

**CHARACTERIZATION OF SOIL SHEAR  
STRENGTH MODEL FROM ELECTRICAL  
RESISTIVITY AND SEISMIC REFRACTION  
METHODS**

**BALARABE BALA**

**UNIVERSITI SAINS MALAYSIA**

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by

**BALARABE BALA**

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## LIST OF SYMBOLS

$A$	Cross-sectional area ( $\text{m}^2$ )
$c$	Total cohesion ( $\text{kN}/\text{m}^2$ )
$E$	Electrical field strength ( $\text{Vm}$ )
$G$	Geophones
$I$	Electrical current (Ampere)
$i_c$	Critical angle of incidence (Deg.)
$j$	Current density ( $\text{Am}^{-2}$ )
$K$	Bulk modulus ( $\text{N}/\text{m}^2$ )
$k$	Geometric factor
$k$	Number of independent variables
$L$	Length (m)
$n$	Number of data points
$R$	Electrical resistance ( $\Omega$ )
$R^2$	Coefficient of determination
$R^2_{adjusted}$	Adjusted coefficient of determination
$u$	Pore water pressure ( $\text{N}/\text{m}^2$ )
$V$	Potential difference/ Voltage (V)
$V_p$	Seismic refraction velocity (m/s)
$V_{pf}$	Pore fluid velocity (m/s)
$V_s$	Seismic surface wave velocity(m/s)
$w$	Moisture content (%)
$\beta_0$	Intercept of regression line
$\beta_i$	Coefficient of the explanatory variables $x_i$
$c'$	Effective cohesion ( $\text{kN}/\text{m}^2$ )

$\varepsilon$	Standard error
$\tau$	Shear stress at failure (N/m <sup>2</sup> )
$\mu$	Shear modulus (N/m <sup>2</sup> )
$\pi$	Pi
$\phi$	Total friction angle (Deg.)
$\phi'$	Effective friction angle (Deg.)
$\rho$	Electrical resistivity ( $\Omega\text{m}$ )
$\rho_a$	Apparent resistivity ( $\Omega\text{m}$ )
$\sigma_c$	Electrical conductivity ( $\text{Sm}^{-1}$ )
$\sigma'$	Effective normal stress (N/m <sup>2</sup> )
$\sigma$	Total normal stress (N/m <sup>2</sup> )
$\sigma_p$	Porosity
$\Delta$	Change

## LIST OF ABBREVIATIONS

ANN	Artificial Neural Network
CBR	California Bearing Ration
CPT	Cone Penetrating Test
DC	Direct Current
D-W	Durbin-Watson
ECDF	Egyptian Code for Deep Foundation
EM	Electromagnetic
ERT	Electrical Resistivity Tomography
GPR	Ground Penetrating Radar
IP	Induced Polarization
MAE	Mean Absolute Error
MASW	Multi-Channel Analysis of Surface Waves
MAPE	Mean Absolute Percentage Error
MLR	Multiple Linear Regression
MT	Magnetotelluric
MSE	Mean Square Error
P-waves	Primary or longitudinal/compressional waves
RES2DINV	Two-Dimensional Resistivity Inversion Software
RMSE	Root Mean Square Error
SAS4000	Signal Averaging System
SCLS	Standard Constraint Least Square
SMI	Soil Moisture Index
SP	Self-potential
SPSS	Statistical Software Package

SPT	Standard Penetrating Test
SRT	Seismic Refraction Tomography
S-WAVE	Secondary or transverse/shear waves
UFC	Unified Facilities Criteria
USM	Universiti Sains Malaysia
VES	Vertical Electrical Sounding
VIF	Variance Inflation Factor
W-S	Wenner-Schlumberger
1D	One-Dimensional
2D	Two-Dimensional
3D	Three-dimensional

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**PENCIRIAN MODEL KEKUATAN RICIH TANAH DARIPADA  
KAEDAH KEBERINTANGAN ELEKTRIK DAN SEISMIK PEMBIASAN**

**ABSTRAK**

Bagi pencirian bawah permukaan yang mantap dan terperinci, kajian ini menentukan model kohesi tanah dan sudut geseran daripada penyongsangan pasca kerintangan elektrik dan set data tomografi biasan seismik, dan data geoteknik menggunakan ujian kekuatan ricih dan kaedah regresi linear berganda (MLR). Ia seterusnya mengaitkan subpermukaan di pelbagai tapak dari perspektif geoteknikal dan geofizik menggunakan model yang dibangunkan dan mengesahkan kebolehpercayaan dan keberkesanan model di lokasi yang berbeza. Oleh itu, tiga model telah dibina; pertama, model linear mudah dicapai antara kekuatan ricih dan parameter lembapan dengan hanya parameter kerintangan. Kedua, kerintangan dan halaju biasan seismik dengan kekuatan ricih dan parameter lembapan ditentukan sebagai model MLR. Dua daripada model MLR, kohesi tanah dan sudut geseran, telah diterima berdasarkan hubungan kukuh antara parameter, seperti pekali penentuan ( $R^2$ ), 0.777 dan nilai- $p$ ,  $<0.050$ , manakala satu lagi ditolak. Pekali yang diperolehi bagi model yang diterima telah dipindahkan dan digunakan untuk anggaran kohesi tanah 2D dan sudut dalaman model geseran untuk pengesahan di kawasan Minden\_USM, Batu Uban, Cahaya Gemilang dan Bukit Gambir. Model yang dibangunkan menunjukkan prestasi yang baik, berdasarkan penilaian ketepatan;  $< 5\%$ , dan  $< 10\%$  untuk punca ralat min kuasa dua (RMSE) dan min ralat peratusan mutlak (MAPE) masing-masing. Pendekatan yang dihasilkan, model geoteknik baharu, membina semula geometri bawah permukaan dalam dua ruang. Model ini menyediakan maklumat yang lebih baik untuk pencirian permukaan dan bawah permukaan dan

boleh digunakan dalam tetapan setanding yang lain, meminimumkan kerosakan tanah dan degradasi alam sekitar. Pendekatan, adalah platform yang melalui anggaran 2D parameter kekuatan ricih pantas boleh dicapai tanpa memilih kaedah penyongsangan sekatan/sendi yang canggih dan kaedah yang merosakkan. Oleh itu, model MLR kekuatan ricih tanah yang baru dibangunkan telah memberikan penerangan berterusan sifat tanah dalam bentuk dua dimensi, dengan itu meningkatkan maklumat bawah permukaan untuk penyiasatan tapak berbanding, kepada maklumat satu dimensi daripada kaedah invasif.

