Second Semester Examination
2022/2023 Academic Session

July/August 2023

EEE276 – Electromagnetic Theory

Duration : 3 hours

Please check that this examination paper consists of **SEVENTEEN (17)** pages of printed material including appendix before you begin the examination.

Instructions : This paper consists of **THREE (3)** questions. Answer **THREE (3)** questions.

1. a) Consider a vector wave that is propagating in air. If the vector field is given in phasor form as:

$$\widetilde{E}(x,y,z) = (\hat{\mathbf{x}}e^{j\pi/4} - \hat{\mathbf{z}}e^{-j\pi/4})e^{j\pi y}$$

Determine the following quantities:

(i) The wavelength of the plane wave.

(5 marks)

(ii) The frequency of the plane wave.

(5 marks)

(iii) The corresponding time domain expression.

(10 marks)

(iv) The propagation direction of the wave $\vec{E}(x, y, z, t)$.

(5 marks)

b) A 100 Ω lossless transmission line terminated in a load with impedance $Z_L = 60 - j100\,\Omega$. The wavelength is 16 cm. Calculate:

(i) The reflection coefficient at the load.

(10 marks)

(ii) The standing wave ratio on the line.

(5 marks)

(iii) The position of the current minimum nearest to the load.

(10 marks)

c) (i) Given that two points in space P (-3, 4, 5) and Q (2, 0, -1). Express a vector that extends from P to Q in rectangular coordinate and spherical coordinates. Show that each of these vectors has the same magnitude.

(15 marks)

(ii) The surface $\rho = 3$ and 5, $\varphi = 100°$ and 130°, and $z = 3$ and 4.5 identify a closed surface. Find the enclosed volume of the enclosing surface.

(10 marks)

(iii) A vector field $\mathbf{D} = \hat{\mathbf{r}}r^3$ exists in the region between two concentric cylindrical surfaces defined by r – 1 and r = 2, with both cylinders extending between z = 0 and z = 5. Verify the divergence theorem by evaluating $\oint_s \mathbf{D}.ds$ and $\int_v \nabla.\mathbf{D}dv$.

(25 marks)

2. a) (i) Starting from Coulomb's Law, relating the force between two point charges to the magnitude of the charges and the distance between them, develop expressions for the electric field density, D and electric field strength, E, at a distance of r meter from a charge of magnitude $+q_1$ Coulomb.

(10 marks)

(ii) Hence evaluate and show that the absolute potential at a distance of r meter from a point charge of + q_1 Coulomb, in air is given by

$$V = \frac{q_1}{4\pi\varepsilon_0 r}$$

where ε_0 is the permittivity of free space.

(5 marks)

(iii) Two identical positive point charges of magnitude 20 nC are situated at the points whose coordinates are given by (0,0) and (2,0).

(iii.a) Determine the magnitude and direction of the resulting electric field strength at the point whose coordinates are (1,2).

(10 marks)

(iii.b) Determine the absolute potential at this point. Assume that the charges are in the air.

(10 marks)

(v) A line of charge with uniform line charge density ρ_l = 1 x 10-6 C/m exists in air along the y-axis between y = -10 m and y = 10 m. Find ***E*** at (0, 0, 10). (Hint: $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x/a^2}{\sqrt{x^2+a^2}}$)

(15 marks)

b) (i) A 10 nC charge with velocity 100 m/sec in the z direction enters a region where the electric field intensity is $\hat{x}$ 800 V/m and the magnetic flux density $\hat{y}$ 12.0 Wb/m^2. Determine the force vector acting on the charge.

(10 marks)

(ii) A circular conducting loop of radius 40 cm lies in the xy plane and has a resistance of 20 Ω. If the magnetic flux density in the region is given as:

$\mathbf{B} = 0.2\,\hat{\mathbf{x}}\,cos500t + 0.75\,\hat{\mathbf{y}}\sin 400t + 1.2\hat{\mathbf{z}}\,cos314t$ (T)

Determine the effective value of the induced current in the loop.

