Second Semester Examination
2022/2023 Academic Session

July/August 2023

EEE377 – Digital Communications

Duration : 2 hours

Please check that this examination paper consists of **TWELVE (12)** pages of printed material including appendix before you begin the examination.

Instructions : This paper consists of **FOUR (4)** questions. Answer **ALL** questions.

Answer **ALL** questions.

1. a) Figure 1.1 below shows a sample and hold the receiver of a Pulse Amplitude Modulation (PAM) baseband transmission receiver. The received signal is sampled at the center of the bit period $t_0 = (i-1)T_b + \frac{T_b}{2}$ and the sample is compared to a 0V threshold for a decision of the demodulated binary data.

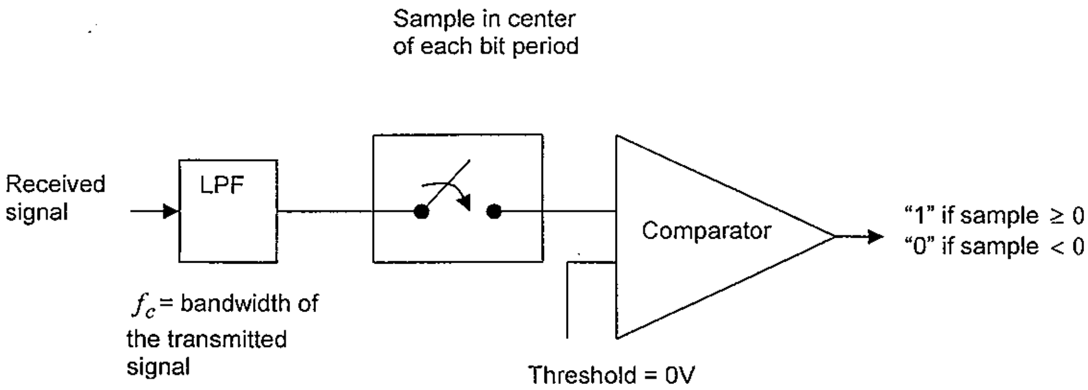

Figure 1.1: Sample and hold receiver for PAM baseband transmission.

The received signal is corrupted by noise which has Gaussian probability density function given as;

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)} \qquad (1.1)$$

Given the probability of the i^{th} bit demodulated in error, P_b as;

$$P_b = \begin{Bmatrix} Value\ of\ Gaussian \\ Random\ variable > \gamma A \end{Bmatrix} = \int_{\gamma A}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_o} e^{-\left[\frac{(X-\mu_o)^2}{2\sigma_o^2}\right]} \qquad (1.2)$$

Based on the above information;

(i) Comment on the randomness of noise before and after the Low Pass Filter (LPF). State the properties of randomness that exist there.

(10 marks)

(ii) Draw the average normalized spectral power density of noise before and after the LPF.

(10 marks)

(iii) Explain the use of LPF in the receiver design. [Hint: Calculate and compare the *total noise average normalized power* before (σ_n^2) and after (σ_o^2) LPF. Then, explain the effect of *total noise average normalized noise power* to the probability of bit error i.e.equation (1.2)]

(20 marks)

b) The source data are modulated using PAM scheme, where binary "1" is represented with a pulse +1 Volt and binary "0" is represented by pulse -1 Volt. The above receiver (Figure 1.2) is used to detect the transmitted signal. The signal is corrupted with the thermal noise. When the signal reaches the receiver, it is attenuated by 60% of the original signal.

(i). Calculate the probability of bit error P_b of the system.

(20 marks)

(ii). If a different type of noise that has probability density function as shown in Figure 1.2 now influence the transmission, calculate the new probability of bit error.

(20 marks)

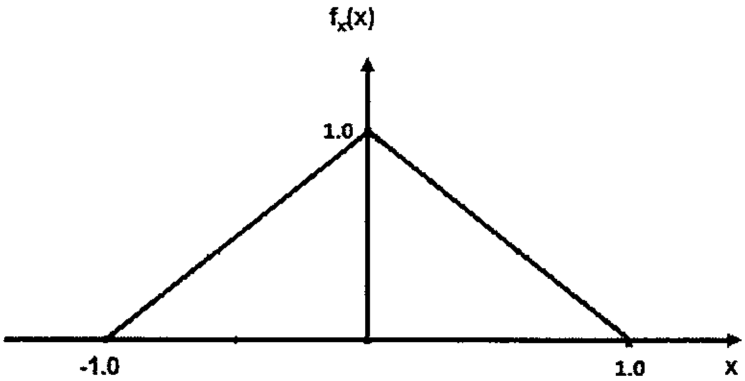

Figure 1.2: A new noise probability density function

c) An optimum receiver has been designed using the **symmetric** rectangular pulses to represent the binary sequence. Binary "1" is represented by +A Volt and binary "0" is represented by -A Volt. The transmitter transmits the binary sequence with the transmission rate of 1,000 Mbits/sec, and the signals are attenuated by a factor of $\frac{1}{4}$ at the receiver. Given that the noise normalized power spectral density as $(\frac{N_o}{2} = 1.6 \times 10^{-9}\ \frac{\text{volts}^2}{\text{Hz}})$. It has been obtained that the minimum probability of bit error $P_b = 0.3483$. Determine the amplitude of the transmitted signal A.