# CHARACTERIZATION OF SOIL SHEAR STRENGTH MODEL FROM ELECTRICAL RESISTIVITY AND SEISMIC REFRACTION METHODS

## ABSTRACT

For a robust and detailed subsurface characterization, the present study characterizes soil cohesion and friction angle models from post inversions of electrical resistivity and seismic refraction tomographic datasets, and geotechnical data using shear strength test and multiple linear regression (MLR) methods. It further correlates the subsurface at various sites from geotechnical and geophysical perspectives using the developed models and validates the reliability and efficacy of the models at different locations. Three models were therefore built; firstly, simple linear models were achieved between shear strength and moisture parameters with only resistivity parameter. Secondly, resistivity and seismic refraction velocity parameters with the shear strength and moisture parameters were determined as the MLR models. Two of the MLR models, soil cohesion and friction angle, were accepted based on the strong relationships among the parameters, such as coefficient of determination ( $R^2$ ), 0.777 and  $p$ -values,  $<0.050$ , while the other rejected. The obtained coefficients of the accepted models were transferred and applied for the estimations of 2D soil cohesion and internal angle of friction models for validation at Minden\_USM, Batu Uban, Cahaya Gemilang and Bukit Gambir areas. The developed models demonstrated good performance, based on the accuracy assessments;  $< 5\%$ , and  $< 10\%$  for the root mean square error ( $RMSE$ ) and mean absolute percentage error ( $MAPE$ ) respectively. The approach generated, new geotechnical models, rebuilding of subsurface geometries in two-space. The models provide improved information for both surface and subsurface characterization and employable in other comparable settings, minimize soil damage,

and environmental degradation. The approach, a platform through which fast 2D estimation of shear strength parameters can be achieved without opting to sophisticated constrained/joint inversions and destructive methods. Therefore, the new developed soil's shear strength *MLR* models have provided continual description of soil properties in two-dimensional form, thereby enhancing the subsurface information for site investigations as compared, to one-dimensional information from the invasive method.

# CHAPTER 1

## INTRODUCTION

### 1.1 Background

Geophysics as a discipline has continually been used to addressing community-environment challenges throughout the globe. These challenges are addressed through deployment of different methods for instance magnetic, gravity, electromagnetic, electrical resistivity, seismic, ground penetrating radar, etc. The contribution of various geophysical methods for geophysical site characterizations has been increasing over several years. Different geophysical data types are being acquired and integrated to single interpretation capable of identifying target locations and potentials.

In geophysics, structures or materials that are buried underneath the ground are usually distinguished or detected based on the differences of physical properties of those structures or bodies compared with that of the surrounding environment. Structural response of physical input depends solely on the characteristics of its physical properties. For instance, the speed of propagation of a sound wave sent into the material. The susceptibility of a material when considered for electrical conductivity. The level of magnetisation of a material by application of a magnetic field. Adequate understanding of the material properties is basically crucial, as this detailed information could be utilized to select the optimal geophysical investigation technique to address a given geophysical or geotechnical or geological problem. Geophysical methods are applied to determine specific properties of targeted body in relation to its surrounding. Consider for example, that the magnetic and gravity potential field methods can only provide sufficient information if there are exist variations in magnetic susceptibility within the crustal structure (Dalan *et al.*, 2017; Martorella, 2021) or in density (Wada *et al.*, 2017) or in both magnetic and density

respectively (Monyano Nieto & Prieto, 2021; Shaole *et al.*, 2021). In a similar fashion, ground penetrating radar (GPR) is another useful geophysical method which is commonly used for detection and imaging of objects and structures at shallow subsurface (Yucel, 2021). GPR uses microwaves (radar waves) produced at the surface and are reflected at the shallow borders, which separated objects of distinct dielectric and conductivity physical properties and are reverted to the same surface. It is commonly applied in various environmental studies (Medhat *et al.*, 2020; Cui *et al.*, 2021). Also, for the self- potential (SP) technique; it is only applicable when there is occurrence of natural voltages within the subsurface owing to passage of fluids or chemical reactions between fluids and minerals of soils. SP is prominent in detection of seepages within natural (landslides) and artificial (dams, canals, etc.) structures as well as fault mapping and mineral prospecting (Adamo *et al.*, 2020).

The application of a single geophysical method for site investigations may be adequate if there exist a strong contrast in the observed geophysical properties between the targeted structure and its surrounding. However, in the event that a single geophysical technique is employed to characterize an unknown geological structure or environment, then there is a great possibility that the technique could omit or skip portion of the target or misrepresent that structure entirely (Dutta *et al.*, 2013). Thus, it implies that individual geophysical technique cannot return the best possible details about the shallow subsurface, integrating more than one geophysical technique, therefore leads to improved understanding and adequate characterization of a subsurface target (Sauvin *et al.*, 2013). It also mitigates the limitations, and inherent uncertainties to the interpretation and inversion, occasioned by an individual approach (Márquez *et al.*, 2007). Consequently, a better approach is to harmonize the advantages

of individual techniques for the much-needed results and improved information, which are not possible in employing a single method.

