Second Semester Examination
2022/2023 Academic Session

July/August 2023

EEE354 – Digital Control System

Duration : 3 hours

Please check that this examination paper consists of **NINE (9)** pages of printed material including 3 appendices before you begin the examination.

Instructions : This paper consists of **THREE (3)** questions. Answer **ALL** questions.

1. a.) The difference equation for an open loop discrete control system is given by,

$$6y[k-2] - 5y[k-1] + y[k] = 2x[k-2] - 3x[k-1] + x[k]$$

From the problem above and by stating your assumptions, solve the given statement's difference equation, $y[k]$, using the z transform if the input is as below. State the output in $y[k]$ as inverse z-transform.

i) $x[k] = 3u[k]$

(20 marks)

ii) $x[k] = r[k]$

(10 marks)

b.) For the system shown in Figure 1, the sampling rate is 1 Hz. The digital filter solves the difference equation given as below:

$$e(k) - e(k-1) = x(k) - 3x(k-1)$$

The plant transfer function is given by:

$$G_p(s) = \frac{1}{s+1}$$

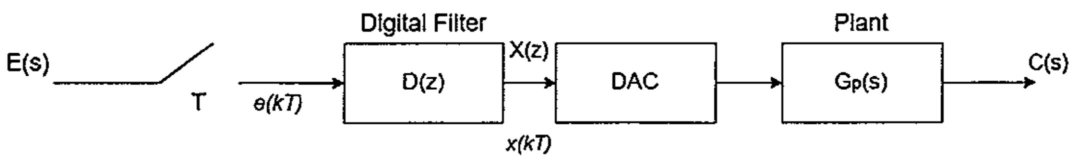

Figure 1

i) Find the system's transfer function, $\frac{C(z)}{E(z)}$.

(40 marks)

ii) Determine the system response, *c(k)* at the sampling interval for a unit step input.

(30 marks)

2. Figure 2 shows a block diagram for a temperature control system for a large test chamber where $D(z)$ is the digital controller, T is the sampling time, and

$$G_p(s) = \frac{2}{s+1}; \quad G_d(s) = \frac{3}{s+1}; \quad H_k = 0.2$$

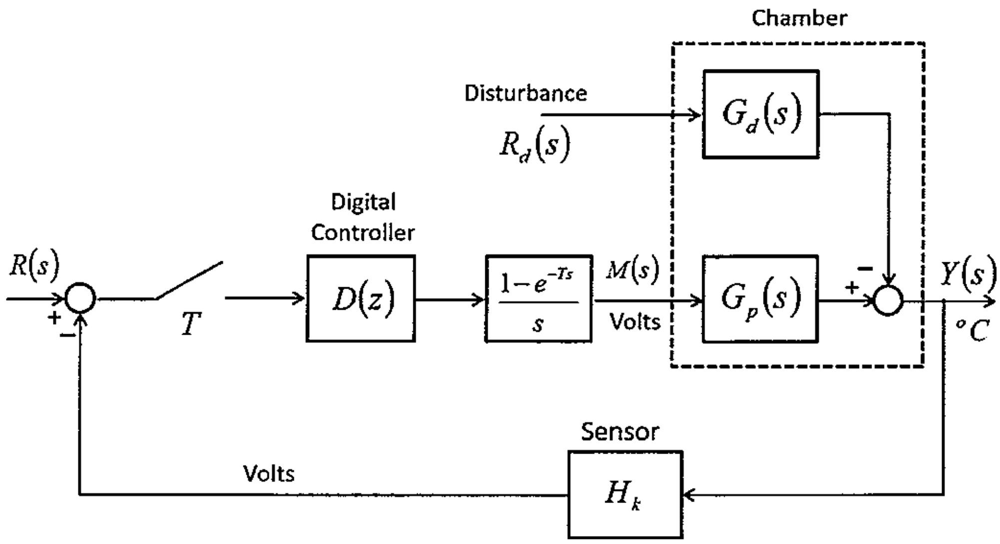

Figure 2

a) Let $D(z)$ be a variable gain K, and R_d be a unit step function. Using the superposition theorem, find the complete expression of $Y(z)$ in terms of K, T and R.

(50 marks)

b) If the control system in Figure 2 is to command a 10°C step in the output, find the structure and the unit of $R(s)$ to achieve this objective.

(10 marks)

c) Following the objective in (b), if $K = 1$ and $T = 0.1\ s$, calculate $Y(z)$ and plot the system response $y(nT)$. Assume $R_d(s) = 0$ for this case.

(30 marks)

d) Verify your answer in (c) using the final value theorem.

(10 marks)

3. a.) Given below are characteristic equations of discrete systems,

i) $z^3 - 1.25z^2 - 1.375z - 0.25 = 0$

ii) $z^2 - 3.3z^2 + 4z + 0.8 = 0$

By stating all assumptions made, determine whether each system (in i. and ii.) is stable, marginally stable or unstable. Include all workings in your solution.