(20 marks)

2. a) A bandpass channel passes frequencies in the range $150\ kHz \leq f \leq 350\ kHz$, is used to transmit the binary **PSK** modulated (with carrier signal such as $A\sin(2\pi f_c t + \theta)$). A specific carrier frequency (f_c) of 250 kHz is used to transmit data at a speed of 50,000 bits/s. Suppose an application requires at least 95% in-band power;

(i) Draw the timing sequence of binary $\{1\ 0\ 1\ 1\}$ and show how these bits are modulated with the carrier signal. [Indicate the number of cycles per bit period].

(10 marks)

(ii) Draw the average normalized power spectral density of a typical transmitted signal.

(20 marks)

(iii) What bandwidth is required if at least 95% of the signal's power must be in-band?

(10 marks)

(iv) Is the signal satisfactorily designed for the channel? [Hint: comments in terms of whether the signal can pass through the channel bandwidth range].

(10 marks)

b) A PSK system transmits 50,000 bits/sec using a signal with peak amplitude of 0.1 volt. By the time the signal arrives at the receiver, its voltage is only 40% of the transmitted voltage. The noise at the input to the correlation receiver is additive white Gaussian noise with an average normalized power spectral density $\left(\frac{N_0}{2}\right)$ of $1.6 \times 10^{-9}\ volts^2/Hz$. Assuming that the receiver is perfectly synchronized, determine;

(i) The accuracy of the system.

(20 marks)

(ii) The minimum average normalized power for the transmitted signal that will provide an average accuracy of one error or less per 100,000 bits transmitted.

(30 marks)

3. a) Consider a random message, V with 4 possible symbols having the following probabilities as shown in Table 3.

Table 3.

Message	X_1	X_2	X_3	X_4
Probability	0.5	0.2	0.15	0.15

Generate 1 code using Huffman coding method. Calculate the efficiency of this code.

(40 marks)

b) The extension code is designed using the symbols in 3(a). Compare the efficiency of the extended codes with the one in 3(a). In your own words, what can you conclude from this comparison?

(60 marks)

4. Construct a Trellis diagram for the convolutional encoder shown in Figure 4.1. This should include state diagram and truth table. Find the encoded code sequence for message sequence of 10101.

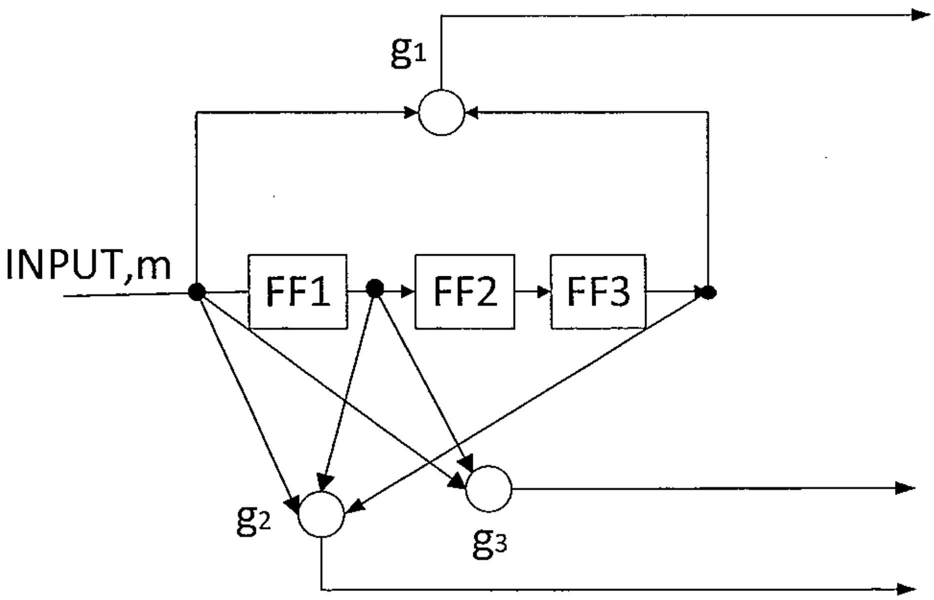

Figure 4.1: A (3,1,4) convolutional encoder.