To achieve a more proper shallow subsurface characterization, especially in a geological environment that is heterogeneous, the different results generated concurrently or otherwise, from the geophysical techniques need to be combined. Examples of such integration started long ago but a few of the earlier works include; Garambois *et al.* (2002) combined three different methods of seismic, resistivity and ground penetration radar for assessment of properties of a porous formation against water infiltration. Demanet *et al.* (2001) employed multiple geophysical profiling methods of electrical resistivity tomography (ERT), seismic refraction, electromagnetic (EM), GPR and seismic reflection surveys, to image the subsurface faults in Roer Graben, Belgium. Recently, Meric *et al.* (2005) tested the usefulness of multiple geophysical methods to delineate a huge movement in geologic crystalline formations, which include EM profiling, ERT, seismic and SP. To detect fault and fracture zones characterized by low velocity and resistivity values as potential paths for water movement, Gan *et al.* (2017), applied *mise-a-la-masse* electrical, seismic refraction tomography. Similarly, Anukwu *et al.* (2020), delineated the near-surface geologic structures within premises of a hot spring for geothermal development with seismic refraction and Multi-Channel Analysis of Surface Waves (MASW) procedures. On the other hand, joint inversions of different datasets have been investigated by geoscientists which include: seismic and EM (Um *et al.*, 2014); magnetotelluric (MT) and resistivity (Amatyakul *et al.*, 2017); seismic travel times and gravity (Shi *et al.*, 2018); electrical resistivity and seismic refraction (Hellman *et al.*, 2017; Pasquale *et al.*, 2019).

Further to the above, the present study employs two geophysical properties of resistivity and seismic energy, to characterize the shallow subsurface. For the resistivity method, electrical current is often injected into the ground through planted electrodes called the current electrodes and the resultant potential difference, between the current electrode pair is then measured. Consequently, estimate of electrical resistivity distribution is made based on the distinction between material-target and its surrounding geology at shallow depths (Fasani *et al.*, 2013). The method uses a known value of input current which makes easier the calculation of the true resistivity. The measured resistivity values are referred to as apparent resistivities; implying weighted means of the resistivity values recorded within the investigative area. Therefore, the reconstruction of a subsurface model of the resistivity variation, through inversion of data, which can produce the measured data, with minimal errors, making the method successful for decades.

ERT technique, has been extensively exploited for various applications for instance, in engineering field characterizations (Gupta *et al.*, 2019; Medhat *et al.*, 2020; Zhou & Che, 2020), mapping of buried cavities (Carollo *et al.*, 2020), aquiferous characterization (Mezquita Gonzalez *et al.*, 2021; Kumar *et al.*, 2020), hazard assessment (Röhling *et al.*, 2019; Hariri *et al.*, 2020), mineral exploration (Loke *et al.*, 2013; Nthaba *et al.*, 2020; Gabarron *et al.*, 2020) and archaeological studies (Balkaya *et al.*, 2018). Due to the robustness, and low susceptible to foreign disturbances compared to, for instance, EM induction, seismic and potential field methods, ERT is broadly used by a large cross-section of engineering geophysical companies for it requires less skilled labour (Fasani *et al.*, 2013).

Despite the merits of ERT method, it however, has little sensitivity to electric and EM disturbances in comparison with the mentioned geophysical techniques

(Cardarelli *et al.*, 2008), in addition to its resultant inherent interpretative ambiguities. For instance, it failed to distinguish an interface between layers of shale and a clay, characterized by a small resistivity contrast. Consequently, seismic method could potentially sense this interface attributable to a great velocity contrast (Hellman *et al.*, 2017). Thus, ERT has increasingly been used in combination with especially seismic refraction tomography (SRT), making a viable approach for field investigation at preparatory stage of tunnelling and road constructions (Dahlin *et al.*, 1999; Dastanboo *et al.*, 2020) as well as providing to a greater extent of robust geophysical interpretation.

Furthermore, SRT technique is one of the current advances in shallow geophysics for environmental and engineering characterizations. It is a well-established method that provides subsurface information which includes depth and mechanical properties of soils and rocks. The technique employs acoustic energy, which is recorded at surface after propagating through the subsurface along refracted lines (Kearey *et al.*, 2002). The waves energy is categorized as either primary compressional waves (*P*-waves) or secondary shear waves (*S*-waves). The *P*-waves are often detected by grounded geophones as the first arrivals since they always arrive earlier than the *S*-waves. After being processed, the first arrival data are presented in 2-dimensional form and interpreted to deduce the subsurface information. However, this unique feature of *P*-waves makes it suitable and predominantly utilized for near-surface geophysical studies; in tectonic delineation of deep-seated landslides (Mebrahtu *et al.*, 2020; Imani *et al.*, 2021), ground assessment for subsurface construction (Ronczka *et al.*, 2018), imaging underground fluids and rock properties (Mollaret *et al.*, 2020), fault identification (Gan *et al.*, 2017), peat deposits imagery (Suhip *et al.*, 2020); archaeological studies (Imposa *et al.*, 2018). However, SRT

method underperforms in a situation, in which, a low velocity layer of rocks/soils is overlaid by a high velocity layer, rendering the low layer hidden. Moreover, the technique provides no additional information at a position lower than the upper boundary of a high velocity layer, implying low depth estimates in cases involving shallow hard rocks (Sjögren 1984; Ronczka *et al.*, 2018). Therefore, the best possible approach is to integrate the merits of the separate method to generate outputs that are significantly higher informational as compared to when a single method is deployed. Since the fundamental motivation for integrating two or more methods of disparate physical properties is the practicability to scale down the intrinsic ambiguities of individual method (Hellman *et al.*, 2017). Therefore, combination of the different results that are derived from multiple techniques or instrumentation simultaneously or otherwise, is needed with a view to conduct a wholistic, reliable and appropriate site-specific subsurface study. Geophysical research works that involved the integration of multiple geophysical techniques are mentioned but a few: GPR, seismic refraction and DC resistivity methods (Schrott & Sass, 2008); ERT and SRT methods (Imani *et al.*, 2021); ERT and tunnel seismic prediction methods (Dastanboo & Gharibdoost, 2020); GPR and ERT surveys (Balkaya1 *et al.*, 2018); Deiana, 2019); Induced polarization (IP) and ERT methods (Sono *et al.*, 2020). Furthermore, joint inversion of two or more individual geophysical data could further improve the total resolution, generating models that have good mutual agreement, thereby resulted in enhanced representation/interpretation. The jointly inverted datasets ideally give rise to a unique model of the subsurface explaining the exploited datasets (Lines *et al.*, 1988). There are numerous approaches developed to accomplish joint inversions of different datasets (e.g. Lines *et al.*, 1988; Gallardo & Meju, 2011; Meqbel & Ritter, 2015; Shi *et al.*, 2017; Mollaret *et al.*, 2020). Parsekian *et al.* (2014) conducted a multiple