(Hint: Faraday's Law (Transformer/Loop): $V_{emf}^{tr} = -N \oint_S \frac{\partial B}{\partial t} \cdot ds$)

(20 marks)

(iii) Magnetic boundary condition is very important for practical applications. Let say a magnetic field intensity is given as $H_1 = \hat{x}6 + \hat{y}2 + \hat{z}3$ (A/m) in a medium with μ_{r1} = 6000 that exists for ***z < 0***.

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(iv) Find its magnetic field intensity, H_2 if passes through another medium with μ_{r2} = 3000 for **z >0.**

(20 marks)

3. a) (i) Describe Faraday's Law in a time-varying EM field by providing an example of application. By using a galvanometer, draw a suitable diagram to illustrate the principle of Faraday's Law.

(15marks)

(ii) The square loop shown in Figure 3 below is coplanar with a long straight wire carrying a current that is given by $I(t) = 25\cos(2\pi \times 10^4 t)$ (A). Determine the **EMF** induced across a small gap created in the loop.

(15 marks)

(iii) By referring to Question 3(a)(ii), determine the direction and magnitude of the current that would flow through a 10 Ω resistor connected across the gap. The loop has an internal resistance of 2 Ω.

(10 marks)

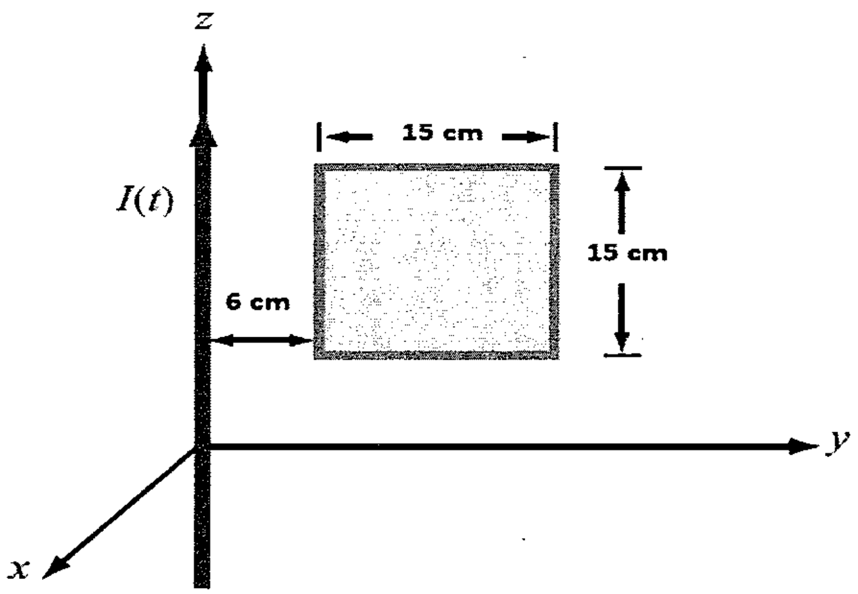

Figure 3

b) (i) Describe Ampere's Law in a time-varying EM field by providing an example of application. State the differential form of the Ampere's Law formula.

(10 marks)

(ii) The electric field phasor of a uniform plane wave is given by $\tilde{\mathrm{E}} = \hat{\mathbf{y}}30e^{j0.5z}$ V/m. If the phase velocity of the wave is 3.2 x 10^8 m/s and the relative permeability is μ_r = 1.9, find the wavelength λ, frequency f of the wave, relative permittivity of the medium ε_r and magnetic field **H** (z,t).

(20marks)

(iii) A satellite communication system is transmitting an elliptically polarized plane of electric field wave given by

$$\mathbf{E}(z,t) = [-\hat{\mathbf{x}}15\sin(\omega t - kz - 45^0) + \hat{\mathbf{y}}30\cos(\omega t - kz)]$$

Determine the polarization angles (γ, χ) and verify the rotation direction.

(10 marks)

c) (i) What is the difference between EMC and EMI concepts?

(5 marks)

(ii) Electronics are everywhere these days, from homes to cars to workplaces. Imagine if a car's radio caused interference while using the GPS system on the mobile phone while driving on the highway. Describe the importance of conducting EMC testing in this scenario.

