UNIVERSITI SAINS MALAYSIA

Peperiksaan Akhir
Sidang Akademik 2007/2008

April 2008

JIM 417 – Persamaan Pembezaan Separa

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan.

Jawab SEMUA soalan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

1. (a) Dapatkan persamaan pembezaan separa peringkat pertama jika penyelesaian amnya diberikan seperti berikut:

$$2u^2 = \alpha x^2 + \beta y^2 + 2$$

dengan α dan β adalah pemalar.

(30 markah)

(b) Tunjukkan bahawa

$u(x, y) = x\,f(x + 3y) + yg(x + 3y)$ menepati persamaan

$$9\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(30 markah)

(c) Selesaikan persamaan pembezaan separa berikut:

$$\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = xu.$$

(40 markah)

2. Diberi persamaan pembezaan separa berikut:

$$u_{xx} + 4u_{xy} + 4u_{yy} + u_y = 10.$$

Bagi persamaan ini,

(a) tentukan jenis,

(10 markah)

(b) dapatkan koordinat cirian dan bentuk berkanun,

(70 markah)

(c) cari penyelesaian am.

(20 markah)

3. Dengan menggunakan kaedah pemisahan pembolehubah, selesaikan masalah nilai awal-sempadan berikut:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \quad , \qquad 0 < x < 1,\ t > 0$$
$$u(x,0) = x(1-x), \qquad 0 \le x \le 1$$
$$u(0,t) = u(1,t) = 0, \qquad t \ge 0.$$

(100 markah)

4. Dengan menggunakan jelmaan Laplace, selesaikan masalah nilai awal-sempadan berikut:

(a) $$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \qquad x > 0,\ t > 0$$

$$u(x,0) = 2e^{-3x}.$$

(50 markah)

(b) Cari siri Fourier bagi fungsi berikut:

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ 1, & 0 \le x < \pi. \end{cases}$$

(50 markah)

5. Dapatkan penyelesaian $u(r, \theta)$ bagi persamaan Laplace

$$r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$$

di dalam suku bulatan $0 < \theta < \frac{\pi}{2}$, $0 < r < 1$ jika syarat-syarat sempadan diberikan oleh

$$u(r,0) = u\left(r, \frac{\pi}{2}\right) = 0, \ 0 < r < 1$$

$$u(1,\theta) = u_o, \ 0 < \theta < \frac{\pi}{2}.$$

(100 markah)

Senarai Rumus

$$u_x = u_\xi \xi_x + u_\eta \eta_x$$

$$u_y = u_\xi \xi_y + u_\eta \eta_y$$

$$u_{xx} = u_{\xi\xi}\xi_x^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\xi\xi_{xx} + u_\eta\eta_{xx}$$

$$u_{xy} = u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}\left(\xi_x\eta_y + \xi_y\eta_x\right) + u_{\eta\eta}\eta_x\eta_y + u_\xi\xi_{xy} + u_\eta\eta_{xy}$$

$$u_{yy} = u_{\xi\xi}\xi_y^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}\eta_y^2 + u_\xi\xi_{yy} + u_\eta\eta_{yy}$$

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left[a_n \text{ kos}\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

dengan

$$a_o = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x) \text{ kos}\left(\frac{n\pi x}{L}\right)dx,\ n = 1, 2, 3, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right)dx,\ n = 1, 2, 3, \ldots$$

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \text{ kos}\left(\frac{n\pi x}{L}\right)$$

dengan

$$a_n = \frac{2}{L}\int_{0}^{L} f(x) \text{ kos}\left(\frac{n\pi x}{L}\right)dx,\ n = 1, 2, 3, \ldots$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

dengan

$$b_n = \frac{2}{L}\int_0^L f(x)\left(\frac{n\pi x}{L}\right)dx, \quad n = 1, 2, 3, \ldots$$

$$f(x) = \frac{1}{2}\sum_{-\infty}^{\infty} c_n e^{inx}$$

dengan

$$c_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx, \quad n = 0, \pm 1, \pm 2, ..$$

$\dfrac{d^2y}{dx^2} - \alpha^2 y = 0$ mempunyai penyelesaian

$$y = Ae^{\alpha x} + Be^{-\alpha x}$$

$\dfrac{d^2y}{dx^2} + \alpha^2 y = 0$ mempunyai penyelesaian

$$y = A \text{ kosh } \alpha x + B \text{ sinh } \alpha x$$

$r^2\dfrac{d^2R}{dr^2} + r\dfrac{dR}{dr} - n^2R = 0$ mempunyai penyelesaian

$$R_n = C_n r^n + \frac{D_n}{r^n}$$

$r\dfrac{d^2R}{dr^2} + r\dfrac{dR}{dr} = 0$ mempunyai penyelesaian

$$R = A + B \ln r$$

$$\mathfrak{F}\left[f(t)\right] = F(\alpha) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{i\alpha t} dt$$

$$f(x) = \mathfrak{F}^{-1}[F(\alpha)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha) e^{-i\alpha x}\, dx$$

$$\mathfrak{F}[f(x)] = F_s(n) = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx, \; n = 1, 2, \ldots$$

$$f(x) = \mathfrak{F}^{-1}[F_s(n)] = \sum_{n=1}^{\infty} F_s(n) \sin \frac{n\pi x}{L}$$

$$\mathfrak{F}[f(x)] = F_c(n) = \frac{2}{L} \int_0^L f(x) \operatorname{kos} \frac{n\pi x}{L} dx, \quad n = 1, 2, \ldots$$

$$f(x) = \mathfrak{F}^{-1}[F_c(n)] = \frac{F_c(0)}{2} + \sum_{n=1}^{\infty} F_c(n) \operatorname{kos} \frac{n\pi x}{L}$$

$$\mathfrak{F}[f''(x)] = \frac{2n}{\pi}\left[f(0) - (-1)^n f(\pi)\right] - n^2 F_s(n)$$

$$\mathfrak{F}[f''(x)] = \frac{2}{\pi}\left[(-1)^n f'(\pi) - f'(0)\right] - n^2 F_c(n)$$

$$\mathscr{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t)\, dt$$

$$\mathscr{L}[e^{\alpha t} f(t)] = F(s - \alpha)$$

Jika $g(t) = \begin{cases} 0 & , \quad t < \alpha \\ f(t-\alpha), & t > 0 \end{cases}$

maka

$$[f(t)] = e^{-\alpha s} F(s)$$

$$\mathscr{L}\,[f^n(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathscr{L}[tf(t)] = -F'(s) = \frac{d}{ds}\, \mathscr{L}\,[f(t)]$$

$$\mathscr{L}\left[\int_0^t f(u)\, du\right] = \frac{F(s)}{s}$$

$$\mathscr{L}^{-1}[F(s)G(s)] = \int_0^t f(u) g(t-u)\, du = f * g$$

Jadual Jelmaan Laplace

$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
1	$\frac{1}{s}$
$t^n, n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
kos at	$\frac{s}{s^2+a^2}$
sin at	$\frac{a}{s^2+a^2}$
kosh at	$\frac{s}{s^2-a^2}$
sinh at	$\frac{a}{s^2-a^2}$
t kos bt	$\frac{s^2-a^2}{(s^2+b^2)^2}$
t sin bt	$\frac{2bs}{(s^2+b^2)^2}$
e^{at} kos bt	$\frac{s-a}{(s-a)^2+b^2}$
e^{at} sin bt	$\frac{b}{(s-a)^2+b^2}$

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