geophysical approach for imaging of the critical zone, that is, detailed information of the Earth's crust near-surface. Similarly, so many studies have been carried out integrating geophysical data with geotechnical or in-situ data for geological disaster risk management, safety, environmental and engineering investigations (Schütze *et al.*, 2012; Osman *et al.*, 2016; Ashraf *et al.*, 2019; Nai *et al.*, 2019; Röhling *et al.*, 2019; Saddek *et al.*, 2019; Röhling *et al.*, 2019; Owusu-nimo & Ko, 2020; Zhou & Che, 2020).

The application of geophysical techniques on the Earth's surface for shallow subsurface details are at times non-objective on account of ambiguities which are inherently linked to data measurements, processing/treatments, and descriptions. It is thus imperative to quantitatively present the shallow crustal information using integrated means. Integrating different datasets may often yield details which might aid to mitigate ambiguities in data measurement. Such as, measurements of seismic refraction velocity and electrical resistivity provides complementary information as refraction velocity is highly adequate for detection and imaging of subsurface boundaries, thicknesses of weathered/fractured regions, which has a good correlation with variation in porosity or density. While ERT offers relatively better resolution of subsurface shallow structures based on the resistivity distribution, strongly connected to variations in moisture content and clay types. The strengths of SRT and ERT in terms of spatial resolutions and capacity to respond to targeted properties (e.g., moisture content), propelled their combined applications in many research works (Linder *et al.*, 2010; Juhojuntti & Kamm, 2015; Hellman *et al.*, 2017; de Pasquale *et al.*, 2019).

Geological subsurface structures are modelled and characterized based on the surface measurement, however, modelling and characterization have been intrinsically

challenging, because of the inadequate accessibility and indirect measurement of heterogeneous and complex subsurface geology. Regardless, a combined use of disparate details of information into a simple model could lead to an increased knowledge of the heterogeneous subsurface, especially for electrical resistivity and compressional seismic velocity, which has long been established as able to improve resolution and reduce interpretable ambiguities (Gallardo & Meju, 2004). However, the procedures used in geophysics to integrate multiple datasets remain an issue. Amongst the limitations, is that integrated studies handled various parameters, degree of measurement, lithologies, or the observed physical properties of the geophysical methods. Besides, absence of direct and accurate theoretical relationships among dissimilar geophysical parameters, for example, seismic compressional velocity and electrical resistivity (Haber & Oldenburg, 1997; Meju & Gallardo, 2003). However, the development of reliable and accurate empirical relations among various subsurface geophysical parameters continues to be a challenge (Hellman *et al.*, 2017; Carollo *et al.*, 2020). In the context of the constraints linked to the approaches for geophysical data integration and establishment of valid empirical models, the present study seeks to develop and assess soil strength and moisture content multiple linear regression models from the measured subsurface seismic refraction velocities and electrical resistivities.

## **1.2 Problem statements**

The use of single geophysical technique for geophysical prospecting and shallow investigations, encountered limitations especially when there is low variation between the host environment and the targeted material. For instance, electrical resistivity tomography technique is one of the modern advances in near surface geophysics that has been utilized in many geophysical prospecting applications

(Nthaba *et al.*, 2020; Gonzalez *et al.*, 2021; Szokoli *et al.*, 2017; Amini & Ramazi, 2016) . However, ERT could not distinguish boundaries of structures with low resistivity contrast (layers of shale and a clay) (Hellman *et al.*, 2017). Thus, the integration of different geophysical methods to mitigate the inherent limitations of single technique and offer a robust characterization/ investigation of shallow subsurface geomaterials (rocks/soils). In this light, many integrated methods have been developed and or employed to address the inherent ambiguities associated with a single method, such as joint inversion approaches (Haber & Oldenburg, 1997; Gallardo & Meju, 2016; Zhdanov & Lin, 2017) and statistical clustering approaches *Hellman et al.*, 2017; Carollo *et al.*, 2020; Du *et al.*, 2021). However, the studies could not account for specific shear strength parameters in terms of soil cohesion and friction angle, which are critical to robust and detailed site exploration or characterization

Geomaterials (rocks/soils) performance and reliable subsurface data/information have traditionally been obtained through field or site exploration methods such as drilling and laboratory procedures. Traditional methods, on the other hand, are limited by several constraints, including high cost, timeframe, data spatial coverage, and environmental stability. Geotechnical techniques based on laboratory tests are primarily used to determine soil parameters. Soil sampling using a drilling technique necessitates many drilling positions for detailed subsurface information/investigation or characterization, which increases the duration of time, costs, and is considered unsustainable to the environment. due to the invasive nature of the exploration procedure. The techniques are deficient in providing shear strength and moisture subsurface information in 2-dimensionsal forms, thus could not offer continuous descriptions of soils' strength properties at the subsurface. To address these

challenges, integration and regression of geotechnical and geophysical datasets are greatly required.

Three models were built in phases; in the first phase, simple linear models were achieved between shear strength and moisture parameters with only one geophysical resistivity parameter. The second phase had two geophysical parameters of resistivity and seismic refraction velocity with the shear strength and moisture parameters as the MLR models. Multiple regression constants generated by application of linear least squares approach can serve as a proxy avenue for fast calculation of soil cohesion, internal friction angle and moisture content from the measured electrical resistivity and seismic refraction velocity values in geophysical field surveys, which requires linear behaviour of the interplay/measured parameters. To eliminate the effects of the nonlinear behaviour of the complex subsurface velocity and resistivity distributions, log-transforming those datasets is more imperative than considering their raw values.