(15 marks)

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APPENDIX A

$$\mathbf{F}_{e21} = \hat{\mathbf{R}}_{12} \frac{q_1 q_2}{4\pi\varepsilon_0 R_{12}^2} \quad \text{(N)} \quad \text{(in free space)},$$

$$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon_0 R^2} \quad \text{(V/m)} \quad \text{(in free space)}, \qquad \mathbf{D} = \varepsilon \mathbf{E} \quad (\text{C/m}^2)$$

Table 1-4: Constitutive parameters of materials.

Parameter	Units	Free-space Value
Electrical permittivity ε	F/m	$\varepsilon_0 = 8.854 \times 10^{-12}$ (F/m) $\simeq \frac{1}{36\pi} \times 10^{-9}$ (F/m)
Magnetic permeability μ	H/m	$\mu_0 = 4\pi \times 10^{-7}$ (H/m)
Conductivity σ	S/m	0

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$

$\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$	$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
$\mathbf{z} = x + jy = \lvert\mathbf{z}\rvert e^{j\theta}$	$\mathbf{z}^* = x - jy = \lvert\mathbf{z}\rvert e^{-j\theta}$
$x = \Re\mathfrak{e}(\mathbf{z}) = \lvert\mathbf{z}\rvert\cos\theta$	$\lvert\mathbf{z}\rvert = \sqrt[+]{\mathbf{z}\mathbf{z}^*} = \sqrt[+]{x^2 + y^2}$
$y = \Im\mathfrak{m}(\mathbf{z}) = \lvert\mathbf{z}\rvert\sin\theta$	$\theta = \tan^{-1}(y/x)$
$\mathbf{z}^n = \lvert\mathbf{z}\rvert^n e^{jn\theta}$	$\mathbf{z}^{1/2} = \pm\lvert\mathbf{z}\rvert^{1/2} e^{j\theta/2}$
$\mathbf{z}_1 = x_1 + jy_1$	$\mathbf{z}_2 = x_2 + jy_2$
$\mathbf{z}_1 = \mathbf{z}_2$ iff $x_1 = x_2$ and $y_1 = y_2$	$\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2)$
$\mathbf{z}_1\mathbf{z}_2 = \lvert\mathbf{z}_1\rvert\lvert\mathbf{z}_2\rvert e^{j(\theta_1+\theta_2)}$	$\dfrac{\mathbf{z}_1}{\mathbf{z}_2} = \dfrac{\lvert\mathbf{z}_1\rvert}{\lvert\mathbf{z}_2\rvert} e^{j(\theta_1-\theta_2)}$
$-1 = e^{j\pi} = e^{-j\pi} = 1\angle\pm180^\circ$	
$j = e^{j\pi/2} = 1\angle 90^\circ$	$-j = e^{-j\pi/2} = 1\angle{-90^\circ}$
$\sqrt{j} = \pm e^{j\pi/4} = \pm\dfrac{(1+j)}{\sqrt{2}}$	$\sqrt{-j} = \pm e^{-j\pi/4} = \pm\dfrac{(1-j)}{\sqrt{2}}$

Time & Phasor Domain:

$x(t)$		$\mathbf{X}$
$A\cos\omega t$	$\Longleftrightarrow$	A
$A\cos(\omega t+\phi)$	$\Longleftrightarrow$	$Ae^{j\phi}$
$-A\cos(\omega t+\phi)$	$\Longleftrightarrow$	$Ae^{j(\phi\pm\pi)}$
$A\sin\omega t$	$\Longleftrightarrow$	$Ae^{-j\pi/2}=-jA$
$A\sin(\omega t+\phi)$	$\Longleftrightarrow$	$Ae^{j(\phi-\pi/2)}$
$-A\sin(\omega t+\phi)$	$\Longleftrightarrow$	$Ae^{j(\phi+\pi/2)}$
$\frac{d}{dt}(x(t))$	$\Longleftrightarrow$	$j\omega\mathbf{X}$
$\frac{d}{dt}[A\cos(\omega t+\phi)]$	$\Longleftrightarrow$	$j\omega Ae^{j\phi}$
$\int x(t)\,dt$	$\Longleftrightarrow$	$\frac{1}{j\omega}\mathbf{X}$
$\int A\cos(\omega t+\phi)\,dt$	$\Longleftrightarrow$	$\frac{1}{j\omega}Ae^{j\phi}$

Table 1-5: Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\tilde{Z}$, where $z(t)=\Re\mathfrak{e}\,[\tilde{Z}e^{j\omega t}]$.