(100 marks)

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Appendix 1

1. **Fourier Series (Trigonometric form)**

$S(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t))$	
$f_0 = \frac{1}{T}$	$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} s(t)dt$
$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} s(t)\cos(2\pi n f_0 t)dt$	$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} s(t)\sin(2\pi n f_0 t)dt$

2. **Fourier Series (One-sided form)**

$S(t) = X_0 + \sum_{n=1}^{\infty} X_n \cos(2\pi n f_0 t + \phi_n)$		
$X_0 = a_0$	$X_n = \sqrt{a_n^2 + b_n^2}$	$\phi_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$

3. **Fourier Series (Double-sided form)**

$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t}$	$c_n = \frac{1}{T}\int_{t_o}^{t_o+T} s(t) e^{-j2\pi n f_o t} dt$

4. **Conversion between 2-sided to 1-sides Fourier Series**

$n > 0$	$c_n = \frac{1}{2}(a_n - jb_n)$	$c_{-n} = \frac{1}{2}(a_n + jb_n)$	$c_{-n} = c_n^*$
	$\|c_n\| = \|c_{-n}\| = \frac{X_n}{2}$	$\angle c_n = \phi_n$	$\angle c_{-n} = -\phi_n$
$n = 0$	$c_o = a_o = X_o$		

5. **Fourier Transform and Inverse Fourier Transform**

$s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f t} df$	$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$

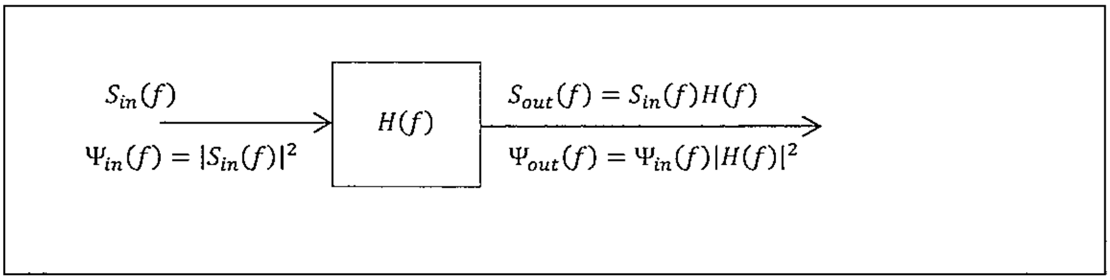
6. **Parseval 's Theorems**

Parseval's Power Theorem	$P_s = \frac{1}{T}\int_{t_0}^{t_0+T} s^2(t)dt = X_0^2 + \sum_{n=1}^{\infty}\frac{X_n^2}{2}$ *time domain* *frequency domain*
Parseval's Energy Theorem	$E_s = \int_{-\infty}^{\infty} s^2(t)dt = \int_{-\infty}^{\infty}\lvert S(f)\rvert^2 df$ *time domain* *frequency domain*

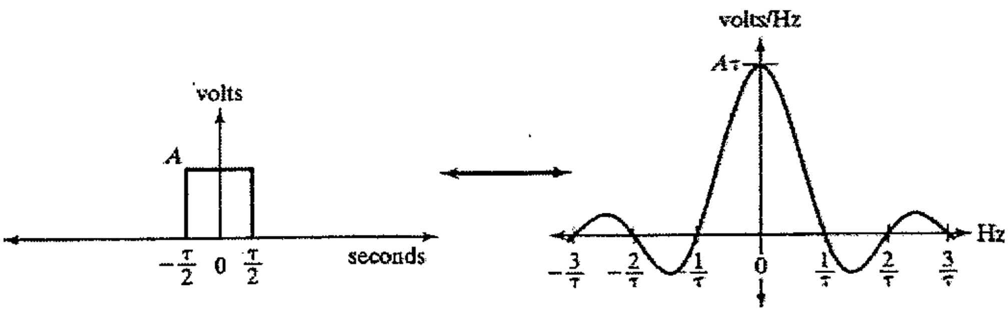
7. **Normalized energy spectral density**

$$\Psi(f) = |S(f)|^2 \left[\frac{volts^2}{Hz^2}\right] \; or \; \left[\frac{volts^2 - \sec}{Hz}\right]$$

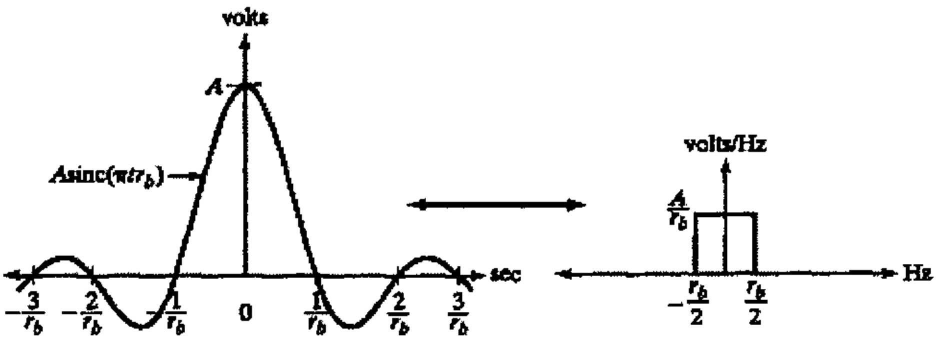
8. **Signal Energy through linear system**