### **1.3 Objectives of the research**

This research aimed at characterizing soil shear strength model from two geophysical and a geotechnical method with the following specific objectives.

- i. To determine soil cohesion and friction angle models from post inversions of electrical resistivity and seismic refraction tomographic models using multiple linear regression (MLR) technique.
- ii. To correlate the subsurface at various sites from geotechnical and geophysical perspectives using the developed models and the geophysical methods.
- iii. To validate the reliability and efficacy of the developed models at different locations of comparable geological settings.

#### 1.4 Scope of the study

In this work, the statistical analysis based on the Multiple Linear Regression (MLR) principles were conducted. MLR soil cohesion, friction angle and moisture content estimation models utilizing electrical resistivity and seismic refraction tomography measurements have been established and validated. Wenner-Schlumberger (W-S) array was employed, as the array configuration that has potential to resolve vertical and horizontal subsurface resistivity changes (Bery & Ismail, 2018; Muhammad & Saad, 2018; Telford *et al.*, 1990). While in seismic refraction tomography, the fixed geometry approach was joined with its resistivity counterpart for good coverage of survey specification. The two geophysical parameters; true resistivity ( $\rho$ ) and seismic refraction velocity ( $V_p$ ) were used in deriving the new models which served as the independent variables, with soil's strength parameters; soil cohesion, friction angle and moisture content as the dependent variables. The three parameters (models) are obtained from the processing of the two measurable geophysical parameters at tomographic levels after inversion and therefore, offered comprehensive picture of the whole survey measurement. They can be viewed as outcomes of ERT and SRT and permit presentation in 2-dimensional pseudo sections. Soil samples was used for the direct shear laboratory geotechnical test. The research survey was conducted at Universiti Sains Malaysia (USM), Pulau Pinang, Malaysia. As reported by Muhammad (2017) that the study area consists of residual homogeneous soil, hence the suitability for the intention for which the research is hoped to accomplish. The choice of appropriate survey area was prudently made as a good data source for modelling and validation.

## **1.5 Significance and novelty of the study**

This research study attempts for the first time to establish multiple linear regression models for predicting soil strength parameters (soil cohesion and friction angle) and moisture content parameter from electrical resistivity and seismic refraction tomography measurements derived using both Werner-Schlumberger array and fixed geometry approach, respectively. The models generated can provide accurate computation of geotechnical attributes of soils, which are crucial and of high-priority for appropriate design and successful construction of any structure (Cosenza *et al.*, 2006; Siddiqui & Osman, 2013). Thus, gives an edge by providing more information from seismic parameter over models involving only subsurface resistivity values. Specific soil attributes (soil cohesion and friction angle) and moisture content parameters can be estimated directly from the proposed models likely leading over joint inversion of both resistivity and seismic velocity data.

A profound intuition on surveying methods is highly needed to both service providers and the experts using the outcomes of geophysical assessments for the formation of the conceptual geological and geotechnical models. The significance of geophysical methods for environmental and geotechnical characterization is associated with rebuilding of subsurface geometries (layering, inclusions, lateral variability) and determine directly or indirectly physical and mechanical properties of interest for the geotechnical model. Therefore, the proposed novel geotechnical models can be viewed as a significant contribution to rebuilding of subsurface geometries from resistivity and seismic velocity values for engineering and environmental applications.

The proposed MLR subsurface models involving resistivity and seismic data can be employed in areas with comparable geological settings for subsurface characterization provided the needed geoelectrical and seismic parameters are given.

The new estimated models are significant and suitable in both surface and subsurface (2D form) investigations, and minimal level of damages, in the calculation of subsurface parameters by geophysical measurements, compensating the drawbacks of drilling methods such as limited data coverage, high cost, destruction, laborious field and laboratory soil tests.

This approach can enhance the reliability and accuracy of geophysical result descriptions as against a single method interpretation with or without contribution of geotechnical data. Also, it generates unified MLR soil strength and moisture content models and present them in 2-dimensional forms, thus offer continuous descriptions of soils' strength properties at the subsurface, as against the traditional geotechnical methods, such as drilling and excavation operations, which only provide 1-dimensional subsurface information based on point measurements. It is also aspired to keep to minimum soil damage, environmental degradation, whenever large surveys are to be conducted, against large invasive drilling operations.

In conclusion, these models would provide robust approach to subsurface characterization and investigations, especially for complementary function from one single geophysical method. When compared with other MLR models involving laboratory test and resistivity data sets, the new models developed supply extra data that are associated with vital information lacking in the former and mitigate ambiguities of results/interpretation inherent in a single-method geophysical result. And most importantly is the generation of 2D geotechnical models of the subsurface

for engineering and environmental applications. The most challenging aspect of community geophysics is having access to instrumentation.

## **1.6 Thesis layout**

This thesis has been categorized into five different chapters; each chapter is explained as below to provide an overview about flow of the thesis.

Chapter one takes introductory chapter, in which the background of the study is clearly introduced. The subsequent item explained the problem statement as an outcome of the knowledge gap observed in the literature. To handle the problems, specific objectives are therefore defined in the next item. Finally, importance of the research is detailed, and prominence is given to the novel contributions of the research work to knowledge.

Chapter two discusses the concepts and fundamentals of both seismic refraction and electrical resistivity methods, as well as various electrode configurations. Also, the basic regression theory and its applicability are explained being instrumental to the study experimental procedure. buttresses review of relevant literature, particularly on joint interpretation of geotechnical and geophysical (ERT and SRT). methods. The chapter explained and presented summaries of previous research works targeted at tackling integrated resistivity/seismic refraction and geotechnical related problems. As such research gaps have been discovered, thereby forming the foundation of the study

Chapter three discusses the general principles of both resistivity and seismic refraction tomography methods. The outlines of data acquisitions and array configuration for the two methods are stated. A geotechnical direct shearing test method is also explained. The principles and applications of multiple linear regression equations are then elaborated. The methodology developed to realize the stated

objectives of the research is spelt out and discussed. Prominent is also tied to data processing, modelling and accuracy assessment approaches. Lastly, geological settings of the study area are also explained.