$z(t)$		$\tilde{Z}$
$A\cos\omega t$	$\Longleftrightarrow$	A
$A\cos(\omega t+\phi_0)$	$\Longleftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t+\beta x+\phi_0)$	$\Longleftrightarrow$	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t+\beta x+\phi_0)$	$\Longleftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	$\Longleftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t+\phi_0)$	$\Longleftrightarrow$	$Ae^{j(\phi_0-\pi/2)}$
$\frac{d}{dt}(z(t))$	$\Longleftrightarrow$	$j\omega\tilde{Z}$
$\frac{d}{dt}[A\cos(\omega t+\phi_0)]$	$\Longleftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\Longleftrightarrow$	$\frac{1}{j\omega}\tilde{Z}$
$\int A\sin(\omega t+\phi_0)\,dt$	$\Longleftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

Chapter 1 Relationships

Electric field due to charge q in free space

$$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon_0 R^2}$$

Magnetic field due to current I in free space

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r}$$

Plane wave $y(x,t) = Ae^{-\alpha x} \cos(\omega t - \beta x + \phi_0)$

- $\alpha = 0$ in lossless medium
- phase velocity $u_p = f\lambda = \frac{\omega}{\beta}$
- $\omega = 2\pi f$; $\beta = 2\pi/\lambda$
- ϕ_0 = phase reference

Complex numbers

- Euler's identity

  $e^{j\theta} = \cos\theta + j\sin\theta$

- Rectangular-polar relations

  $x = |z|\cos\theta, \quad y = |z|\sin\theta,$

  $|z| = \sqrt[+]{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x)$

Phasor-domain equivalents

Table 1-5

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Chapter 3 Relationships

Distance Between Two Points

$$d = [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{1/2}$$

$$d = \left[r_2^2 + r_1^2 - 2r_1 r_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2\right]^{1/2}$$

$$d = \left\{R_2^2 + R_1^2 - 2R_1 R_2[\cos\theta_2 \cos\theta_1 + \sin\theta_1 \sin\theta_2 \cos(\phi_2 - \phi_1)]\right\}^{1/2}$$

Coordinate Systems Table 3-1

Coordinate Transformations Table 3-2

Vector Products

$$\mathbf{A} \cdot \mathbf{B} = AB \cos\theta_{AB}$$

$$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}}\, AB \sin\theta_{AB}$$

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Divergence Theorem

$$\int_V \nabla \cdot \mathbf{E}\, dV = \oint_S \mathbf{E} \cdot d\mathbf{s}$$

Vector Operators

$$\nabla T = \hat{\mathbf{x}}\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\frac{\partial T}{\partial z}$$

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla \times \mathbf{B} = \hat{\mathbf{x}}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

(see back cover for cylindrical and spherical coordinates)

Stokes's Theorem

$$\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{s} = \oint_C \mathbf{B} \cdot d\mathbf{l}$$

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} R & \hat{\boldsymbol{\phi}} R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Chapter 4 Relationships

Maxwell's Equations for Electrostatics

Name	Differential Form	Integral Form
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$
Kirchhoff's law	$\nabla \times \mathbf{E} = 0$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$

Current density	$\mathbf{J} = \rho_v \mathbf{u}$
Poisson's equation	$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$
Laplace's equation	$\nabla^2 V = 0$
Resistance	$R = \frac{-\int_l \mathbf{E} \cdot d\mathbf{l}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{s}}$
Boundary conditions	Table 4-3
Capacitance	$C = \frac{\int_S \varepsilon \mathbf{E} \cdot d\mathbf{s}}{-\int_l \mathbf{E} \cdot d\mathbf{l}}$
RC relation	$RC = \frac{\varepsilon}{\sigma}$
Energy density	$w_e = \frac{1}{2}\varepsilon E^2$