(40 marks)

b.) The Bode plot of a digital system is given in Figure 4. Analyze the system and state the numerical value of the following:

i) Gain margin

ii) Phase margin

iii) Gain cross-over frequency

iv) Phase cross-over frequency

(40 marks)

Please state whether the system is stable or not. Suppose a Proportional controller with a gain K is added to improve the transient performance of the control system in question, what is the range of K that still guarantees system stability.

(20 marks)

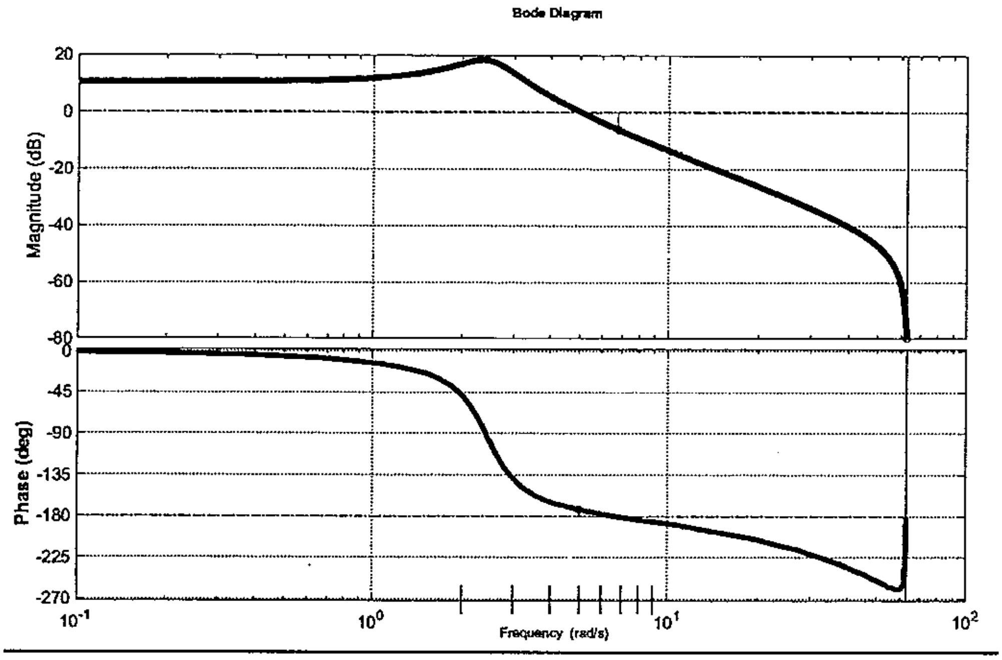

Figure 4: Bode plot of a control system

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APPENDIX A

Course Outcomes (COs) – Programme Outcomes (POs) Mapping

QUESTION	CO	PO
1	1	2
2	2	4
3	3	2

APPENDIX B

z transform of common functions

	$F(s)$	$f(t)$	$f(kT)$	$F(z)$
1.			$\delta_0(k)$ $1, \quad k=0$ $0, \quad k \neq 0$	1
2.			$\delta_0(n-k)$ $1, \quad n=k$ $0, \quad n \neq k$	$\frac{1}{z^k}$
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{z}{z-1}$
4.	$\frac{1}{(s+a)}$	e^{-at}	e^{-akT}	$\frac{z}{z-e^{-aT}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz}{(z-1)^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z(z+1)}{(z-1)^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z(z^2+4z+1)}{(z-1)^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z}{(z-e^{-aT})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{z^2-(1+aT)e^{-aT}z}{(z-e^{-aT})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(z+e^{-aT})z}{(z-e^{-aT})^3}$
13.	$\frac{a}{s^2(s+a)}$	$t-\frac{1-e^{-at}}{a}$	$kT-\frac{1-e^{-akT}}{a}$	$\frac{z[(aT-1+e^{-aT})z+(1-e^{-aT}-aTe^{-aT})]}{a(z-1)^2(z-e^{-aT})}$
14.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{z(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})}{(z-1)(z-e^{-aT})}$

APPENDIX C

TRIGONOMETRY — LAWS AND IDENTITIES

TANGENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

RECIPROCAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

PERIODIC IDENTITIES

$$\sin(\theta + 2\pi n) = \sin\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta$$

$$\tan(\theta + \pi n) = \tan\theta$$

$$\csc(\theta + 2\pi n) = \csc\theta$$

$$\sec(\theta + 2\pi n) = \sec\theta$$

$$\cot(\theta + \pi n) = \cot\theta$$

EVEN/ODD IDENTITIES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

HALF ANGLE IDENTITIES

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

PRODUCT TO SUM IDENTITIES

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

SUM TO PRODUCT IDENTITIES

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

LAW OF SINES

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

LAW OF TANGENTS

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$

$$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$$

$$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$$

SUM/DIFFERENCES IDENTITIES

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

MOLLWEIDE'S FORMULA

$$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha-\beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$$

COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$