Chapter four presents' outcomes of the research findings. It illustrates the performance and accomplishment of data conversion from geophysical parameters into geotechnical parameters. It displays the new soil's strength parameters and moisture content models developed and explains their efficacy to predicting the geotechnical parameters. Application of the new models at various sites has also been carried out for good performance and accuracy analyses.

Chapter five concludes the main findings of the research and presents recommendations for future works.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

In this chapter, concepts and fundamentals of both seismic refraction and electrical resistivity methods, as well as various electrode configurations, are firstly discussed to put forward the comprehension of the theories behind the research. Also, the basic regression theory and its applicability are explained being instrumental to the study experimental procedure. Literature review of the related present and previous geotechnical, geophysical, and integrated (geophysical and geotechnical) research works essential to soil properties are presented. This chapter contains five subsections in the order of; introduction, electrical resistivity tomography method, seismic refraction method, regression method, previous related research works and concluded with chapter summary. Description of the general content of the chapter is presented in the first section followed by detailed presentation of the utilized methods, explanations of previous research works related to subsurface soil characterization based on the employed techniques as geotechnical, geophysical, and integrated approaches. Last section of the chapter provided a summary of previous studies reported, hence lead to identification of study gaps.

#### **2.2 Electrical resistivity tomography (ERT)**

The electrical resistivity method is among the oldest and increasingly used geophysical surveying methods (Dahlin & Loke, 2018; Loke *et al.*, 2013; Reynolds, 2011). Considering that the electrical resistivity parameter has wide range of values among the subsurface geological materials (Figure 2.1), the technique could be more susceptible to subsurface changes as compared to other surveying techniques. For this reason and others for instance its simple concept, low cost and automated data

measurement systems, the application of ERT technique has continued to grow in shallow subsurface investigations including but not limited to averagely complex terrains (Falae *et al.*, 2019; Loke *et al.*, 2018), groundwater exploration (Dahlin & Loke, 2018; Yen *et al.*, 2019), mineralization potentials (Gupta *et al.*, 2019), monitoring of embankment (Hojat *et al.*, 2020), archeological studies (Nasha *et al.*, 2020; Luca *et al.*, 2020). Recently, ERT method has also proven to be highly applicable in addressing aspects of environmental, engineering and environmental challenges (Lech *et al.*, 2020; Zhou & Che, 2020; Loke *et al.*, 2018). In ERT method, artificial electric current is driven into subsurface between two current electrodes placed on the ground surface and the resultant potential difference is measured through other electrodes called potential electrodes. The resultant potential difference is subsequently processed into electrical sounding graphs or curves of apparent resistivity values, indicating resistivity contrasts at the subsurface for various geological bodies (Zhou, 2018). Analyzing such data, unearth the subsurface resistivity anomalous bodies or any geologic structure.

Accurate numerical modeling of underground resistivity field and acquisition of vast quantities of data have been made possible due to the growth in computing and numerical computational approaches. Thus, the transformation of ordinary direct current resistivity method into electrical resistivity tomography method, in which large scale surveys are normally presented in 2 or 3 dimensions using automated electrode selector system and inversion schemes to reconstruct underground resistivity structure with the measured data (Loke *et al.*, 2013). The observed data are related with varied depths which are interpreted as lithologic and hydrological models of the subsurface. Besides the reconstruction of subsurface models from the observed resistivity data, the non-invasive ERT continues to feature in obtaining more accurate data at low

surveying rate (Lech *et al.*, 2020; Loke *et al.*, 2013). Thus, the deployment of ERT technique in this research.

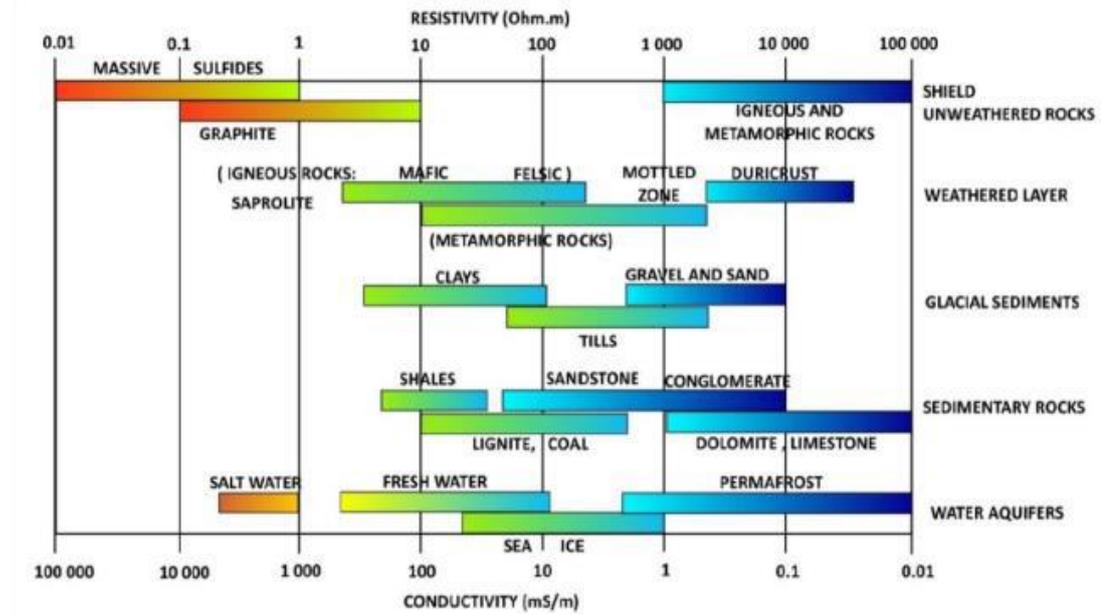


Figure 2.1 Estimated resistivity values of subsurface structures (Palacky, 1987).