	Electric Field
Point charge	$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon R^2}$
Many point charges	$\mathbf{E} = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{q_i(\mathbf{R} - \mathbf{R}_i)}{\lvert\mathbf{R} - \mathbf{R}_i\rvert^3}$
Volume distribution	$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{V'} \hat{\mathbf{R}}' \frac{\rho_v \, dV'}{R'^2}$
Surface distribution	$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{S'} \hat{\mathbf{R}}' \frac{\rho_s \, ds'}{R'^2}$
Line distribution	$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{l'} \hat{\mathbf{R}}' \frac{\rho_\ell \, dl'}{R'^2}$
Infinite sheet of charge	$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_s}{2\varepsilon_0}$
Infinite line of charge	$\mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0} = \hat{\mathbf{r}} \frac{D_r}{\varepsilon_0} = \hat{\mathbf{r}} \frac{\rho_\ell}{2\pi\varepsilon_0 r}$
Dipole	$\mathbf{E} = \frac{qd}{4\pi\varepsilon_0 R^3}(\hat{\mathbf{R}}\, 2\cos\theta + \hat{\boldsymbol{\theta}} \sin\theta)$
Relation to V	$\mathbf{E} = -\nabla V$

Table 4-3: Boundary conditions for the electric fields.

Field Component	Any Two Media	Medium 1 Dielectric ε_1	Medium 2 Conductor
Tangential E	$\mathbf{E}_{1t} = \mathbf{E}_{2t}$	$\mathbf{E}_{1t} = \mathbf{E}_{2t} = 0$	
Tangential D	$\mathbf{D}_{1t}/\varepsilon_1 = \mathbf{D}_{2t}/\varepsilon_2$	$\mathbf{D}_{1t} = \mathbf{D}_{2t} = 0$	
Normal E	$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$	$E_{1n} = \rho_s/\varepsilon_1$	$E_{2n} = 0$
Normal D	$D_{1n} - D_{2n} = \rho_s$	$D_{1n} = \rho_s$	$D_{2n} = 0$
Notes: (1) ρ_s is the surface charge density at the boundary; (2) normal components of $\mathbf{E}_1$, $\mathbf{D}_1$, $\mathbf{E}_2$, and $\mathbf{D}_2$ are along $\hat{\mathbf{n}}_2$, the outward normal unit vector of medium 2.			

SECOND PART

(i) $B = \mu H$

(ii) $Biot - Savart\ Law$: $\boldsymbol{H} = \frac{I}{4\pi}\int_l \frac{dl \times \hat{R}}{R^2}$ (A/m)

(iii) Moment, $\boldsymbol{m} = \hat{\boldsymbol{n}}NIA$ $(A.m^2)$

(iv) For Toroidal coil: $H = \frac{NI}{2\pi r}$ $and\ \boldsymbol{H} = -\hat{\Phi}H$ (for $a \le r \le b$), $\Phi = \int \widehat{\boldsymbol{B}} \cdot \overrightarrow{ds}$

(v) Magnetic Energy Density: $w_m = \frac{1}{2}\mu H^2$ (J/m^3)

(vi) Faraday's Law (Motional): $V_{emf}^m = \oint_c (\boldsymbol{u} \times \boldsymbol{B}) \cdot \boldsymbol{dl}$

(vii) Faraday's Law (Transformer/Loop): $V_{emf}^m = -N \oint_S \frac{\partial B}{\partial t} \cdot \boldsymbol{ds}$

(viii) Lossless Medium:

(ix) $k = \omega\sqrt{\mu\varepsilon}$, $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ (Ω), $u_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$ (m/s), $\lambda = \frac{2\pi}{k} = \frac{u_p}{f}$ (m)

(x) Power Density, $S_{av} = \frac{1}{2}\,\mathfrak{Re}\left|\hat{E} \times \hat{H}^*\right|$ (W/m^2)

Chapter 5 Relationships

Maxwell's Magnetostatics Equations

Gauss's Law for Magnetism

$$\nabla \cdot \mathbf{B} = 0 \quad \Longleftrightarrow \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

Ampère's Law

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \Longleftrightarrow \quad \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I$$

Lorentz Force on Charge q

$$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

Magnetic Force on Wire

$$\mathbf{F}_m = I \oint_C d\mathbf{l} \times \mathbf{B} \quad \text{(N)}$$

Magnetic Torque on Loop

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \quad \text{(N·m)}$$

$$\mathbf{m} = \hat{\mathbf{n}} NIA \quad \text{(A·m}^2\text{)}$$

Biot–Savart Law

$$\mathbf{H} = \frac{I}{4\pi} \int_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \quad \text{(A/m)}$$

