac{9}{4}z^{-1} - \frac{1}{4}z^{-2}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

(40 marks/*markah*)

(c) The transfer function of a discrete-time system is given below. Draw a realization structure of the filter using cascade of second order system in direct form II and write down the set of difference equations that correspond to this implementation.

Sistem pemindahan sistem masa diskret diberikan di bawah. Lukiskan struktur realisasi penapis menggunakan lata sistem tertib kedua dalam bentuk langsung II dan tuliskan set persamaan perbezaan yang sepadan dengan pelaksanaan ini.

$$H(z) = \frac{(1 + z^{-1})^4}{(1 - z^{-1} + \frac{7}{8}z^{-2})(1 + 2z^{-1} + \frac{3}{4}z^{-2})}$$

(30 marks/*markah*)

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APPENDIX A

LAMPIRAN A

Table A.1: Some commons z-transform pairs.

	Signal $x[n]$	z-transform, $X(z)$	ROC
1	$\delta[n]$	1	All z
2	$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3	$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4	$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
5	$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
6	$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7	$(\cos\omega_0 n)u[n]$	$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
8	$(\sin\omega_0 n)u[n]$	$\frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
9	$(a^n\cos\omega_0 n)u[n]$	$\frac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
10	$(a^n\sin\omega_0 n)u[n]$	$\frac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

APPENDIX B

LAMPIRAN B

Table B.1: List of formulas for filter design

Fourier coefficient	$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n} d\omega$
Impulse response for band-stop FIR filter	$h(n) = \begin{cases} \frac{\pi - \omega_H + \omega_L}{\pi} & for\ n = \alpha \\ \frac{\sin((n-\alpha)\omega_L)}{(n-\alpha)\pi} - \frac{\sin((n-\alpha)\omega_H)}{(n-\alpha)\pi} & for\ n \neq \alpha \end{cases}$ where $\alpha = \frac{M-1}{2}$
Normalize frequency (rad/sec)	$\omega_d = 2\pi f T$
Window function	$W_{hamming} = 0.54 - 0.46\cos(\frac{n\pi}{\alpha})$ $W_{hanning} = 0.5 - 0.5\cos(\frac{n\pi}{\alpha})$ $W_{Blackmann} = 0.42 - 0.5\cos\left(\frac{n\pi}{\alpha}\right) + 0.08\cos(\frac{2n\pi}{\alpha})$
Transfer function	$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n}$
Analog Low Pass Prototype Conversion for band stop	$Vs = \frac{\omega_{apH} - \omega_{apL}}{\omega_{asH} - \omega_{asL}}$ $H(s) = H_p(S)\Big\vert_{S=\frac{sW}{s^2+\omega_0^2}}$ $\omega_0 = \sqrt{\omega_{aH} * \omega_{aL}}$ $W = \omega_{aH} - \omega_{aL}$
Wrapping Frequency	$\omega_a = \frac{2}{T} \times \tan(\frac{\omega_d T}{2})$
Stop-band attenuation	$A_s = -20\log(\delta_s)$
Pass-band attenuation	$A_p = 20\log(1 - \delta_p)$
Butterworth filter order	$n \geq \frac{log_{10}(\frac{10^{0.1A_s} - 1}{\varepsilon^2})}{2log_{10}(v_s)}$ $\varepsilon^2 = 10^{0.1A_p} - 1$

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Table B.2: Frequency-domain characteristics of some window functions

Window function	Main lobe width, $\Delta\omega_{mainlobe}$	Transition width, $\Delta\omega$	Stopband Attenuation:
Rectangular	$4\pi/M$	$1.8\pi/M$	21 dB
Bartlett	$8\pi/M$	$6\pi/M$	25 dB
Hanning	$8\pi/M$	$6.2\pi/M$	44 dB
Hamming	$8\pi/M$	$6.6\pi/M$	53 dB
Blackmann	$12\pi/M$	$11\pi/M$	74 dB

APPENDIX C

LAMPIRAN C

Question	Course Outcome (CO)	Programme Outcome (PO)
1	1	PO4
2	2	PO4
3	3	PO3
4	4	PO3