### 2.2.1 Theoretical formulations of electrical resistivity method

The basic physics law that governs the electrical resistivity method is the Ohm's Law which asserts that the electrical resistance,  $R$  of a given conductor is expressed as in Equation 2.1;

$$R = \frac{V}{I} \quad (2.1)$$

Where,

$V$  = potential difference across the cylindrical conductor in volts (V)

$I$  = electric current measured in ampere (A)

For simple body case, resistivity,  $\rho$  measured in ohm meter ( $\Omega.m$ ) can be mathematically expressed as in Equation 2.2;

$$\rho = R \left( \frac{A}{L} \right) \quad (2.2)$$

$A$  = cross-sectional area of the cylindrical conductor measured in meter-square ( $\text{m}^2$ ).

$L$  = length of the conductor measured in meter (m).

Electrical property is also usually explained by conductivity parameter  $\sigma$  ( $\text{Sm}^{-1}$ ) equivalent to the reciprocal of resistivity of geomaterial. Hence,  $\sigma$  is expressed as in Equation 2.3;

$$\sigma = \frac{1}{\rho} \quad (2.3)$$

In other words, Ohm's Law explains the mathematical relationship between current density,  $J$  (in  $\text{Am}^{-2}$ ) and electric field strength,  $E$  (in  $\text{Vm}$ ) with electrical conductivity of the medium,  $\sigma$  as the proportionality constant, this relation is stated in Equation 2.4;

$$J = \sigma E \quad (2.4)$$

The electric field vector,  $E$  in Equation 2.4 is expressed as the gradient of the electric potential function,  $\nabla V$ ;

$$E = -\nabla V \quad (2.5)$$

Equation 2.4 of current density, becomes Equation 2.6 after substitution of Equation 2.5, as Equation 2.6;

$$J = -\sigma \nabla V \quad (2.6)$$

Adopting divergence condition, that is charge free zone (situation where no charge sources or sinks in medium, for instance the earth),  $\nabla \cdot J = 0$ , as expressed in Equation 2.7;

$$\nabla \cdot J = -\nabla \cdot (\sigma \nabla V) = 0 \quad (2.7)$$

Multiplying a vector operator,  $\nabla$  to the product of  $\sigma \nabla V$ ,  $\sigma$  being a scalar function as Equation 2.8;

$$\nabla \sigma \cdot \nabla V + \sigma \nabla^2 V = 0 \quad (2.8)$$

Equation 2.8 is called Poisson's relation, describing the electrical flow in a heterogeneous medium (in this case, the earth).

Consider the case of an isotropic homogeneous earth having a constant conductivity,  $\sigma$ . Thus, Equation 2.8 narrows to Equation 2.9;

$$\nabla^2 V = 0 \quad (2.9)$$

This is a Laplace equation (Equation 2.9) valid only for homogeneous medium, which  $\nabla^2$  is a second derivative operator on  $V$ , the scalar potential.

Assume a situation in which a single point source of current (current electrode) placed in an isotropic homogeneous medium and the other current electrode required to obtain a complete circuit was at a distant (infinity) so that its influence is not significant (Figure 2.3). With this condition, the electric current moves radially outward from the source producing it (current electrode), and perpendicular to the equipotential surface.

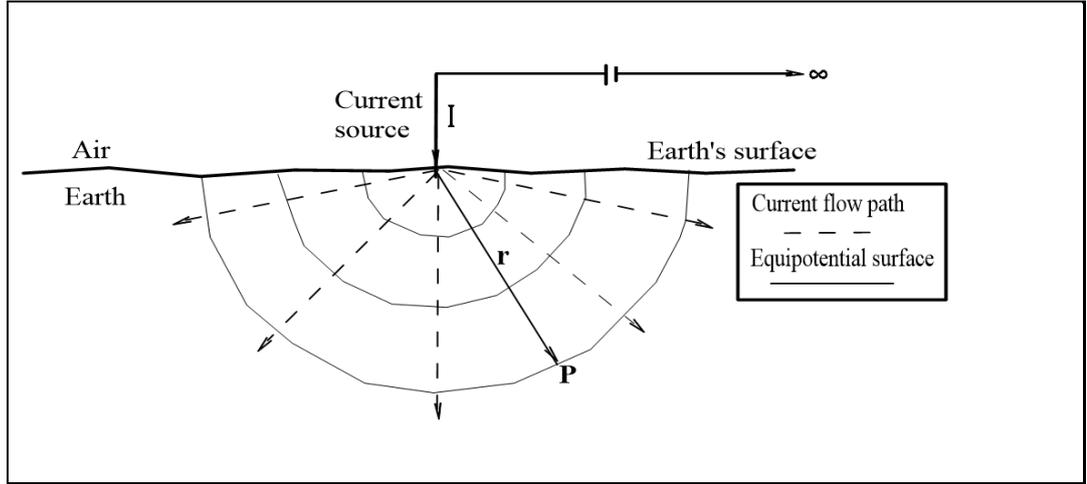


Figure 2.2 Current flow for a single-point current electrode source with equipotential surfaces and direction of flow (Modified from (Kearey *et al.*, 2002).