Magnetic Field

Infinitely Long Wire $\quad \mathbf{B} = \hat{\boldsymbol{\phi}} \dfrac{\mu_0 I}{2\pi r} \quad$ (Wb/m^2)

Circular Loop $\quad \mathbf{H} = \hat{\mathbf{z}} \dfrac{Ia^2}{2(a^2+z^2)^{3/2}} \quad$ (A/m)

Solenoid $\quad \mathbf{B} \simeq \hat{\mathbf{z}} \mu n I = \dfrac{\hat{\mathbf{z}} \mu N I}{l} \quad$ (Wb/m^2)

Vector Magnetic Potential

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \text{(Wb/m}^2\text{)}$$

Vector Poisson's Equation

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

Inductance

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{1}{I} \int_S \mathbf{B} \cdot d\mathbf{s} \quad \text{(H)}$$

Magnetic Energy Density

$$w_m = \frac{1}{2} \mu H^2 \quad \text{(J/m}^3\text{)}$$

Table 6-1: Maxwell's equations.

Reference	Differential Form	Integral Form	
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$	(6.1)
Faraday's law	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \dfrac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$	(6.2)*
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$	(6.3)
Ampère's law	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{s}$	(6.4)

*For a stationary surface S.

Chapter 6 Relationships

Faraday's Law

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} = V_{\text{emf}}^{\text{tr}} + V_{\text{emf}}^{\text{m}}$$

Transformer

$$V_{\text{emf}}^{\text{tr}} = -N\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s} \qquad (N \text{ loops})$$

Motional

$$V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

Charge-Current Continuity

$$\nabla\cdot\mathbf{J} = -\frac{\partial \rho_v}{\partial t}$$

EM Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla\times\mathbf{A}$$

Current Density

Conduction $\mathbf{J}_c = \sigma\mathbf{E}$

Displacement $\mathbf{J}_d = \dfrac{\partial \mathbf{D}}{\partial t}$

Conductor Charge Dissipation

$$\rho_v(t) = \rho_{vo}e^{-(\sigma/\varepsilon)t} = \rho_{vo}e^{-t/\tau_r}$$

Chapter 7 Relationships

Complex Permittivity

$$\varepsilon_c = \varepsilon' - j\varepsilon''$$

$$\varepsilon' = \varepsilon$$

$$\varepsilon'' = \frac{\sigma}{\omega}$$

Lossless Medium

$$k = \omega\sqrt{\mu\varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \quad (\Omega)$$

$$u_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} \quad (\text{m/s})$$

$$\lambda = \frac{2\pi}{k} = \frac{u_p}{f} \quad (\text{m})$$

Wave Polarization

$$\widetilde{\mathbf{H}} = \frac{1}{\eta}\hat{\mathbf{k}}\times\widetilde{\mathbf{E}}$$

$$\widetilde{\mathbf{E}} = -\eta\,\hat{\mathbf{k}}\times\widetilde{\mathbf{H}}$$

Maxwell's Equations for Time-Harmonic Fields

$$\nabla\cdot\widetilde{\mathbf{E}} = 0$$

$$\nabla\times\widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}}$$

$$\nabla\cdot\widetilde{\mathbf{H}} = 0$$

$$\nabla\times\widetilde{\mathbf{H}} = j\omega\varepsilon_c\widetilde{\mathbf{E}}$$

Lossy Medium

$$\alpha = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{1/2} \quad (\text{Np/m})$$

$$\beta = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right]\right\}^{1/2} \quad (\text{rad/m})$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \quad (\Omega)$$

$$\delta_s = \frac{1}{\alpha} \quad (\text{m})$$

Power Density

$$\mathbf{S}_{\text{av}} = \frac{1}{2}\mathfrak{Re}\left[\widetilde{\mathbf{E}}\times\widetilde{\mathbf{H}}^*\right] \quad (\text{W/m}^2)$$

APPENDIX B

Question	Course Outcome (CO)	Programme Outcome (PO)
1	1	PO3
2	2	PO3
3	4	PO6