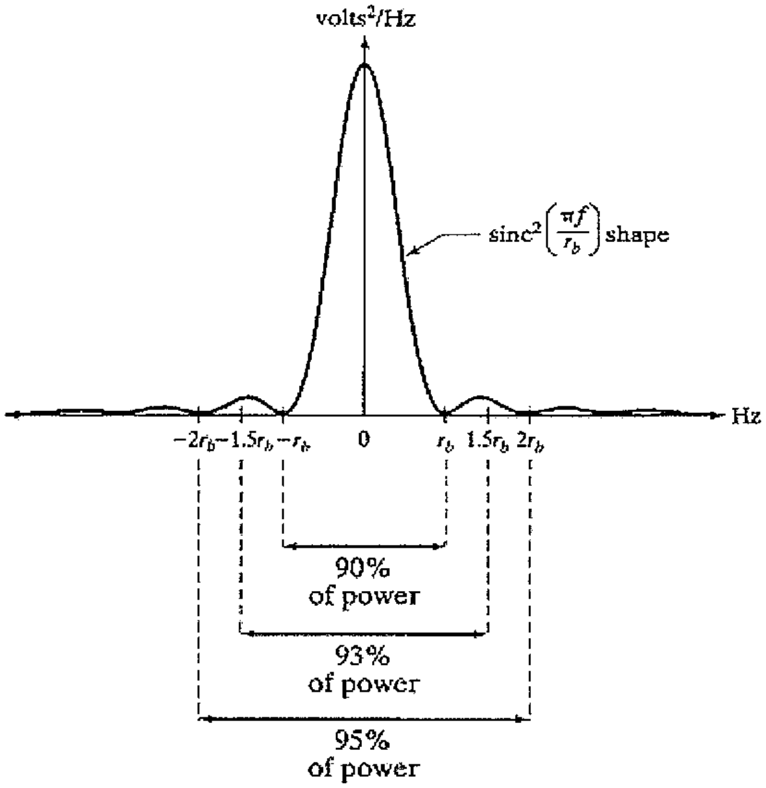
9. **Rectangular-sinc transforms pair**



10. **Sinc-shaped transform pair**



11. Average normalized power spectral density *G(f)* of the output system



12. Stochastic Relations

i. Probability Distribution Function

$$F_X(a) = P\{X \le a\} = \int_{-\infty}^{a} f_X(x)\,dx$$

ii. Probability Density Function

$$f_X(x) = \frac{d}{dx} F_X(x)$$

iii. Gaussian probability Density function

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

iv. Other Relationships

$$P\{a < X \le b\} = P\{X \le b\} - P\{X \le a\} = F_X(b) - F_X(a) = \int_{a}^{b} f_X(x)\,dx$$

$$P\{X > c\} = 1 - P\{X \le c\} = \int_{c}^{\infty} f_X(x)\,dx$$

13. Receiver design

Simple (Sample and hold) receiver $(t_0 = (i-1)T_b + \frac{T_b}{2})$	$P_b = Q\left(\frac{\gamma A - \mu}{\sigma}\right)$	
Optimum receiver $(t_0 = iT_b)$	Symmetric	$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ $E_b = \gamma^2 \int_{-\infty}^{\infty} \lvert S(f) \rvert^2 df = \gamma^2 \int_{(i-1)T_b}^{iT_b} s^2(t) dt$
	Asymmetric	$P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$ $E_d = \gamma^2 \int_{-\infty}^{\infty} \lvert S_1(f) - S_2(f) \rvert^2 df = \gamma^2 \int_{(i-1)T_b}^{iT_b} \lvert s_1(t) - s_2(t) \rvert^2 dt$

APPENDIX 2

The Q Function (Gaussian distribution with $\mu = 0$ and $\sigma = 1$

$$Q(a) = \int_a^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx \qquad Q(a) \cong \frac{1}{a\sqrt{2\pi}} e^{\frac{-a^2}{2}} \text{ for } a \geq 3$$

	Third Significant Digit									
a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002

Q(a)	*a*
10^{-4}	3.73
5×10^{-5}	3.90
10^{-5}	4.27
5×10^{-6}	4.43

Q(a)	*a*
10^{-6}	4.76
10^{-7}	5.20
10^{-8}	5.61
10^{-9}	6.00