The potential is measured at every point (**P**) with respect to the distance (**r**) from the current source electrodes. In the context of the spherical symmetry in relation to the Earth's homogeneous subsurface, the scalar potential is only obtainable as a function of length of the source current electrode. Consequently, Equation 2.9 is expressible as Equation 2.16;

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin^2 \varphi} \left( \frac{\partial^2 V}{\partial \theta^2} \right) + \frac{1}{r^2 \sin \varphi} \frac{\partial}{\partial \theta} \left( \sin \varphi \frac{\partial V}{\partial \varphi} \right) = 0 \quad (2.16)$$

Consider a point source of current, due to the complete symmetry of the current propagation with reference to  $\theta$  and  $\varphi$  directions. Hence, the derivatives in relation to these angles are zeros, so that Equation 2.16 reduces to Equation 2.17;

$$\nabla^2 V = \frac{d}{dr} \left( r^2 \frac{dV}{dr} \right) = 0 \quad (2.17)$$

By integrating Equation 2.17 to arrive at Equation 2.18;

$$r^2 \frac{dV}{dr} = C \quad (2.18)$$

Further integration of Equation 2.18 produces Equation 2.19;

$$V = \int \frac{dV}{dr} dr = \int \frac{C}{r^2} dr = -\frac{C}{r} + D \quad (2.19)$$

After integration and further simplifications, Equation 2.20 was obtained as the apparent resistivity formular;

$$\rho = \frac{2\pi rV}{I} \quad (2.20)$$

For practical purposes, any geophysical resistivity field survey demands the use of at least more than one current electrode with a definite distance. In the case of two source point electrodes (Figure 2.4) placed on the Earth's surface with an electrical resistivity of  $\rho$ . The two-point source current electrodes, with one of the electrodes as a source electrode which transmits electric current into the ground, a flux of electric field lines in outward direction while the other, a sink electrode that receives the transmitted current, as a flux of electric field lines inwardly (Lowrie, 2007). Semi-spherical equipotential surfaces are generated around the two current electrodes, both of which affect the potential at every single close surface point.

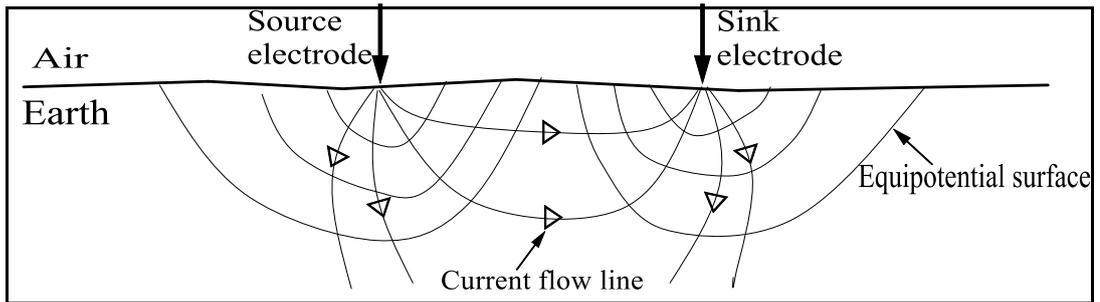


Figure 2.3 Current flow lines and equipotential potential surfaces generated by two-point source electrodes (adopted from Lowrie, 2007).

Fundamentally, in every single geophysical resistivity survey, measurement of potential difference between two given electrode points is performed (Figure 2.5). The simple electrode arrangement comprises of a pair of current electrodes, designated as  $C_1$  and  $C_2$ , and another pair of potential electrodes, represented as  $P_1$  and  $P_2$ , used to record the potential difference. Since the current at the source electrode is positive ( $+I$ ) and at the sink electrode is negative ( $-I$ ).

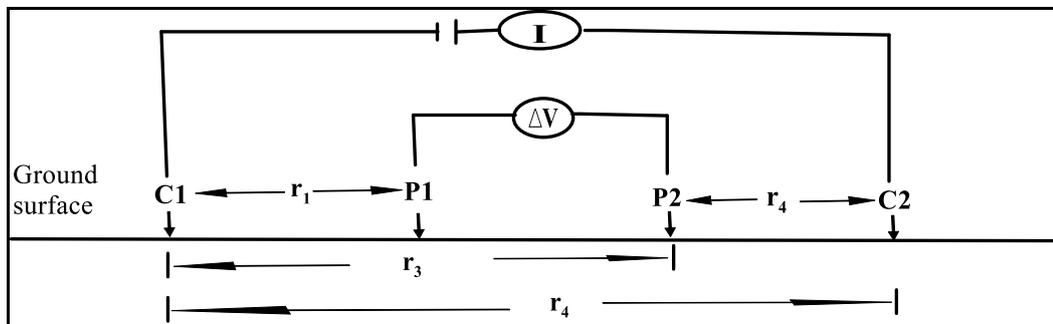


Figure 2.4 The conventional set up of geophysical resistivity measurement (adopted from (Reynolds, 2011).

Since the current at the source electrode is positive ( $+I$ ) and at the sink electrode is negative ( $-I$ ), the resultant potential  $V_{P_1}$  at  $P_1$  due to  $C_1$  ( $+I$ ) and  $C_2$  ( $-I$ ) is expressible as Equation 2.27;

$$V_{P_1} = V_1 + V_2 = \frac{I\rho}{2\pi r_1} + \frac{-I\rho}{2\pi r_2} \quad (2.27)$$

Or

$$V_{P_1} = \frac{I\rho}{2\pi} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \quad (2.28)$$

Equally, the resultant potential  $V_{P_2}$  at  $P_2$  due to  $C_1 (+I)$  and  $C_2 (-I)$  is as Equation 2.29;

$$V_{P_2} = \frac{I\rho}{2\pi} \left( \frac{1}{r_3} - \frac{1}{r_4} \right) \quad (2.29)$$

Against this background, the total potential difference ( $\Delta V$ ) between the two potential electrodes ( $P_1$  and  $P_2$ ) is measurable as expressed in Equation 2.30;

$$\Delta V = \frac{I\rho}{2\pi} \left[ \left( \frac{1}{r_1} - \frac{1}{r_2} \right) - \left( \frac{1}{r_3} - \frac{1}{r_4} \right) \right] \quad (2.30)$$

For measurement of electrical resistivity in geophysical survey, the needed parameters include the supplied current ( $I$ ), the electrode separation distances  $r_1$ ,  $r_2$ ,  $r_3$ ,  $r_4$  and the measured potential difference ( $\Delta V$ ). Thus, the resistivity or apparent resistivity ( $\rho_a$ ) can be determined as in Equation 2.31, considering the heterogeneity of the Earth's subsurface.

$$\rho_a = \frac{2\pi\Delta V}{I} \frac{1}{\left( \frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4} \right)} \quad (2.31)$$

Where,